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ALTERNATING-CURRENT ELECTRICITY

AND

ITS APPLICATIONS TO INDUSTRY

SECOND COURSE

BY

W. H. TIMBIE

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FIRST EDITION
FIRST THOUSAND

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PREFACE

In presenting a foreword to this volume, the authors have little to add to the statements made in the preface of the First Course. They have tried faithfully to fulfill in content and in spirit the plan there outlined.

Of the First Course, the chapters on Modern Systems of Power Transmission, Single-phase Alternators, Polyphase Alternators, The Generation of Proper E.M.F. Wave-form, and Armature Windings were written by Mr. Higbie. The chapters on Fundamental Ideas, Series and Parallel Circuits, Power and Power-Factor, Inductive Reactance, and Simple Trigonometric Functions were written by Mr. Timbie. The Introduction to Chapter II was kindly contributed by Mr. A. L. Williston.

Of the Second Course, the chapters on The Regulation and Control of Alternators, Parallel Operation of Alternators, Fundamental Principles of the Transformer, Operation and Polyphase Connections of Transformers, and Asynchronous Motors were written by Mr. Higbie. The chapters on Short Transmission and Distribution Lines, Long Transmission Lines and Capacitance, Synchronous Motors, Converters and Rectifiers were written by Mr. Timbie. The authors, however, have coöperated in the criticism, correction and elaboration of all parts of the text.

They again wish to express their appreciation to Mr. J. M. Jameson for his efficient editing and to Mr. Bates for his valuable suggestions on the text and for the solution of the problems and the checking of the data and the examples.

W. H. T.
H. H. H.

Boston, Mass.,
Ann Arbor, Mich.,
January, 1916.

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Alternating-current Electricity
and
Its Applications to Industry

SECOND COURSE

CHAPTER I

PERFORMANCE OF ALTERNATING-CURRENT GENERATORS. REGULATION AND CONTROL.

The armature windings of alternators have been taken up in detail in the "First Course" as a practical means of studying the current and voltage relations in polyphase circuits. It is now necessary to note the performance of the alternator as a whole when in service.

The purchaser and the operator of an alternating-current generator are concerned about the following features:

First. How does the generator behave when it delivers power?

Second. How much power can a given generator deliver without endangering its own safety; or, how large a generator is required to carry a given load?

Third. How does the generator interact with other parts of the system to which it must be connected?

These actions or properties are more or less interdependent, and it is impossible to discuss any one of them without involving all. The discussions in the two following chapters are prepared for the man who is responsible for the maintenance of good service in the electric plant or system and for the man who pays the bills and seeks the profit, rather than for the designer.

1. Effects of Loading an Alternator. As we increase the current taken from an alternator, by increasing the load which is connected across its terminals, we notice the following effects:

First. The armature windings heat up, and there is in consequence a general temperature rise all over the machine.

Second. The efficiency changes, at first rising as the load increases, but falling off again for very large loads.

Third. The terminal voltage tends to fall; to keep it constant requires adjustment, either manually by an operator, or automatically by a "voltage regulator."

Fourth. The form of the e.m.f. wave may change.

These effects have been stated in the usual order of their importance. All of them are troublesome or undesirable in one way or another. If any one of them were to proceed far enough it might set a limit to the amount of power that could be taken from the alternator, although usually the heating limit is reached first.

2. Heating of the Alternator. All of the losses in an electrical machine appear as heat, in one part or another. These losses are:

First. I^2R in the armature winding.

Second. I^2R in the field winding.

Third. Hysteresis and eddy-current losses ("iron losses") in the armature core.

Fourth. Friction, of the shaft in the bearings, of brushes on collecting-rings, and of all moving parts against air.

Fifth. Eddy-current loss in the faces of the poles.

When heat energy is developed in or applied to any body, the temperature of the body is raised.* The body tends to lose heat energy to its surroundings in proportion to the excess of its own temperature above the temperature of the surroundings. The temperature of the body will, therefore, rise until the rate of losing heat is equal to the rate of developing or supplying heat in the body. When this state of equilibrium is attained, the temperature ceases to rise and is steady. It is as if we were pumping water into a tank with a hole in its base; the level will rise until the pressure or "head" due to it forces water out through the hole as fast as it is being pumped in. Then the level becomes stationary. The head, or pressure, is a measure of the outflow of water and corresponds to the excess of temperature of a body above its surroundings.

---

* This statement does not apply to bodies which are in process of changing from solid to liquid, or liquid to gas, or vice versa.

If we keep the size of outlet from the tank fixed, and increase the rate of pumping, the head increases. If we keep the rate of pumping fixed and reduce the outlet, the head also increases. Similarly, if we increase the rate of heat development in a machine by operating it in a manner to increase any of its losses, leaving the surroundings unchanged, the temperature rises; and conversely, if the losses be decreased, the temperature falls. Also, if we do anything which facilitates the escape of heat, the temperature corresponding to any given rate of loss will be lowered, and if we retard the escape of heat, the temperature will be raised.

Temperature has a very profound and serious effect upon most materials which are suitable to be used as insulators in an electrical machine. Such materials are paper or sheet fiber, and fabrics of cotton, linen or silk. These are impregnated with shellacs or varnishes made of natural-oil resins or gums, such as are used in insulating paints and compounds. At comparatively low temperatures the resins and gums become partial conductors and allow leakage currents to flow within the machine from one conductor to another. This develops more heat, which very quickly ruins the windings and makes a short-circuit. The papers and fabrics at high temperatures quickly become charred and crumble, allowing the conductors to come in contact with one another and with the frame of the machine. The highest permissible temperature to which ordinary insulations may be subjected continuously is about 90° to 100° C. Below this temperature, the insulation is not permanent, but deteriorates very slowly, breaking down in perhaps twenty to fifty years. Above this temperature, the rate of deterioration increases very rapidly; on short-circuit, it might be completely ruined in a few seconds.

The time required by the generator to reach this maximum allowable temperature depends upon many conditions. If the machine is heavy and contains a great quantity of material, its heat-storage capacity is large, and it may take 12 hours continuous operation at full load to reach a steady temperature. If it is a small high-speed machine for the same capacity and having approximately equal losses, it may reach a steady temperature in 6 hours. This time depends also upon the ratio between power losses and radiating surface.

In most modern machines a great increase of output is obtained from a given size and weight by using fans or vanes to force air currents through "ventilating ducts" in the machine. The amount of losses or heat that can be carried away from each square inch of surface without exceeding the highest permissible temperature, is thus increased enormously. If anything should happen to restrict the quantity of air blown through this machine, such as a stoppage of the fans, the temperature would rise very rapidly, probably beyond the allowable limit in a few minutes. In some locations, dirt in the atmosphere sticks to the ventilating passages and contracts them so that the machine must be dismantled and cleaned periodically, notwithstanding the facts that the air velocities in the passages are cyclonic and that the air is often cleansed before being taken to the machine.

Variations of the output of a machine also have an important effect upon both the rate and ultimate amount of its temperature rise. The average power output depends directly upon the average value of the effective current; but the average rate of heating depends upon the average square of the effective current (I^2R). If the power or amperes output varies, we shall find that the average value of (I^2R) is greater than the value of (Average $I)^2 \times R$. This means, that **if the temperature of a generator or other machine depends upon the average rate of heating, or upon the average losses, the temperature will be higher on a fluctuating load than on an equivalent steady load.**

However, if the variations of load are very slow, and particularly if the high loads are persistent, the danger of overheating may not be determined by the **average watts** lost, but by the watts lost at the **high loads**. Thus, it is frequently specified or guaranteed that a generator shall be able to carry rated full-load continuously, 25 per cent overload for two hours, 50 per cent overload for one minute, and 100 per cent overload momentarily, all without danger to its insulation. Evidently the temperature rises at a much greater rate than the load.

The operating temperature of the generator depends also upon the temperature of its surroundings and upon the temperature of the air supplied to it for ventilation. The **load** determines the power **losses**, and these determine the **temperature difference** between the machine and its surroundings. But the **danger to the insulation** depends upon the actual **maximum temperature** of the insulation. If the operating room in general, and the ventilating air in particular, has a high temperature, this will reduce the permissible margin of the temperature rise, which limits the rate of heat loss, and therefore the load. If the air temperature is low, the generator may carry a larger load with safety. Sometimes, particularly in small plants, the capacity of the generators is reduced, or their depreciation at rated full load is increased, by locating them in a hot or poorly-ventilated room. Heating is a condition which aggravates itself, because a rise in temperature of the windings increases their resistance, so that the same current produces a greater I^2R loss, and this tends to increase the temperature still further.

This matter of heating is too important to be left to the varying judgment of individuals, and therefore definite rules as to the amount of temperature rise permissible, and the conditions under which it should be measured, have been agreed upon by electrical engineers and manufacturers of electrical machinery. These are to be found among the "Standardization Rules" of the American Institute of Electrical Engineers, which specify the minimum requirements of good practice in the operating and testing of electrical machinery. The rules cover in detail many points other than the heating and rating of machines, and are too extensive to be reviewed here. They may be found in any electrical handbook, and should be referred to whenever the rating or performance of any electrical machine is in question.

Example 1. The Standardization Rules state that the temperature of the field and armature should not be permitted to rise more than 55° C. above the room temperature. The armature temperature of a certain generator rises 70° C. when delivering a steady load of 200 kv-a. What is the greatest load in kilovolt-amperes which this generator should be permitted to carry continuously at this voltage?

Note. (*a*) The rate of heat loss is approximately proportional to the difference between machine temperature and room temperature.

(*b*) It is safe to assume that half the heat loss with a 55° rise in temperature is due to hysteresis and eddy currents in the machine and is constant at all loads. The other half is due to the I^2R loss in the copper and is thus proportional to the square of the current.

Applying (*a*),

$$\frac{\text{loss with } 55^\circ \text{ rise}}{\text{loss with } 70^\circ \text{ rise}} = \frac{55}{70} \text{ or } \frac{11}{14}.$$

Applying (*b*),

let

$$I = \text{current with } 55^\circ \text{ rise},$$
$$I_1 = \text{current with } 70^\circ \text{ rise}.$$
$$\text{Loss with } 55^\circ \text{ rise} = \text{iron loss} + I^2r \text{ loss}.$$

But

$$\text{iron loss} = I^2r \text{ loss}.$$

Thus

$$\begin{aligned} \text{loss with } 55^\circ \text{ rise} &= 2\ I^2r \text{ loss}. \\ \text{Loss with } 70^\circ \text{ rise} &= \text{iron loss} + I_1^2r \text{ loss} \\ &= I^2r \text{ loss} + I_1^2r \text{ loss}. \end{aligned}$$

Thus

$$\frac{\text{loss with } 55^\circ \text{ rise}}{\text{loss with } 70^\circ \text{ rise}} = \frac{2\ (I^2r \text{ loss})}{(I^2r \text{ loss}) + (I_1^2r \text{ loss})} = \frac{11}{14},$$

or

$$28\ (I^2r \text{ loss}) = 11\ (I^2r \text{ loss}) + 11\ (I_1^2r \text{ loss})$$
$$17\ (I^2r \text{ loss}) = 11\ (I_1^2 \text{ loss}).$$

Thus

$$\frac{I^2r \text{ loss}}{I_1^2r \text{ loss}} = \frac{11}{17}.$$

That is, the heat loss due to the current in the armature windings must be reduced so that it is only $\frac{11}{17}$ of its present value, if the temperature is not to rise more than 55°.

This heat loss is proportional to the square of the current, thus

$$\frac{I^2r \text{ loss}}{I_1^2r \text{ loss}} = \frac{I^2}{I_1^2} = \frac{11}{17}.$$

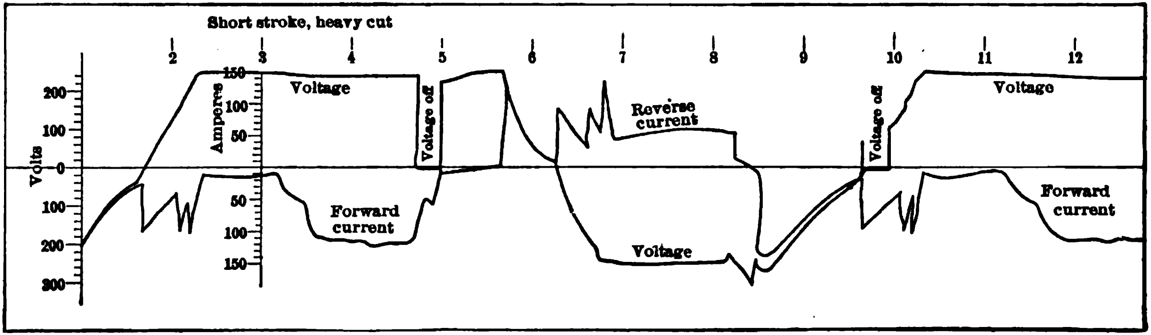

FIG. 1. Oscillograph record of cycle of operation of a motor driven planer. This is a copy of part of a long film covering several cycles of operations. *Electric Journal*, May, 1912.

But the kilovolt-amperes delivered by the generator at a given voltage is proportional to the current in armature windings. Thus

$$\frac{\text{kv-a. (with 55° rise)}}{\text{200 (with 70° rise)}} = \frac{I}{I_1} = \sqrt{\frac{11}{17}} = \frac{8}{10}.$$

Therefore the output of the generator which will cause a 55° rise only is $\frac{8}{10}$ of the output which causes a 70° rise, or $\frac{8}{10}$ of 200 = 160 kv-a.

The generator could thus be run as a 160-kv-a. machine and not overheat.

Prob. 1-1. When operated continuously at the full-load rating on the name plate, the armature of a certain generator attains a constant temperature of 40° C. above the room temperature. On the basis of the assumptions of Example 1, calculate:

(*a*) By what percentage the total heat losses in the armature may safely exceed those at rated full load.

(*b*) By what percentage the current output may safely exceed the rated full-load current?

Prob. 2-1. Fig. 1 shows the variations of current input to a motor driving an automatically-controlled planer, during a little more than one cycle of operation. Calculate:

(*a*) The numerical average value of the current, regardless of direction.

(*b*) The effective value of the current.

(*c*) The ratio between the I^2R loss in the motor during one cycle due to the actual current as shown by Fig. 365, and the I^2R loss that would occur during the same time if the current were steady at the numerical average value.

Prob. 3-1. Fig. 2 shows approximately the variation of current supplied to an 800-horse-power 2200-volt 225-r.p.m. three-phase induction motor, during one complete cycle of operation of a water-hoist to which it is geared, in a mine. Calculate:

(*a*) Average value of I^2.

(*b*) Square of average value of I.

(*c*) Percentage by which actual watts I^2R loss during one cycle of operation is greater or less than the I^2R loss would be if the motor were to take an equivalent average current.

Prob. 4-1. The rotating-field coils of a certain alternator rise 45° C. above the room temperature of 25° C., when carrying such current that full rated voltage is obtained from armature terminals at zero load, rated frequency. The field current of this alternator must be increased 15 per cent to deliver the same (rated) voltage across armature terminals at full load, same frequency. Under ordinary conditions, the excess of machine temperature above room-temperature is directly proportional to the heat losses in the machine. What will be the temperature rise, and the actual temperature, of the field coils when the alternator operates steadily at full load? (For an approximate solution, the increase in resistance of the field coils due to temperature may be neglected.)

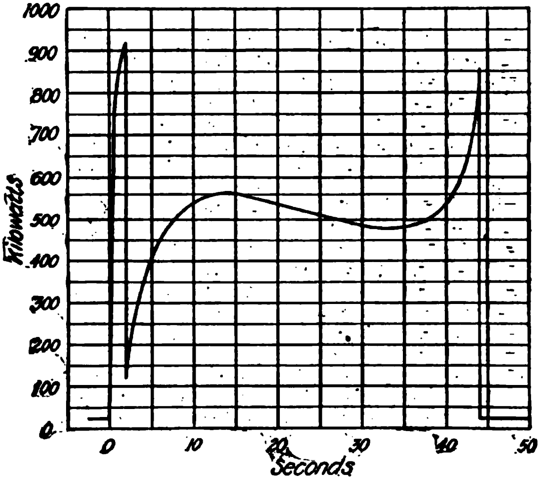

FIG. 2. Curve of current taken by an 800-h.p., 2200-volt, 225-r.p.m. three-phase induction motor, geared direct to a mine water-hoist, having a capacity of 250,000 gal. per hr. The curve shows one hoisting cycle. *General Electric Co.*

3. Advantages of High Efficiency. Efficiency expressed exactly, as a number, is the ratio between the output of and the input to a given machine or process. Thus:

Instantaneous efficiency =

$$\frac{\text{real power output in watts, kw. or h.p.}}{\text{real power input, in same units at same instant}}.$$

All-day efficiency =

$$\frac{\text{energy output during one day in kw.-hours or h.p-hours}}{\text{energy input during same day, in same units.}}.$$

Efficiency may be calculated in any one of several ways, depending upon the form in which it is convenient or necessary to obtain the data. Thus:

Efficiency =

$$\frac{\text{output}}{\text{input}} = \frac{\text{output}}{\text{output} + \text{total losses}} = \frac{\text{input} - \text{total losses}}{\text{input}}.$$

It is not within the scope of this book to explain methods for measuring the quantities which must be known in order to calculate the efficiency. But we are concerned in finding what conditions promote the highest efficiency of the alternator, and what it costs us to permit a lower efficiency than might be. A higher efficiency means:

1. Less cost of input to produce a given output, or more of valuable output from a given expenditure for input.
2. Larger and more costly engines required to deliver a given amount of electric power. More fixed charges and more space required.

Prob. 5-1. When the total losses of a machine are 10 per cent of its **output**, what is its efficiency?

Prob. 6-1. When the total losses of a machine are 10 per cent of its **input**, what is its efficiency?

Prob. 7-1. An alternator is delivering 250 kv-a. at 85 per cent power-factor, and its efficiency at this load is 92 per cent. How many horse-power must the direct-connected engine deliver at the shaft, to drive this alternator?

4. Relation of Efficiency to Load on Alternator. The efficiency is lowered by any condition which increases the ratio of losses to output. The **losses** in an alternator are listed in Art. 2. The **power output** is directly proportional to the **amperes** output from the terminals, to the **volts** between terminals, and to the **power-factor** of the receiving circuit, or the phase difference between the current and pressure at the terminals.

Suppose we increase the amperes output while the terminal volts and power-factor remain constant. The watts output will increase in direct proportion to the amperes output, under these conditions. But the I^2R loss in the armature will increase in proportion to the **square** of the current. No other loss is affected appreciably by the change, except the

I^2R loss in the field. The voltage tends to fall when the armature current increases, and to keep the terminal voltage constant requires an increase in field current. This increases the field I^2R loss, though not nearly in proportion to the armature I^2R loss. Whether the ratio of the **total losses** to the **output** will be increased or diminished depends upon whether the losses which vary with the load are greater or less than the losses which do not vary with the load. If the fixed losses are greater than the variable losses, an **increase of output** will **raise the efficiency.** But as soon as the variable losses become greater than the fixed losses, a **further increase of output will lower the efficiency.** For illustration, Table I gives results of a test for losses and efficiency on a fairly large alternator; it is taken from an article on "Shop Testing of Alternating-Current Generators and Synchronous Motors" in "The Electric Journal," December, 1913.

TABLE I

Load, per cent of rated	0.0	25.0	50.0	75.0	100.0	125.0
Line amperes per terminal	0.0	49.0	98.0	147.0	196.0	245.0
Field amperes	77.0	81.5	86.5	94.0	101.5	112.0
Terminal volts	6300.0	6300.0	6300.0	6300.0	6300.0	6300.0
Core loss, in kw	39.4	39.5	39.6	39.7	39.8	39.9
Field copper loss, kw	4.68	5.24	5.92	6.98	8.13	9.92
Armature copper loss, kw	0.0	0.42	1.68	3.78	6.72	10.51
Friction and windage, kw	30.25	30.25	30.25	30.25	30.25	30.25
Total losses, kw	74.33	75.41	77.45	80.71	84.90	90.58
Kv-a. output	0.0	534.0	1071.0	1604.0	2140.0	2672.0
Real kw. output	0.0	427.0	857.0	1283.0	1712.0	2137.0
Real kw. input	74.33	502.4	934.4	1363.7	1796.9	2227.6
Efficiency, per cent	0.0	85.0	91.7	94.1	95.3	95.9

At all loads covered by Table I, the sum of fixed losses (consisting of friction, windage and core loss) is larger than the sum of variable losses (field copper loss and armature copper loss), and the efficiency is continually increasing as the output increases. It is apparent, however, that the efficiency is approaching a maximum value, as the variable losses become more nearly equal to the constant losses.

By drawing from the above data a curve with field copper loss and per cent load as coördinates, and extending this curve as far as 260 per cent of rated load, we may arrive at the following approximate values:

At 200 per cent load, output = 3424 kw.; core loss = 40.2 kw.; friction and windage = 30.3 kw.; field copper loss = 20.7 kw.; armature copper loss = 26.9 kw.

Efficiency = 96.6 per cent; "constant" losses = 70.5 kw.; "variable" losses = 47.6 kw.

At 220 per cent load, output = 3770 kw.; core loss = 40.3 kw.; friction and windage = 30.3 kw.; field copper loss = 26.1 kw.; armature copper loss = 32.5 kw.

Efficiency = 96.6 per cent; "constant" losses = 70.6 kw.; "variable" losses = 58.6 kw.

At 240 per cent load, output = 4110 kw.; core loss = 40.4 kw.; friction and windage = 30.3 kw.; field copper loss = 39.3 kw.; armature copper loss = 38.7 kw.

Efficiency = 96.5 per cent; "constant" losses = 70.7 kw.; "variable" losses = 78.0 kw.

At 260 per cent load, output = 4450 kw.; core loss = 40.5 kw.; friction and windage = 30.3 kw.; field copper loss = 59.6 kw.; armature copper loss = 45.4 kw.

Efficiency = 96.1 per cent; "constant" losses = 70.8 kw.; "variable" losses = 105 kw.

These results illustrate several important facts:

1. The efficiency reaches its maximum value at about that load for which the sum of the losses that vary with load is equal to the sum of losses that do not vary with load. This relation is in general only approximate, but it may be proved by mathematics to hold exactly for any case where the total variable loss is proportional to the square of the output. This is true of the armature copper loss but not of the field copper loss. Neither is the field copper loss constant. If the variation of field current, required to keep the terminal voltage constant, were not great, the field copper loss would be grouped with the constant losses

in applying this general relation for maximum efficiency.

2. The core losses are very nearly constant. They consist of hysteresis and eddy-current losses in armature core and pole faces, and their amount depends upon the frequency (speed) and flux density in the cores. The frequency and terminal voltage are assumed to be held constant. As the load or armature current increases, the e.m.f. induced in the armature windings must increase slightly to overcome the voltage drops within these windings and keep the terminal voltage constant. Consequently, the flux (and therefore the flux density) must be increased slightly as the output increases, and the core loss is therefore greater. A part of the increase of field current is for the purpose of producing this increase of flux and induced e.m.f.
3. The friction and windage losses are constant. These depend principally upon the speed, which is constant.
4. The armature copper loss increases as the square of the armature current, or as the square of the kv-a. output if the voltage is constant, or as the square of the kilowatt output if the voltage and power-factor are both constant.
5. The field copper loss increases with the output. In this particular case it increases quite rapidly; in other cases it may be almost constant and independent of the load. To keep the terminal voltage constant, as is usually required, some increase of field current is necessary for reasons cited in (2). If the iron is already nearly saturated with flux, a large increase in field current is necessary to produce a small increase in flux. Alternators are usually designed so that their cores are not nearly saturated at rated full load; but if they are operated at great overloads or much above rated voltage, saturation may be approached.

The load on this particular alternator has a power-factor of 0.80, and therefore the armature current has a considerable reactive component. It will be shown presently that current lagging 90° behind the induced e.m.f. in an alternator exercises a magnetic effect ("armature reaction") which directly opposes that of the field winding and tends to reduce the flux. To keep the voltage constant, therefore, a further increase of field current may be necessary, to neutralize the demagnetizing effect of any lagging wattless component of armature current that exists when the power-factor of the load is less than unity.

The rate at which the field current must increase with the load depends, therefore, very much upon the power-factor of the receiving circuit. It depends also upon the construction of the machine. If the number of turns in the armature coils is large, a comparatively weak field winding will produce enough flux to generate rated voltage; but a given amount of reactive current through this large number of armature turns produces an excessive demagnetizing effect upon the weak field. In such case, a large increase of field current is necessary to compensate the demagnetizing effect of the large number of armature turns. This means a large increase of field copper loss with increase of output, which affects the efficiency as already explained.

Prob. 8–1. Is the alternator whose performance is given in Table I a three-phase or a two-phase machine? Prove your answer.

Prob. 9–1. Assuming the alternator of Table I to be three-phase, star-connected, calculate the effective resistance per phase of the winding. Repeat the calculation, assuming it to be delta-connected.

Prob. 10–1. From Table I draw a set of curves, using per cent of rated output as abscissas, and as ordinates the following:

(*a*) friction and windage loss;

(*b*) friction and windage loss plus core loss;

(*c*) friction and windage loss, plus core loss, plus field copper loss;

(*d*) total losses;

(*e*) efficiency. The ordinates of the loss curves are to be plotted to a scale of kilowatts, and of the efficiency curves to a scale of percentage. Explain how these curves illustrate the points discussed in the preceding article.

Prob. 11–1. (*a*) If the alternator of Table I were to operate at full rated load and 100 per cent power-factor, what losses would be changed, and how?

(*b*) What would be the approximate value of efficiency under this condition?

(*c*) What would be the efficiency when delivering full rated load at 50 per cent power-factor, neglecting change of field current required for constant voltage?

5. What is a Good Efficiency? High efficiency is obtained generally by increasing the first cost of the machine. High efficiency means small losses corresponding to a given output. Small copper-losses mean low resistances, and low resistances mean large sizes of wire which are heavy and cost much more. Small core-losses mean low flux densities, and low densities mean large core-areas to carry enough flux to generate the required e.m.f. at standard frequency. Large core-areas mean large volume and weight of iron in the magnetic circuit, which again increases the cost of the machine. Low core-loss also means high-grade iron or steel, with thin laminations well insulated from one another, and all this increases the cost still further.

It is possible for the designer to build an alternator for almost any desired output with almost any desired efficiency, by being sufficiently liberal with quantity and quality of materials and skilled labor. Experience has indicated, however, that it is not economical to build the average alternator of given size for an efficiency higher than a certain fairly definite value. We have here a distinction between efficiency and economy. Why is it not economical, or why does it not "pay," sometimes to build machines for the highest possible efficiency?

Consider that we have a hydro-electric plant with a **limited amount of power available** (1000 kw.) and a **market for all the energy we can generate** (24 hours per day) at 2 cents per kilowatt-hour. Will it pay to specify a 96 per cent efficiency for the generator, rather than 95 per cent? If we get the higher efficiency alternator, we shall reduce the total losses by 10 kw., or 1 per cent of the input, or we shall have this much more of salable output from the same water power. During one year this represents (24 × 365 × 10) = 87,700 kw-hours, which, at 2 cents per kw-hour, will sell for $1754 per year. Now, every dollar invested in the plant must earn or save at least 13 cents per year. This is the amount necessary to pay "fixed charges" (consisting of interest, depreciation, taxes and insurance) on one dollar of invested capital for one year. Therefore, we can afford to pay $\frac{\$1754}{0.13} = \$13,500$ **more** for the generator having 96 per cent efficiency, than for the generator having 95 per cent efficiency. If the **extra cost** of the **higher efficiency** generator is less than this figure, we should by all means specify a 96 per cent efficiency (or perhaps even higher), because the increase in earnings will pay more than the fixed charges on the additional investment required, or will pay higher dividends than were anticipated. If, on the other hand, the **extra cost** of the 96 per cent generator is **more** than $13,500, it would be unwise (from the view-point of cost alone) to specify 96 per cent efficiency, and some lower value would be more economical.

A wise engineer would consider future prospects, also, to some extent in solving this problem. If a generator were to be used in a water-power plant so located that there would be not even a remote prospect of selling all the power that could be had from the water available, it would be "poor business" to pay more than necessary for the generator by demanding an unusually high efficiency. On the other hand, in locations where the cost of fuel is high and space for generating equipment is expensive (as in steam-electric plants situated in cities), even a small fraction of one per cent increase in efficiency has a very great value, and improvements or refinements which reduce the losses will pay well. It is obvious, from these considerations, that the efficiency should be chosen with regard for the demand for and value of a kilowatt-hour to the plant in question, and also with regard for the increase of first-cost of the alternator to produce a higher efficiency.

Efficiencies at rated full load measured by test on many alternators, large and small, are given in Table II, which is taken from a paper by E. M. Olin, Trans. A.I.E.E., July, 1912. Notice that in general the efficiencies are lower for the smaller machines. Neither the size and weight nor the losses of a dynamo can be reduced in exact proportion to its rated output. A small machine is always heavier and more expensive in proportion to its capacity, and usually has a lower efficiency, than a larger machine, other things being equal (such as quality of materials, workmanship, and the like).

Notice, also, from Table II, that several machines of the same rated output may have efficiencies that differ, although the values are fairly close together. This is due to the influence of conditions already mentioned. The alternators also differ widely in the distribution of losses at their rated full load. For instance, the ratio of variable losses (W_{I^2R}) to constant losses ($W_F + W_R$) varies between $\left(\frac{0.32}{2.77+2.23} = 0.064\right)$ for Alternator No. 30, and $\left(\frac{6.14}{0.82+3.30} = 1.49\right)$ for Alternator No. 2. We have seen that the efficiency is greatest in any one machine when this ratio is approximately unity (1.00). Therefore it appears that Alternator No. 30 (8000 kw.) will reach its maximum efficiency when greatly overloaded, and that Alternator No. 2 reached its maximum efficiency at a load considerably less than its rated output.

The rated full load of an alternator is usually the largest output which it can deliver continuously without exceeding a safe temperature in any part of the machine. It is within the power of the designer to determine whether the maximum efficiency of the alternator shall occur when the output is less than, equal to, or greater than this rated full-load. By being generous with copper and sparing of iron, making the variable losses small in comparison with the fixed losses, the maximum efficiency is caused to happen at a heavy load. By being generous with iron in the magnetic circuits and sparing of copper in the electrical circuits, the variable losses are caused to be large in comparison with the fixed losses, and the maximum efficiency occurs at light load. The same statements apply to electric motors, transformers, and other apparatus.

TABLE II. — EFFICIENCY TESTS ON A-C. GENERATORS*

No.	Size rated kv-a.	Frequency.	Loss, W_{I^2R}.	Loss, W_F.	Loss, W_R.	Total loss.	Efficiency at full-load.
1	100	60	2.78	1.08	3.50	7.36	93.15
2	100	60	6.14	0.82	3.30	10.26	90.7
3	125	60	3.19	3.70	2.97	9.86	91.1
4	200	60	2.58	1.72	2.45	6.75	93.7
5	300	25	1.77	0.71	3.33	5.81	94.5
6	300	60	1.89	1.90	2.40	6.19	94.15
7	333	60	0.83	1.47	2.94	5.24	95.0
8	400	60	2.64	0.61	3.00	6.25	94.15
9	500	25	2.25	0.50	1.42	4.17	96.0
10	500	60	0.65	3.66	4.16	8.47	92.25
11	600	60	0.83	1.79	2.27	4.89	95.4
12	700	60	1.65	0.94	1.86	4.45	95.78
13	725	60	1.52	0.70	1.27	3.49	96.7
14	1 000	60	1.67	1.52	1.83	5.02	95.26
15	1 000	60	0.74	4.10	1.66	6.50	93.9
16	1 250	60	1.25	0.83	2.65	4.73	95.5
17	1 250	60	1.04	0.83	2.59	4.46	95.77
18	1 500	60	0.66	3.16	3.16	6.98	93.5
19	2 000	60	1.03	0.92	2.10	4.05	96.13
20	2 000	60	1.20	1.19	1.83	4.22	96.0
21	2 500	60	0.56	2.00	2.40	4.96	95.3
22	3 000	60	0.95	2.01	2.46	5.42	94.82
23	3 000	60	0.76	1.47	2.57	4.80	95.4
24	3 000	25	0.51	3.73	2.00	6.24	94.15
25	3 000	60	1.18	1.62	1.70	4.50	95.76
26	3 750	60	1.47	0.49	1.46	3.42	96.75
27	4 000	60	0.47	4.13	2.53	7.13	93.3
28	5 000	60	0.91	0.89	1.98	3.78	96.4
29	6 666	60	0.41	2.17	2.55	5.13	95.1
30	8 000	60	0.32	2.77	2.23	5.32	94.9
31	10 000	60	0.40	2.40	1.85	4.65	95.6

* All losses expressed in per cent of output at rated full-load, non-inductive.

W_{I^2R} = loss due to resistance of windings, brushes, and sliding contacts, to useful current flowing, in field and armature;

W_F = loss in friction;

W_R = rotation loss, not frictional, due to hysteresis and eddy currents caused by rotation of iron through the magnetic field, and all I^2R losses due to local or useless currents within the armature.

Prob. 12-1. Calculate the efficiency at $\frac{1}{2}$, $\frac{3}{4}$, $1\frac{1}{4}$, and $1\frac{1}{2}$ times rated full-load, for the 500 kv-a. alternator No. 9 in Table II. Assume that the values given in Table II correspond to rated full-load unity power-factor, and that one-quarter of the total full-load copper-loss remains constant at all loads, while the remainder of the copper loss varies as the square of the current output and that all rotational losses are constant.

Prob. 13–1. For the alternator of Prob. 12–1 draw curves, using per cent of rated full-load kv-a. output as abscissas in all cases, and as ordinates the following:

(*a*) Kw. lost in friction.
(*b*) Kw. lost in friction + rotation losses.
(*c*) Kw. lost in friction + rotation + copper, or total kw. loss.
(*d*) Efficiency in per cent.

Extend these curves if need be, and discuss the relation between the losses when the load is such that the efficiency is maximum.

Assume friction and rotation losses constant at all loads.

Prob. 14–1. For each alternator listed in Table II, calculate each loss as a percentage of the total losses, at full-load. Tabulate these values, showing the distribution of the losses in the various sizes of alternators covered by Table II. Discuss your results.

6. Effect of Load on Voltage of Alternator. Addition of load usually causes the terminal voltage of an alternator to change. If the power-factor of the load is 100 per cent, the voltage falls. If the power-factor is less than 100 per cent and the current **lags**, the voltage falls much more than with non-inductive load. If the power-factor is less than 100 per cent and the current **leads**, the voltage will fall less than with non-inductive load, or may even rise, depending upon the proportion of leading reactive components. Most generators are required to deliver lagging current. Leading loads are the exception, and are met usually in very long distance and very high voltage transmission systems.

The inherent or automatic change of voltage which occurs when the load is reduced from rated full-load to zero load, while the speed and field current are kept constant, is known as the "voltage regulation." It is usually expressed as a percentage, thus:

Voltage regulation, in per cent =

$$\left(\frac{\text{zero-load voltage} - \text{full-load voltage}}{\text{full-load voltage}}\right) \times 100.$$

An ordinary value for the voltage regulation of an alternator would be 8 per cent on non-inductive load and 24 per cent on a lagging inductive load having 80 per cent power-factor. For the average central station load, consisting of lighting and power, a power-factor of 80 per cent should be assumed. Power-factors of 90 to 95 per cent are usually obtained only when the load is entirely incandescent lighting or heating. Power-factors as high as this, or even above 95 per cent, may be obtained if a large part of the load consists of synchronous motors, which are adjusted to take a leading reactive component of current from the system. A power-factor of 70 per cent may be expected in a plant having a large proportion of induction motors, arc lighting, electric furnaces, or electric welding load.* An extensive set of measurements of power-factor on motor service circuits and industrial installations in Cleveland, reported by H. L. Wallau in the *Electrical World*, Nov. 22, 1913, shows maximum and minimum values of 0.95 and 0.20 respectively, with an average of 0.69. The power-factor of the entire station load was 89 per cent by day and 92 per cent by night.

From this data it is evident that the effect of load, and particularly of low power-factor, upon the voltage of alternators must be studied. In many central stations the maximum allowable variation of voltage is considered to be 5 per cent during the daytime and 2 per cent during the night. Sudden variations are much more objectionable than slow variations. Good inherent voltage regulation in the alternator itself must be depended upon to limit these variations of voltage due to very rapid fluctuations of load. Automatic voltage-regulators, operating to adjust the field current, are perfectly capable of preventing the slower variations of voltage. Where the load is principally motors, constancy of voltage is not usually as important as where the load is largely lighting.

Example 2. What is the power-factor of a generator when it is loaded with 400 kw. at 87 per cent lagging power-factor and with 600 kw. at 75 per cent lagging power-factor?

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* See *Electric Journal*, 1911, page 623.

Draw Fig. 3 representing the 400 kw. by the line OA.

Draw OB at an angle of 30° (87 per cent $\doteq$ cos 30°) and draw AB perpendicular to OA.

The line OB represents the kilovolt-ampere load of which the line OA is the effective power and the line AB is the reactive power.

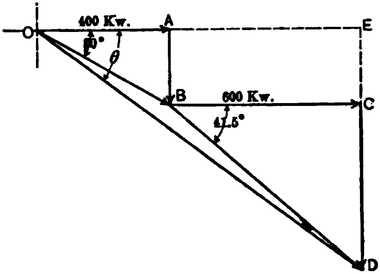


FIG. 3. Topographic vector diagram showing the method of adding loads with different power-factors.

Similarly draw BC from B parallel to OA to represent the effective load of 600 kw. Draw BD and DC to represent the total kv-a. load and the reactive load respectively.

Draw the construction lines AE and CE.

Total effective power $= OA + BC = 1000$ kw.

Total apparent power $= OD = \sqrt{\overline{DE}^2 + \overline{OE}^2}$.

$$\begin{aligned} DE &= EC + DC \\ &= 400 \tan 30° + 600 \tan 41.5° \\ &= 231 + 531 \\ &= 662. \\ OE &= OA + AE \\ &= 400 + 600 \\ &= 1000. \end{aligned}$$

Thus
$$\begin{aligned} OD &= \sqrt{662^2 + 1000^2} \\ &= 1195 \text{ kv-a.} \end{aligned}$$

Power-factor of load
$$\begin{aligned} &= \cos\theta \\ &= \frac{1000}{1195} \\ &= 83.7 \text{ per cent.} \end{aligned}$$

Prob. 15–1. An alternator supplies three feeders, one of which takes 100 kw. at 80 per cent power-factor, another 200 kw. at 85 per cent power-factor and the third 150 kw. at 95 per cent power-factor, all lagging. What load, in kilowatts and kilovolt-amperes, is the alternator delivering and at what power-factor?

Prob. 16–1. (*a*) If a load of 50 kw. at 90 per cent power-factor is suddenly added to the third feeder, by what percentage is the total armature current of the alternator increased, and what does its power-factor become?

(*b*) If, at the same instant, the second feeder drops a load of 100 kw. having 95 per cent power-factor, by what percentage is the armature current of the alternator greater or less than its initial value, and what is its power-factor? Assume terminal voltage to be held constant.

Prob. 17–1. If an alternator supplies two feeders, one at a power-factor of 70 per cent and the other at 95 per cent, what must be the ratio between the power (kilowatts) taken by these two feeders in order that the total power-factor of the alternator may be 85 per cent? If the terminal voltage of the alternator is kept constant when the first feeder is disconnected, by what percentage will the power output and the rate of heating (I^2R) in the armature be reduced?

7. Effects of Unsteady Voltage. Fluctuation of voltage in an electric supply system is a disadvantage to both central station and customer, and there are no compensating advantages. Considering the lamps connected to the system, we find that when they are operated at a voltage above normal their life decreases very rapidly; and this shortening of life is particularly bad with carbon lamps and gem lamps, which it is the practice of central stations to renew free of charge. If the voltage drops below normal, the life is lengthened correspondingly, but the candle-power and the watts consumed by the lamp decrease very rapidly. This is unsatisfactory to the central station, because the consumer takes less kilowatt-hours and the income of the central station is correspondingly reduced. It is also unsatisfactory to the consumer both because the power consumed by the lamps does not decrease in proportion to the candle power and because a flickering or unsteady light is very disagreeable and injurious to the eyes.

Considering motors connected to the circuit, we find that the speed of most types of alternating-current motor is not much affected by changes of voltage. The speed of the synchronous motor would not change at all; the speed of the induction motor would change slightly, and the speed of the series motor considerably more. However, the maximum torque which an induction motor (commonest type of alternating-current motor) can develop varies approximately as the square of the voltage applied. This means that a given percentage decrease of voltage, such as is likely to result from bad regulation of generator and line when the motor is overloaded and is taking a large current at low power-factor, will reduce its capacity for instantaneous overloads by a much greater percentage.

8. Causes of Voltage Variation in Alternator. In order to make the discussion more definite and simple, we shall consider at first a single-phase alternator, or one phase of a polyphase alternator. Also we shall assume that the field current of the alternator is held constant. All alternators are driven at practically constant speed by well-governed engines, because a variation of frequency would cause a proportional change of speed in all motors connected to the alternator, which is objectionable to the consumer. Under these conditions, **changes of terminal voltage must be due to changes of flux or to voltage drops or reactions within the armature windings.** In fact, both of these effects are produced when current flows through the armature windings.

The winding has some resistance, and a part of the e.m.f. which is generated must be used up within to overcome the opposition which this resistance offers to the flow of current.

In Fig. 4, the vector E_i represents the total e.m.f. induced, and I the current flowing, in the winding. The e.m.f. vector rI, opposite to I, represents the reaction against the flow of current due to resistance; r is the effective resistance of the winding (per phase). The **resultant** of the generated e.m.f. (E_i) and the reaction (rI) is the terminal e.m.f. E_t, on the supposition that there is no reaction except that caused by resistance. The induced e.m.f.

may be thought of as composed of two components, as in Fig. 5, the terminal voltage E_t and the voltage (rI) used to overcome the armature resistance. The power-factor of the receiving circuit or load is equal to the cosine of the angle (θ) of phase difference between E_t and I. The **effective resistance** or **alternating-current resistance** (r) is a number which, multiplied by the square of the effective value of current flowing through the winding, gives a product equal to the total average watts lost or transformed into heat, in the armature copper. It is usually about 15 per cent greater than the **direct-current resistance.** The difference between the alternating-current resistance and the direct-current resistance is due to eddy currents in the conductors, and to the tendency (called "skin effect") of alternating-currents to flow near the surface of wires (particularly when the wires are large or the frequency is high).

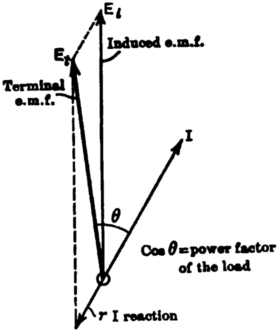


FIG. 4. The terminal e.m.f. E_i is the resultant of the induced e.m.f. and the (rI) armature reaction.

But there are other reactions besides that due to resistance of the armature winding. When current flows through the armature, the conductors build up around themselves local fluxes entirely independent of the main field flux which produces E_i. These local fluxes alternate with the armature current, and this alternation of flux linking with the armature wires induces e.m.f.'s in these wires. This local flux is in phase with the armature current I, and the e.m.f. induced by this flux is 90° out of phase with the flux and lagging behind it. This e.m.f. is the **reaction** or **counter-e.m.f. due to self-inductance of the armature winding,** and it thus **lags 90° behind I.** In Fig. 6, we take account of the reactance (x), due to self-inductance of the armature winding, as well as of the resistance (r). In this figure (xI) is the **induced** e.m.f. or **reaction** due to

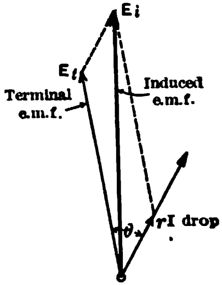


FIG. 5. The induced e.m.f. E_i is composed of two parts, E_t, the terminal e.m.f. and (rI) the volts required to overcome the armature resistance.

change in the number of interlinkages between the armature wires and the flux produced by armature current; rI is the resistance reaction, as in Fig. 4; zI (the vector sum of rI and xI) is the **total reaction,** or impedance voltage, due to resistance and inductive reactance of the armature winding. The resultant of zI and E_i **is the terminal voltage** $\mathbf{E_t}$.

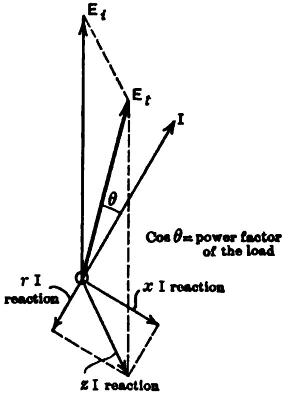
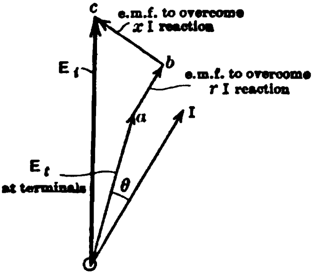


Fig. 6. The terminal voltage E_t is the resultant of the armature reactions (xI) and (zI) and the induced e.m.f. E_i.

A more usual and convenient method which can be used for finding E_i when E_t, I and cos θ (or power-factor of load) are given, is shown in Fig. 7. This is a topographic diagram corresponding exactly to the polar diagram of Fig. 6, except for the fact that it is the reverse process, namely, that of determining what total e.m.f. (E_i) must be induced by the main field, in order to have a given e.m.f. (E_t) between terminals, when a current of I amperes is delivered at a power-factor (cos θ). In this case, we begin by drawing E_t. Then lay off I, lagging behind E_t by an angle whose cosine is the given power-factor. Then lay out from the end of E_t an e.m.f. (ab) parallel to I and in the same direction, and of a length representing (rI) volts (to the same scale that E_t is drawn). This is the **e.m.f. required to overcome the reaction due to resistance** of the armature winding. Now, from the end b lay out a vector (bc), in direction leading I by 90°, and in value equal to (xI) where x is the inductive reactance (in ohms) of the armature winding. This is the e.m.f. which is **required to overcome the reaction due to self-inductance** of the winding. The **vector sum** of E_t, ab **and** bc is the total e.m.f. that must be induced in the



c
e.m.f. to overcome x I reaction
E_i
b
e.m.f. to overcome r I reaction
a
I
E_t at terminals
θ
O

Fig. 7. A topographic vector diagram for determining the induced voltage E_i necessary in order to deliver the terminal voltage E_t and current I at a power-factor of cos θ.

winding by the main field, in order to have E_t volts at the terminals.

The magnetic effect of the current flowing in the armature conductors produces not only a reacting e.m.f. (counter e.m.f. of self-induction) in the conductors, but also produces important **changes in the main field flux.** These magnetic effects of the armature currents upon the main field are known by the general term "**armature reaction.**" The armature reaction may be analyzed or resolved into two effects, namely:

First. A distortion of the main field, causing it to become more dense at some places, and at other places less dense, than it was at the zero load.

Second. A direct weakening or strengthening effect, causing the main field flux to be less or greater than it was at zero load.

The distortion of the main field changes the form of the e.m.f. wave in each armature inductor, and may cause the wave-shape of the terminal e.m.f. to be either further from or closer to a sine wave, when the load is increased (see First Course, Fig. 297). This change of wave-shape may change slightly the effective value of voltage. The weakening or strengthening effect decreases or increases the amount of e.m.f. (E_i) which is induced in the armature, and, therefore, also decreases or increases the terminal e.m.f. (E_t).

We may consider the armature current as consisting of two components which flow through the same winding. One of these, in phase with E_i, has an average effect to **distort** the main field but not to change appreciably the amount of flux or the effective value of E_i. The other component of I, lagging or leading 90° with respect to E_i, has an average effect to reduce or to increase the main flux and, therefore, correspondingly reduce or increase the effective value of E_i and E_t. If the load is inductive and has a low power-factor, the lagging quadrature component of I is relatively large, the weakening of the main field is greater, therefore, E_i and E_t fall more for a given increase of I than would be the case with a higher power-factor. It was the demagnetizing effect due to lagging **reactive** component of I, which caused the voltage regulation of the alternator in Art. 120 to be 24 per cent at 80 per cent power-factor, although it was only 8 per cent at 100 per cent power-factor.

Example 3. What is the voltage regulation of a 500-kv-a. single-phase 2300-volt generator at unity power-factor? Effective armature resistance of 0.5 ohm and a reactance of 4 ohms. Neglect all reactions except those represented by the internal-resistance drop and the reactance drop.

$$\text{The current at unity power-factor} = \frac{500{,}000}{2300} = 217 \text{ amp.}$$

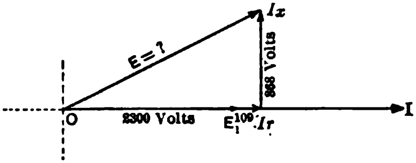


FIG. 8. Topographic vector diagram for determining the voltage regulation of a generator at unity power-factor.

Construct Fig. 8, drawing OE_1 along current vector I to represent the terminal voltage of the generator under full load.

$$\begin{aligned} Ir &= 217 \times 0.5 \\ &= 109 \text{ volts.} \end{aligned}$$

Add vector Ir to E_1 in phase with E_1.

Voltage to overcome armature reactance:

$$\begin{aligned} Ix &= 217 \times 4 \\ &= 868 \text{ volts.} \end{aligned}$$

Add vector Ix leading Ir by 90°.

The vector E represents the e.m.f. which must be generated to supply the terminal voltage E_1, the resistance drop Ir and the reactance drop Ix.

$$\begin{aligned} E &= \sqrt{(2300 + 109)^2 + 868^2} \\ &= 2560 \text{ volts.} \end{aligned}$$

When the load is removed from the generator, the terminal voltage will rise to the value of E or to 2560. Thus the no-load voltage = 2560.

$$\begin{aligned} \text{Regulation} &= \frac{(\text{no-load volts}) - (\text{full-load volts})}{(\text{full-load volts})} \\ &= \frac{2560 - 2300}{2300} \\ &= 11.3 \text{ per cent.} \end{aligned}$$

Prob. 18–1. A three-phase alternator rated 1000 kv-a. 6600 volts has an effective resistance of 0.5 ohm and a reactance of 10.0 ohms per phase of the winding. Assuming these constants to represent all reactions which affect the drop of terminal voltage under load, calculate the following (the phases being Y-connected):

(*a*) Volts required between terminals of each phase of the winding at rated full load, power-factor 1.00.

(*b*) Amperes in each phase of winding, rated full load.

(*c*) Volts required to overcome reactions due to resistance, and to reactance.

(*d*) Total voltage induced in each phase.

(*e*) Increase of voltage when full load is removed.

(*f*) Voltage regulation, per cent. Illustrate solution by vector diagram.

Prob. 19–1. Repeat calculations of Prob. 18–1 on basis of Y-connected alternator supplying load of 80 per cent lagging power-factor.

Prob. 20–1. Repeat Prob. 18–1 for alternator Δ-connected at same phase voltage with load of 100 per cent power factor.

Prob. 21–1. Repeat Prob. 18–1 for alternator Δ-connected with load of 80 per cent power-factor.

9. Armature Reaction in Single-phase Alternators. Some explanation of the bare statement of facts given in the preceding article is necessary to a proper understanding of important operating features of alternating-current generators and motors. Let us first analyze the magnetic actions of the armature of a single-phase alternator, or a polyphase alternator of which only one phase is loaded. In Fig. 9 to 28, this phase occupies two slots per pole of an alternator having 6 slots per pole with any number of poles.

When I is in phase with $\mathbf{E}_i$, Fig. 9 to 12 illustrate conditions at consecutive instants one-quarter period apart, for one complete cycle of e.m.f. In Fig. 9, E_i is maximum, and I is also maximum in same direction. The armature current produces flux represented by lines l and c. The varying amount of l and c as I alternates is the cause of the self-induction reaction denoted by (xI) in previous diagrams. At the instant illustrated by Fig. 9, any armature flux c which passes through the field poles does so in a general direction at right angles to the flux m which is produced by the field windings. It weakens the main field at the leading pole-tip but strengthens it in equal amount at the trailing-tip. The total flux per pole is not altered unless the average permeability of the iron is changed by the redistribution of flux.

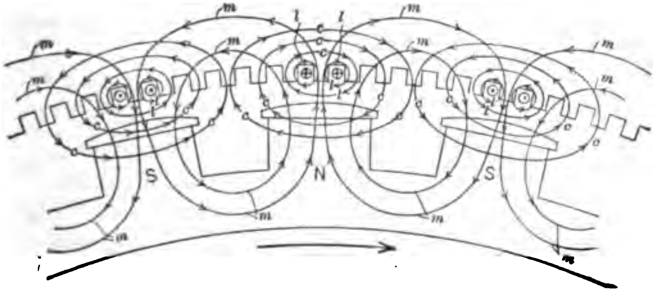


Fig. 9. The e.m.f. and the current in the armature coils are in phase and have a maximum value at this instant. The magnetic lines *c*, *c*, *c*, due to current in armature windings are at right angles to the lines *m*, *m*, *m*, due to the field coils. This weakens the leading pole-tip and strengthens the trailing tip.

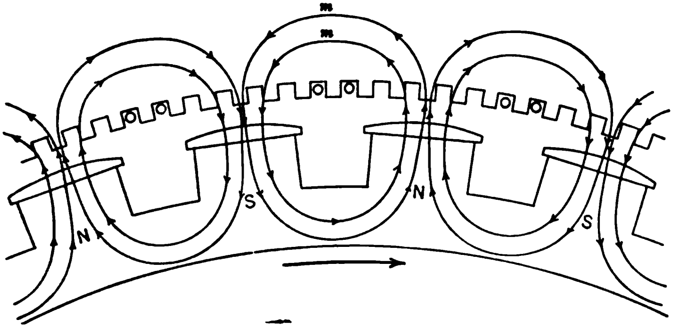


Fig. 10. One-quarter of a cycle later than Fig. 9. The current and e.m.f. of the armature are zero, and the field is unaffected. While passing from position in Fig. 9 to that of Fig. 10 the field was both distorted and weakened.

One-quarter period later the conditions are as represented in Fig. 10. If *I* were still flowing in the same direction as in Fig. 9, the armature flux *c* would pass through the magnetic circuit parallel to the main field *m* and in direct opposition to it — i.e., its entire action upon *m* would be demagnetizing or weakening, instead of cross-magnetizing or distorting. But at this instant (Fig. 10) *I* equals zero because *I* is in phase with E_t which is zero as conductors are cutting no lines. While we are passing from Fig. 9 to 10, the armature reaction is partly distorting and partly weakening. Between Fig. 10 and 11, it is partly distorting and partly strengthening the main field. At the instant of Fig. 11, we have maximum distortion and no weakening or strengthening. Between Fig. 11 and 12, it is partly distorting and partly weakening again. At

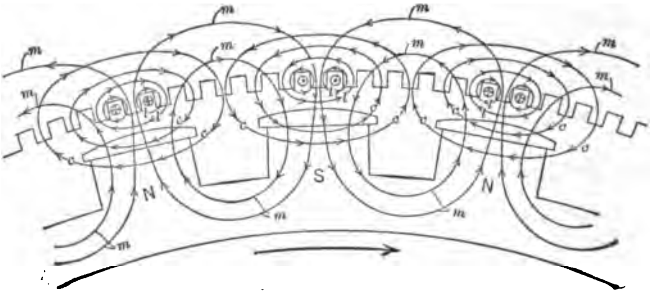


FIG. 11. One-half cycle later than Fig. 9. There is again a maximum distortion of the field, but no weakening or strengthening. Both *I* and *E* are again maximum though in the opposition direction to that of Fig. 9. Field is distorted and strengthened in passing from Fig. 10 to Fig. 11.

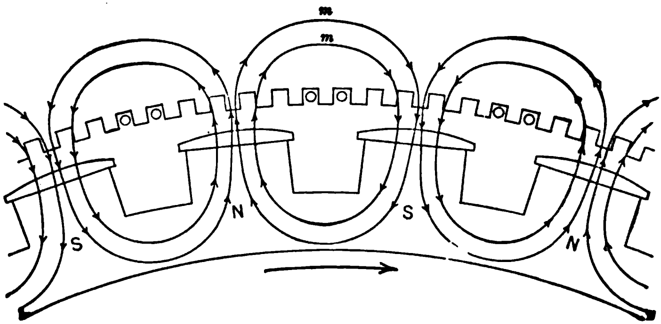


FIG. 12. Three-quarters of a cycle later than Fig. 9. Between this figure and Fig. 11 the effect of the armature reaction was to distort and weaken main field.

the instant of Fig. 12, both effects are zero. Between Fig. 12 and 9 (completing one cycle of e.m.f.) the armature reaction partly distorts and partly strengthens the main field.

Summarizing the effects throughout one cycle of e.m.f. we find that, **when I is in phase with E_i:**

(*a*) There is a distorting effect, varying in intensity from a maximum to zero to maximum to zero (or through two cycles during each cycle of e.m.f.), but always tending to concentrate the flux at the trailing pole-tip.

(*b*) There is a direct effect, alternately weakening and strengthening the main field twice during each cycle of E_i, but having a zero **average** effect upon the amount of flux from each pole and the effective value of E_i.

It follows, therefore, that the decrease of terminal voltage with increase of **non-inductive** load on a single-phase alternator is caused principally by resistance and self-inductance of the armature winding. The armature reaction causes the flux to oscillate across the pole-faces, which would seriously increase the loss due to eddy-currents in the pole-faces if they were not well laminated.

Having seen from this discussion how the magnetic action of armature currents affects the main field of any multipolar alternator, the student is prepared to appreciate and use a very valuable diagram of armature reaction used by Professor Morecroft.* In Fig. 13 to 20, the length and direction of the vector n represent the strength (ampere-turns) and direction of the magnetic action of the armature. The total action of n is equivalent to c ampere-turns acting at right angles to the field ampere-turns (producing distortion of flux), plus w or s ampere-turns acting parallel to the field ampere-turns (decreasing or increasing the main flux). Fig. 13 corresponds to Fig. 9 and Fig. 14 corresponds to an instant or position midway between Fig. 9 and 10. During one cycle of induced e.m.f., the end of vector n moves twice around the dotted

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* See "Continuous and Alternating Current Machinery" by J. H. Morecroft, John Wiley and Sons.

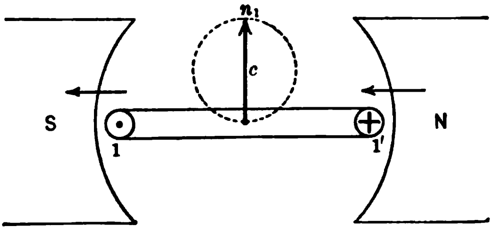

Fig. 13. The line n represents the amount and direction of the magnetic action of the armature. The line c represents that component of n which is at right angles to the main field. This figure corresponds to Fig. 9, and thus $c = n$ since the whole magnetic effect of the armature is to cross-magnetize the poles.

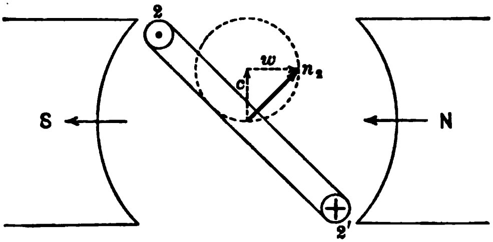

Fig. 14. The condition half-way between Fig. 9 and Fig. 10. The magnetic effect n has become smaller, and has two components, c, tending to cross-magnetize the field, and w, tending to weaken the field.

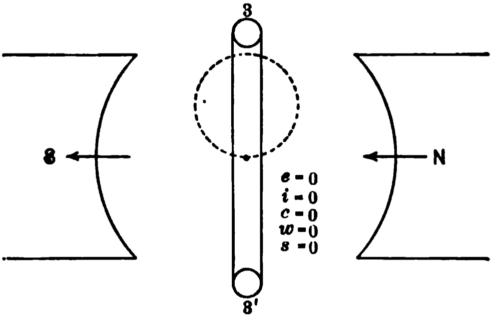

Fig. 15. The condition shown in Fig. 10. The magnetizing effect of the armature is zero.

circle. A careful study of these diagrams will disclose the same actions described from Fig. 9 to 12.

When the armature current (I), lags 90° behind the induced

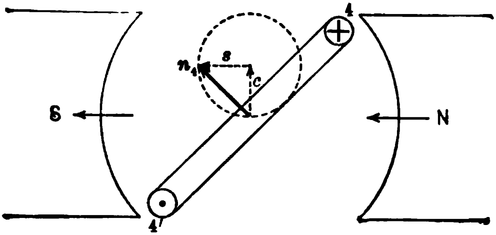

Fig. 16. A condition half-way between that of Fig. 10 and that of Fig. 11. The armature magnetic effect *n* now has two components, *c*, tending to cross-magnetize the field, and *s*, tending to strengthen it.

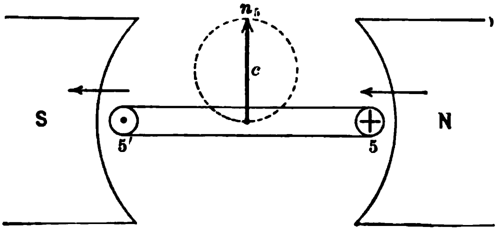

Fig. 17. The condition of Fig. 11. All the armature magnetic effect tends to distort the field by cross-magnetizing it.

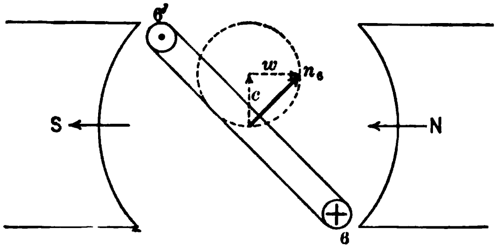

Fig. 18. Half-way between Fig. 11 and Fig. 12. There is a weakening effect *w* and a distorting effect *c*.

e.m.f. (E_i), the effects of armature reaction on the main field may be seen clearly by examining Fig. 21, 22, 23 and 24. The Morecroft diagram corresponding to this case is Fig. 29. If we

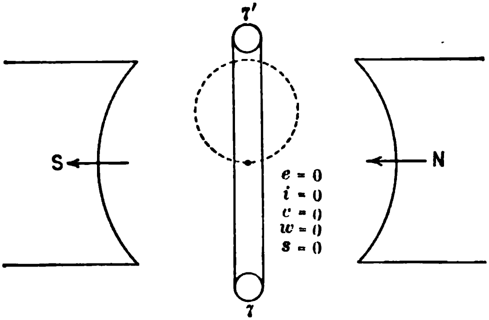

FIG. 19. The same condition as shown in Fig. 12. No magnetic effect from the armature.

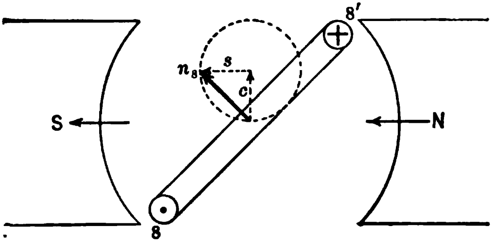

FIG. 20. Half-way between Fig. 12 and Fig. 9. The armature magnetic force now has a distorting effect *c* and a strengthening effect *s*. When the armature coil has completed one cycle, the magnetizing effect of the armature has completed two cycles.

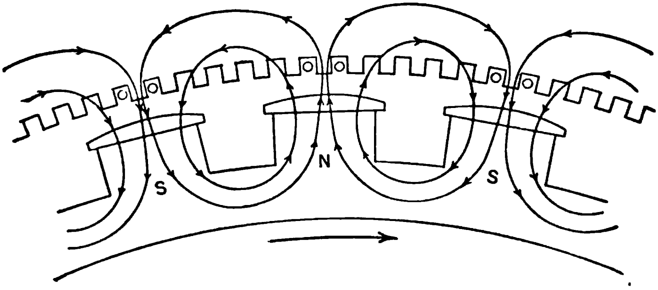

FIG. 21. The e.m.f. is a maximum but the current is zero in the armature because the current lags 90°. No magnetizing effect by the armature.

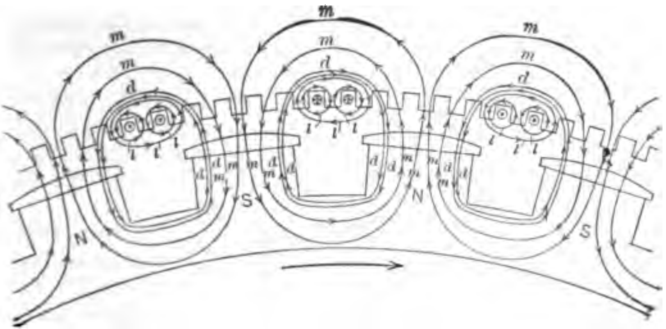

FIG. 22. One-quarter cycle later than Fig. 21. Note that the armature current tends to weaken the field. Fig. 29 shows the Morecroft diagrams for the condition half-way between Fig. 21 and Fig. 22.

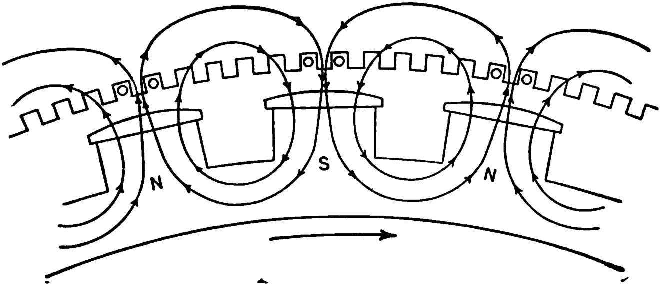

FIG. 23. One-half cycle later than Fig. 21. The current has again become zero, therefore, the magnetizing effect of the armature again becomes zero.

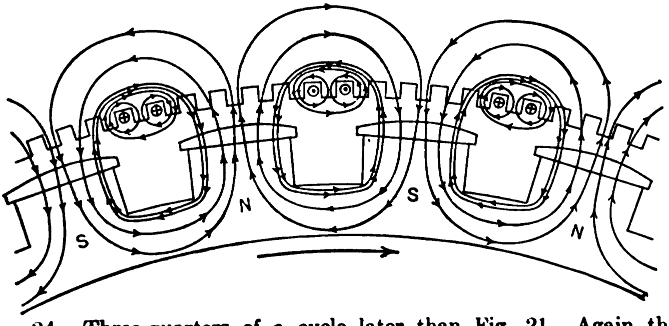

FIG. 24. Three-quarters of a cycle later than Fig. 21. Again the armature reaction weakens the fields.

follow the cross-magnetizing and the weakening or strengthening actions of the armature ampere-turns, through one complete cycle of current, we shall find that:

(*a*) There is a distorting effect which varies cyclically with twice the frequency of the current, pushing the flux out of its normal or zero-load position first toward one pole-tip and then toward the other pole-tip. The **average** distorting effect is zero. The pole-face eddy-current losses are likely to be large.

(*b*) There is a direct effect in line with the ampere-turns of the field winding. This effect varies cyclically between zero and a maximum value, but always acts to **weaken** the main field. There is an average weakening effect, and E_i is reduced. It is evident, therefore, that when I lags 90° behind E_i it causes a larger drop in E_t, and makes the voltage regulation worse than with non-inductive load.

When I leads E_i by 90°, conditions are as represented by Fig. 25, 26, 27, 28 for successive instants, or by Fig. 30

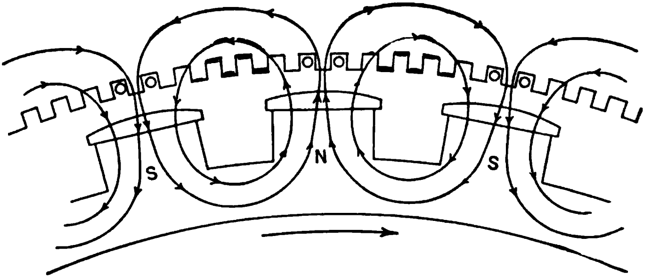

FIG. 25. The current leads the e.m.f. by 90°. With the armature in this position the magnetizing effect of the armature is zero.

for an entire cycle of current. Analyzing the meaning of these diagrams in the manner indicated, we find that:

(*a*) There is a distorting effect similar to that produced when I lags 90° behind E_i. The **average** distortion is zero.

(*b*) There is a pulsating direct effect parallel to the main field, but always in a direction to **increase** the flux. On the average this effect increases E_i and E_t.

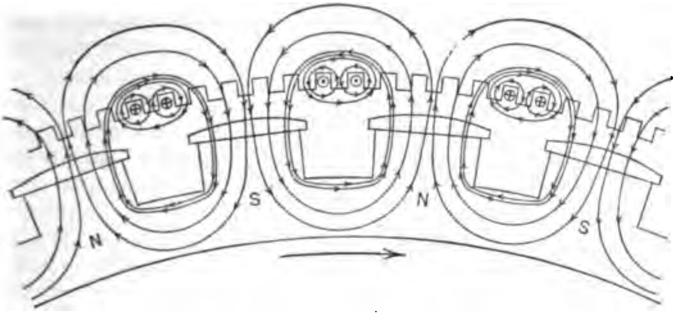

FIG. 26. One-quarter of a cycle later than Fig. 25. The armature reaction strengthens the fields. Fig. 30 is the Morecroft diagram for a condition half-way between this and Fig. 25.

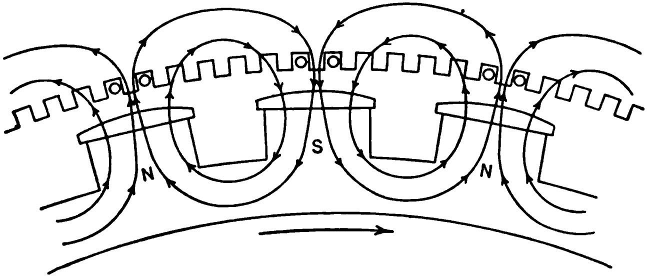

FIG. 27. One-half cycle later than Fig. 25. The e.m.f. being at a maximum the current must be zero, and therefore the armature reaction is zero.

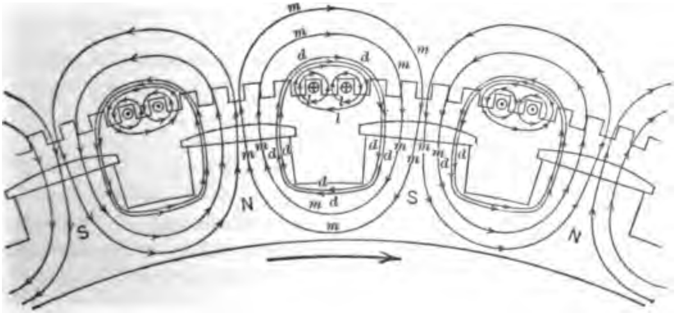

FIG. 28. Three-quarters of a cycle later than Fig. 25. Again the current has become a maximum and the fields are again strengthened by the armature reaction.

Now consider the general case when I lags behind E_t some angle between 0° and 90°, or the power-factor of the entire circuit is between 1.00 and 0.00. Fig. 31 is drawn for a power-factor equal to cos θ. The coil is shown at the instant when I has its maximum value; the total armature ampere-turns is represented by n, which

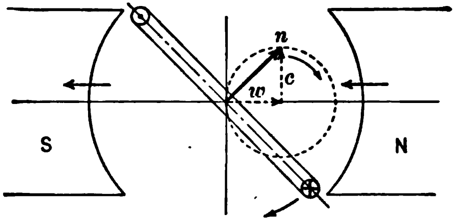

Fig. 29. A condition half-way between Fig. 21 and Fig. 22. The current lags 90° behind the voltage. The armature reaction n tends to cross-magnetize c and weaken w, the main field. Note: The marks ⊙ and ⊕ indicate direction of *current*, not e.m.f.

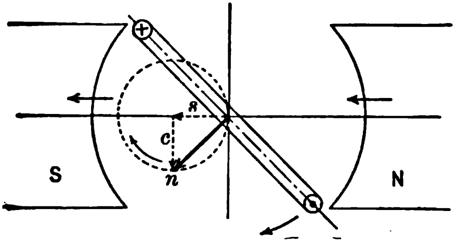

Fig. 30. The current leads the voltage by 90°. The armature reaction has a cross-magnetizing component c and a strengthening component s. The condition half-way between Fig. 25 and 26.

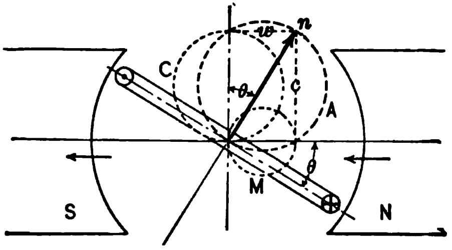

Fig. 31. The armature current lags behind the voltage θ°. The armature reaction n has a field weakening effect w and a cross-magnetizing effect c. The circles C and M are component circles of circle A.

is equivalent to a distorting effort of c ampere-turns plus a weakening effect of w ampere-turns, on the main field. As the coil makes one revolution, the end of the vector n rotates twice around the circle A. The vertical component of n has an average value upward, indicating that the flux is pushed toward the trailing pole-tip most of the time, although it concentrates slightly on the other tip for a small part of each half-cycle of current. The horizontal component of n has an average value opposite to the main field, although it increases the flux for a small part of each half-cycle of current. The analysis is simplified by resolving I into two components, one in phase with E_i and the other 90° behind E_i. The magnetizing action of the former is represented by the circle C and of the latter by the circle M (compare Fig. 14 and 29).

By this analysis we see that whatever average distorting effect there may be, is due to the component of I in phase with E_i, and whatever average weakening effect there may be, is due to the component of I which lags 90° behind E_i. It follows that a given reduction of power-factor will affect the voltage regulation much more seriously when the power-factor is high, than when it is low. Thus, Fig. 32 shows that when we reduce the power-factor from 100 per cent to 90 per cent, the wattless or reactive component c_1I_1 increases from 0 to 43.6 per cent of I, while a further equal decrease of power-factor (from 90 per cent to 80 per cent) increases the wattless component by 16.4 per cent of I (or, to 0.60 I). A reduction of power-factor from 20 per cent or 30 per cent to zero would produce an insignificant change in the quadrature component of I, and therefore would not appreciably alter the flux and the voltage regulation.

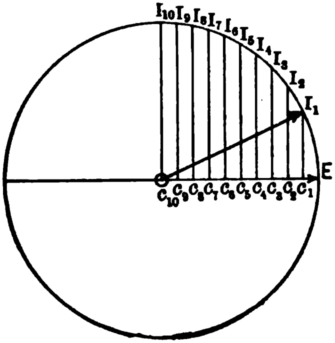
I_{10} I_9 I_8 I_7 I_6 I_5 I_4 I_3 I_2 I_1
E
C_{10} C_9 C_8 C_7 C_6 C_5 C_4 C_3 C_2 C_1

FIG. 32. Note that the decrease of the power-factor from 1.00 to .90 increases the reactive component of the current from zero to I_1c_1, or about 44 per cent of the total value of the current (OI_1).

Prob. 22–1. What factors must be considered as limiting or preventing the rapid displacements of flux across the pole-faces, which theory indicates should result from the varying cross-magnetizing component of armature reaction in the single-phase alternator?

Prob. 23–1. Prove that the path of the end of the vector **n** (Fig. 13 to 20), as the armature winding rotates, is truly a circle.

Prob. 24–1. Describe in detail the magnetizing action of the armature for one complete cycle, when the current lags 90° behind induced e.m.f. See Fig. 21, 22, 23, 24, 29.

Prob. 25–1. Repeat Problem 24–1, but assume the current to **lead** the induced e.m.f. by 90°. See Fig. 25, 26, 27, 28, 30.

Prob. 26–1. The single armature coil in Fig. 13 to 20 has 10 turns and carries an harmonic current whose effective value is 10 amperes. Calculate the maximum and average cross-magnetizing force due to the armature, in ampere-turns. Current is in phase with induced e.m.f.

Prob. 27–1. The armature specified in Prob. 26–1 carries a current which lags 90° behind the induced e.m.f., as illustrated by Fig. 29. Calculate:

(*a*) The maximum and average values of cross-magnetizing ampere-turns.

(*b*) The maximum and average values of demagnetizing ampere-turns.

10. Armature Reaction of Polyphase Alternator. When a polyphase alternator carries a balanced load (equal current at same power-factor in all phases), the magnetic field due to the currents in the armature windings is steady, or constant in value and fixed in direction. **When I is in phase with the $\mathbf{E}_i$** which produces it, the armature reaction pushes the flux toward the trailing tip of each pole and holds it there steadily but **neither increases nor decreases appreciably the amount of flux. When I lags 90° behind $\mathbf{E}_i$,** the **flux is reduced** by the counter magneto-motive force of the armature ampere-turns, but the distribution of flux is nearly the same as at zero-load. Between these two extremes, or when the power-factor of the entire circuit is between 1.00 and zero, the armature current produces both distortion and weakening of the main field (I lagging). Each effect is constant, and depends upon the values of I, $\cos\theta$, and number of turns in the armature winding. High power-factors, few armature turns and strong fields are conducive to good voltage regulation.

Fig. 33 to 38 present an analysis of the armature reaction in a three-phase alternator with balanced load. In Fig. 33, n_1 represents the total ampere-turns of Phase 1, at the instant when the current in it is maximum. The current is in phase with E_i, and the dotted circle represents the path of the end of vector n_1, as the armature rotates (compare Fig. 13). Fig. 34 represents Phase 2, and Fig. 35 represents Phase 3, all at the same instant. If we add vectorially the three armature reactions, n_1, n_2, n_3, as in Fig. 36,

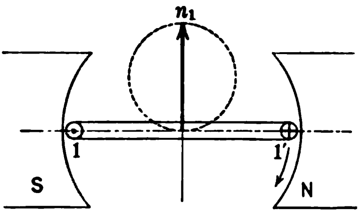

FIG. 33. The vector n_1 represents the armature reaction in Phase 1 of a three-phase generator with balanced load of unity power-factor.

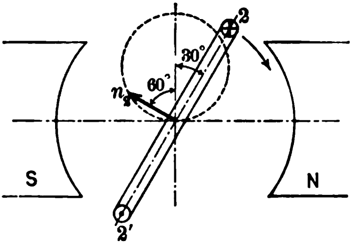

FIG. 34. The vector n_2 represents the armature reaction of Phase 2 of the generator at the same instant as that represented in Fig. 33.

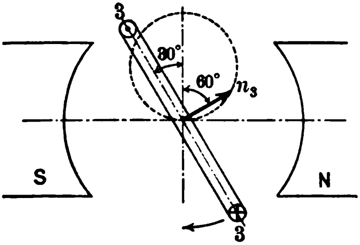

FIG. 35. The vector n_3 represents the armature reaction of Phase 3 at same instant as Fig. 33 and 34.

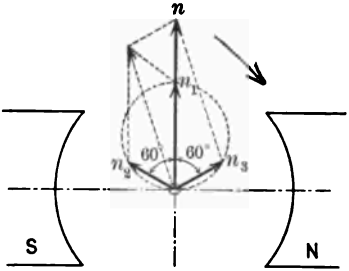

FIG. 36. The vector n represents the resultant armature reaction of the three phases of Fig. 33, 34 and 35.

we find that the vector *On* represents the total ampere-turns of all phases, both as to value and direction. The vector *On* remains the same length and the same direction, for all positions of the armature, provided *I* remains in phase with E_i. To illustrate this, Fig. 37 is drawn, representing n_1, n_2, and n_3 at an instant 30 electrical degrees or $\frac{1}{12}$ period after Fig. 36; at this instant, n_2 has increased, n_1 has decreased, and n_3 has become zero (current in Phase 3 is zero at this instant). When *I* lags $\theta°$ behind E_i (power-factor of entire circuit, including armature winding, equal to $\cos\theta$), conditions are as represented by Fig. 38. The vector *On* represents total ampere-turns of the armature, as to both value and direction. The armature currents produce a constant counter m.m.f. of *Od* ampere-turns in the magnetic circuit, reducing the flux, and E_i. There is a constant cross-magnetizing m.m.f. of *Oc* ampere-turns, distorting the flux. The main difference between the armature reaction of the polyphase and the single-phase alternator is seen to be in the constancy of the former and the double-frequency cyclic variation of the latter.

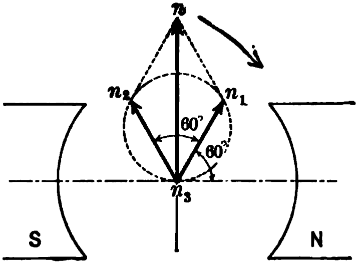

FIG. 37. The vector *n* represents the resultant armature reaction of the three-phase generator at an instant 30° later than that of Fig. 36, when n_3 has become zero. Note that the value of *n* has remained unchanged.

Prob. 28-1. Each phase of the winding in Fig. 33 to 38 has 10 turns and carries an harmonic current whose effective value is 10 amperes. Calculate the maximum ampere-turns of one phase, and the total ampere-turns of all three phases together for the position shown in Fig. 36. Repeat for Fig. 37. Compare these total values and note how they agree with statements in the text.

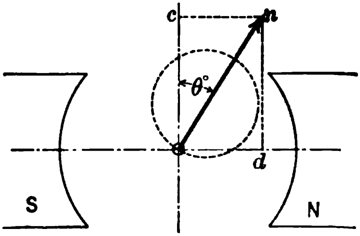

FIG. 38. The vector *n* represents the constant armature reactions of a three-phase generator, when the power-factor is $\cos\theta$.

Prob. 29-1. Can you make a general proof that the total magnetic effect (n) of all phases of a three-phase winding is constant? What is the ratio between the total effect of all phases, and the maximum effect of one phase, in a three-phase alternator?

Prob. 30-1. Draw vector diagrams for a two-phase alternator, after the manner of Fig. 33 to 37.

11. Synchronous Impedance. The effects of armature self-inductance and of armature reaction cannot be measured separately by any convenient commercial test. In rough calculations of voltage regulation of an alternator (by the so-called E.M.F. Method), the effects of these two reactions upon the terminal voltage are combined in a factor known as the "synchronous reactance (X)." The "synchronous impedance" is a factor which measures the combined effect of armature resistance and synchronous reactance, upon the terminal voltage.

Synchronous impedance is always measured (as the name implies) while the alternator (or synchronous motor) is rotating at synchronous speed, i.e., at rated speed. It has little relation to the impedance of the same armature as measured at standstill, even with currents of rated frequency. The armature is short-circuited through an ammeter connected directly across the terminals, the field current being adjusted to a value low enough to produce approximately full-load amperes through the short-circuited armature. After measuring I, the short-circuit is opened, and the open-circuit voltage is measured. This open-circuit voltage must, therefore, be the voltage required to force the full-load current through the armature alone. Then, for the same field current and (rated) frequency:

Synchronous impedance (per phase) =

$$\frac{\text{open-circuit volts, per phase}}{\text{short-circuit amperes, per phase}} = Z.$$

Then we measure the resistance of each phase of the winding, usually with direct current, using the ammeter-voltmeter method. Increasing this by 15 per cent to allow for eddy-currents and skin-effects in windings, we get r, the alternating-current resistance of one phase. From this we get:

Synchronous reactance (per phase) =

$$\sqrt{\underset{\text{(per phase)}}{(\text{Synchronous impedance})^2} - \underset{\text{(per phase)}}{(\text{a-c. resistance})^2}} = X$$

Example 4. A Y-connected alternator rated to deliver 5000 kv-a. at 6600 volts, 60 cycles, yields the following data when tested for synchronous impedance:

Short-circuit armature current = 438 amperes per terminal.

Open-circuit volts between terminals = 2790 for same field current and rated frequency.

Measured resistance of the armature (hot) between any two terminals = 0.0962 ohm.

From these we calculate as follows:

Volts induced in each phase of winding $= \dfrac{2790}{\sqrt{3}} = 1610.$

Amperes in each phase of winding = 438.

Synchronous impedance of each phase of winding $= \dfrac{1610}{438}$ $= 3.68$ ohms.

Direct-current resistance of each phase $= \dfrac{0.0962}{2} = 0.0481$ ohm.
(two phases in series between terminals)

Alternating-current resistance per phase = 115 per cent of 0.0481 = 0.0554 ohms.

Synchronous reactance per phase $= \sqrt{(3.68)^2 - (0.0554)^2}$
$= 3.678$ ohms.

Prob. 31-1. The following test readings show the relation between open-circuit volts and field amperes for a delta-connected alternator rated 2140 kv-a. three-phase 50 cycles 6300 volts 300 r.p.m., and operated at rated speed (see Electric Journal, Nov. 1913, page 1189):

Volts between terminals.	Field amperes.	When the armature is short-circuited the following readings are taken, also at rated speed.		
		Short-circuit amperes per terminal.	Field amperes.	Short-circuit loss, kw.
0	0.0			
1200	13.2			
2400	26.7			
3600	40.5			
4800	55.5			
6000	72.5	0	0.0	0.0
6300	77.1	99	15.3	3.5
6930	90.0	195	30.0	13.5
7800	118.0	294	46.0	30.0
8550	155.0	390	60.5	53.5

The armature resistance per phase as measured by direct current is 0.1025 ohm at 25° C. The field resistance is 0.718 ohm at 25° C. From these data, draw curves with field current as abscissas, open-circuit volts per phase and short-circuit amperes per phase as ordinates. Calculate the synchronous impedance per phase from these curves, corresponding to each value of current observed in the short-circuit test. Does the synchronous impedance appear to be a constant quantity within the range of these observations? Calculate also the synchronous reactance.

12. Calculation of Voltage Regulation by Synchronous Impedance Method. Let it be required to calculate the voltage regulation of the alternator of Example 4, from rated full-load kv-a. at 80 per cent power-factor to zero-load. First consider the case of a lagging current, as in Fig. 39. Lay out a reference vector *OI* of any length, representing the current in one phase. Then lay out *Od*, leading *OI* by $\theta°$ (= angle whose cosine is 0.80, the power-factor). The length of *Od* represents the e.m.f. required to be produced at the terminal of each phase of the winding, which in this case is 3750 volts. Then from *d* lay out the vector *dc* parallel to *OI* and in the same direction, and of length representing (rI) volts. This is the e.m.f. required to overcome the resistance reaction in each phase.

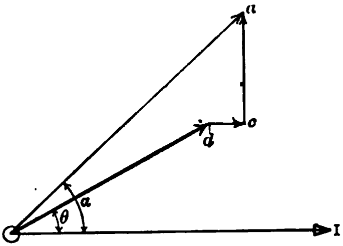

FIG. 39. The vector *Oa* represents the e.m.f. which must be generated to supply a terminal voltage of *Od*, at a power-factor of $\cos\theta$.

$$Oa = Od \oplus dc \oplus ca.$$

$$I \text{ (per phase)} = \frac{5000 \text{ kv-a.}}{\sqrt{3} \times 6.6 \text{ kv.}} = 437 \text{ amperes.}$$

$$rI \text{ (per phase)} = 0.0554 \times 437 = 24.2 \text{ volts.}$$

Then from *c* we lay out *ca* in a direction leading *OI* by 90°, and of a length proportional to (XI), the e.m.f. required to overcome the reactions represented by the synchronous reactance.

$$XI \text{ (per phase)} = 3.678 \times 437 = 1608 \text{ volts.}$$

The resultant vector Oa represents the e.m.f. that should be generated by a field excitation sufficient to overcome the reactions or voltage drops due to resistance, inductance and armature reaction, and still have Od volts remaining between the terminals. We calculate Oa as follows:

Component of Oa in phase with $OI = Ob + dc =$

$$(Od \cos\theta + rI) = (3750 \times 0.8) + 25 = 3024 \text{ volts.}$$

Component of Oa in quadrature with $OI = bd + ca =$

$$(Od \sin\theta + XI) = (3750 \times 0.6) + 1608 = 3858 \text{ volts.}$$

$$Oa = \sqrt{(3024)^2 + (3858)^2} = 4900 \text{ volts.}$$

If the field current and speed are kept constant, presumably we should get this voltage between the terminals of each phase when the current is reduced to zero, since then there would be no reactions of any sort within the armature. As the winding is Y-connected, the pressure between terminals at zero-load should be $\sqrt{3} \times 4900 = 8500$ volts. The rise in voltage from full-load to zero-load would be $(8500 - 6600) = 1900$. Then:

Voltage regulation at 80 per cent power-factor
by synchronous impedance method

$$= \frac{1900}{6600} = 28.8 \text{ per cent.}$$

Fig. 40 shows the method of computing the voltage regulation for a leading current.

When tested under actual full-load conditions, alternators nearly always show a lower or better voltage regulation than is obtained by calculation according to the synchronous impedance method explained above. This may be due to a number of causes. X is computed from short-circuit readings taken under conditions of low flux densities in the iron and very low power-factor, yet we assume it to retain the same value at much higher power-factors, and much higher degrees of saturation necessary to produce full rated voltage. Moreover, in our diagrams (Fig. 39 and 40) we draw the synchronous reactance e.m.f. (XI) perpendicular to I just as if it were all due to genuine inductive reactance; nevertheless we know that the change of E_i due to field-weakening effect of armature reaction is in phase with E_i, and only the amount of it depends upon the phase of I. However, this method is very easily applied, the data are easy to obtain, and the error is on the safe side. The student should be thoroughly familiar with this method because it applies to transmission lines even better than to alternators and is theoretically correct in all respects.

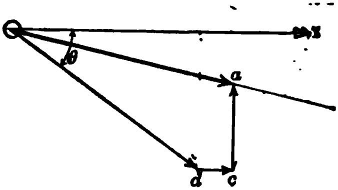

FIG. 40. The current leads the voltage by $\theta°$. The generated e.m.f. represented by Oa may be less than the terminal voltage Od.

Prob. 32–1. Calculate (by the synchronous impedance method) the voltage regulation of the alternator specified in Prob. 31–1, when delivering rated full-load output at 100 per cent power-factor. Draw complete vector diagram to illustrate your solution.

Prob. 33–1. (*a*) Solve Problem 32–1 using a load of 80 per cent lagging power-factor.

(*b*) 80 per cent leading power-factor.

Prob. 34–1. What would be the voltage regulation in Problem 33–1 if the resistance drop were neglected, and the value of synchronous impedance were used in place of the value of synchronous reactance, thus eliminating the vector *dc* and slightly lengthening *ca*, in Fig. 39 and 40? (The Ir drop in this armature is really much smaller than is usual.)

Prob. 35–1. Using the synchronous impedance method, calculate the terminal voltage of the alternator specified in Problem 31–1, when the current output is reduced to one-half of rated full-load value, while keeping the field excitation constant at full-load value. Power-factor of load is constant at 80 per cent.

13. Calculation of Voltage Regulation by A.I.E.E. Method. The following method is prescribed by the

Standardization Rules of the A.I.E.E. (see Proceedings A.I.E.E., Aug. 1914), because experience proves that values of regulation obtained by it are in close agreement with those obtained by load test. The most trustworthy value of voltage regulation is obtained by taking real loads from the generator, but on account of the amount of power required this is not practicable with very large machines.

This method consists in computing the regulation from experimental data of the open-circuit saturation curve and the zero power-factor saturation curve. The latter curve, or one approximating very closely to it, can be obtained by supplying full-load current of the generator to a load of idle-running synchronous motors adjusted to take lagging current at lowest possible power-factor. The power-factor under these conditions is very low and the load saturation curve approaches very closely to what it would be at zero power-factor with the same current delivered. From this curve and the open-circuit curve, points for the load saturation curve for any power-factor can be obtained by means of

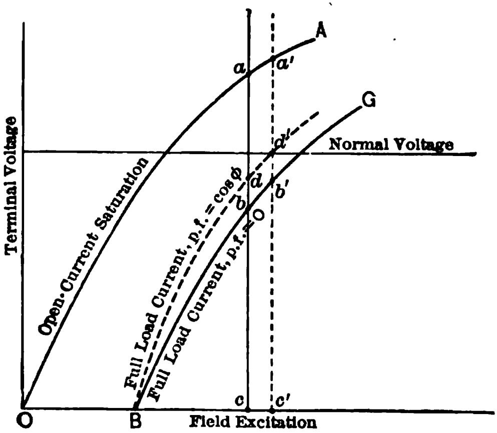

FIG. 41. Curves *OA* and *BG* are drawn from data obtained on a no-load run and a full-load zero power-factor run. The curve *Bd'* is plotted from values obtained from *OA* and *BG* by means of Fig. 42.

vector diagrams. From the saturation curves the voltage regulation may be calculated.

The alternator is operated at constant rated frequency with I equal to zero, and the voltage is measured corresponding to various values of field current. These are plotted as "open-circuit saturation" curve *OA*, Fig. 41. The armature is then connected to a load capable of taking the full rated-load current of the alternator at or near zero power-factor. Power-factors of 10 per cent or 20 per cent are practically equivalent to zero, as shown by means of Fig. 32. The current and power-factor are kept constant while the field current is varied, and the corresponding values of volts at terminals are plotted as curve *BG*, Fig. 41, usually called the "load saturation curve" at zero power-factor.

At any given field excitation *Oc*, the voltage that would be induced on open circuit is *ac*. The terminal voltage at zero power-factor and full-load current is *bc*, and the apparent internal drop is *ab*. The terminal voltage *dc* with full-load current of any other power-factor can then be found by drawing an e.m.f. diagram as in Fig. 42, where ϕ is an angle the cosine of which is the power-factor of the load. The line *be* is the resistance drop (*IR*) in the armature winding, *ba* is the total internal drop and *ac* is the total induced voltage, *ba* and *ac* being laid off to correspond with the values obtained from Fig. 41. The terminal voltage at power-factor cos ϕ is then *cb* of Fig. 42, which, laid off in Fig. 41, locates the point *d*. By finding a number of such points, the curve *Bdd'* for power-factor cos ϕ is obtained, and we have:

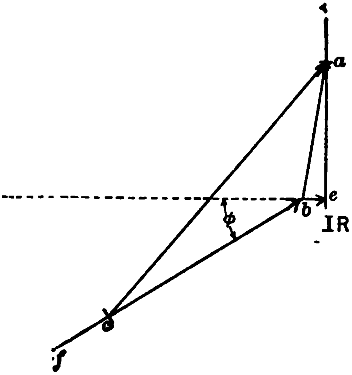

FIG. 42. Lay off *be*. Draw the vertical line from *e*. Draw *bf* at angle ϕ to *be* and swing an arc with a radius equal to *ab*, Fig. 41, until it cuts the vertical at *a*. From *a* swing an arc with a radius of *ac*, Fig. 41, to cut *bf* at *c*.

$$\textbf{Regulation} \text{ in per cent (at power-factor } \cos\phi) = \frac{a'd'}{d'c'} \times 100,$$

since *a'd'* is the rise in voltage when we throw off the rated full-load current at rated voltage *c'd'* and power-factor cos ϕ.

Generally the resistance drop (RI) may be neglected, as it has very little influence on the regulation, except in very low speed machines where the armature resistance is relatively high, or in some cases where the regulation at unity power-factor is being estimated; for low power-factors, its effect is negligible in practically all cases. If resistance is neglected, the simpler e.m.f. diagram of Fig. 43 may be used to obtain points on the load saturation curve for the power-factor under consideration. Where it is not possible to obtain by test a zero power-factor saturation curve, this curve may be estimated closely from open-circuit and short-circuit curves, by a method explained in Proceedings A.I.E.E., Aug. 1914, page 1263.

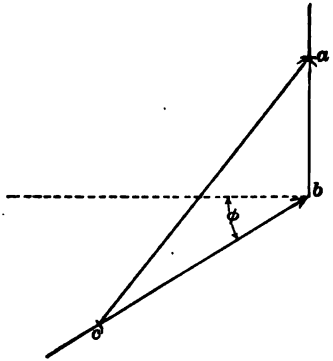

FIG. 43. The diagram of Fig. 42 takes this simple form when the resistance drop *be* is not considered.

Example 5. An alternator has saturation curves as shown in Fig. 44, which are reported in Trans. A.I.E.E., 1908, page 1065. Calculate the voltage regulation of this generator for a power-factor of unity.

From full-load curve at 1.00 power-factor in Fig. 44, we find that there must be 235 amperes in the field to produce the normal voltage of 12,000 volts.

From the no-load saturation curve we find that these 235 amperes in the field will produce a terminal voltage of 12,550 volts, when the load is thrown off. The rise in voltage from full-load to no-load is 12,550 − 12,000 = 550.

$$\text{The regulation at 1.00 power-factor} = \frac{550}{12{,}000} = 4.6 \text{ per cent.}$$

Example 6. Calculate the regulation of the above generator for a power-factor of 0.70. Neglect armature resistance.

Select a field current of 300 amperes as a value which will probably produce full-load voltage of about 12,000 volts at this power-factor.

The difference in voltage between no-load and full-load at zero power-factor is represented by the distance *ab* in Fig. 44. Lay off this distance as *ab* in Fig. 45. Draw the line *bc* of indefinite length at an angle of 45.5° to *bx*. (45.5° is the angle of which

0.70 is the cosine.) In Fig. 44, the length *ac* is the no-load voltage for this field current. Take off this distance in Fig. 44 with compasses and draw an arc in Fig. 45, with the point *a* as the center, until it cuts the line *cb* at *c*. The line *cb* represents the full-load voltage at a power-factor of 70 per cent. Lay this distance off from *c* to *d* on the line *ca* of Fig. 44. Draw a line through *d* parallel to the curve for the full-load current at zero power-factor, and thus determine the point *d′* which shows what field current is needed to produce a full-load voltage of 12,000 volts at 70 per cent power-factor. The corresponding point *a′* for the same field current on the no-load curve shows the value to which this terminal voltage will rise when the load is taken off and this field current continued.

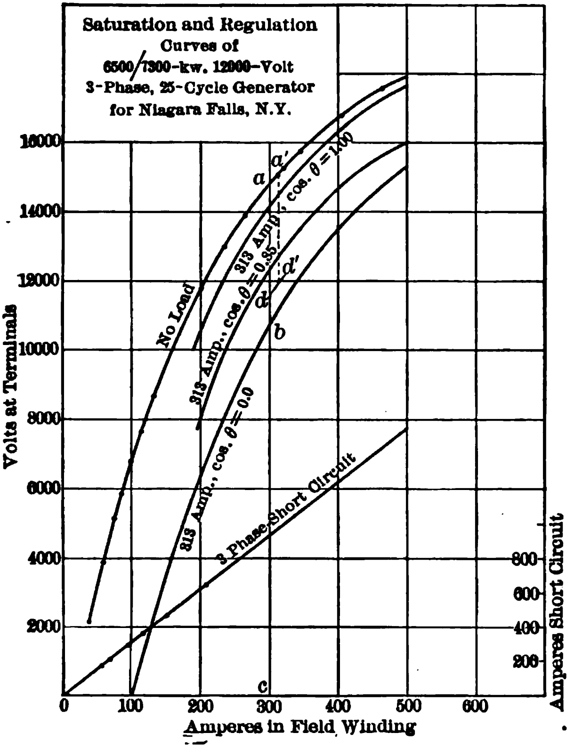

FIG. 44. Typical test curves for an alternating-current generator.

The distance $a'd'$, therefore, represents the voltage rise from full-load to no-load at this power-factor, equal to 14,600 – 12,000, or 2600 volts.

Regulation at 70 per cent power-factor $= \dfrac{a'd'}{12,000} = \dfrac{2600}{12,000} = 21.7$ per cent.

Prob. 36–1. Calculate the voltage regulation of the alternator of Example 5 for a power-factor of 85 per cent from these curves.

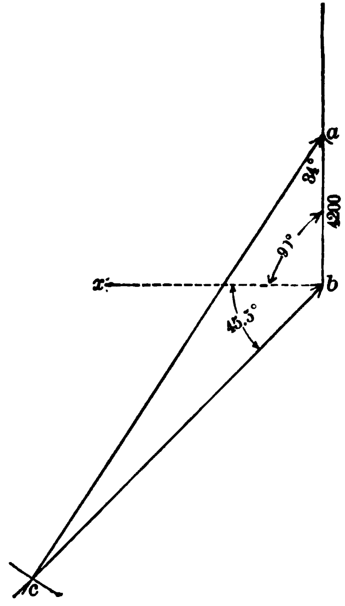

FIG. 45. The line bc represents the terminal voltage of the generator at full-load at 70 per cent power-factor. The scale is the same as in Fig. 44.

Prob. 37–1. From the data in Fig. 44, calculate the synchronous impedance of this alternator using the data of the short-circuit test and the open-circuit saturation curve. Calculate the voltage regulation by the synchronous impedance method for 85 per cent power-factor and for 100 per cent power-factor. Neglect the resistance of the armature. Compare with the results of Problem 36–1 and Example 5.

Prob. 38–1. The data in the table below for curve of full-load saturation at zero power-factor were taken on the alternator specified in Problem 31–1. Draw the zero-load and the full-load zero power-factor saturation curves for this alternator. Calculate enough of the saturation curve for 80 per cent power-factor to enable you to calculate the per cent voltage regulation at this power-factor, by the A.I.E.E. method. Compare with the result of Problem 33–1.

Prob. 39–1. Calculate the voltage regulation of the alternator specified in Problems 31–1 and 38–1, by the A.I.E.E. method, at a power-factor of 100 per cent, and compare the result with that obtained in Problem 31–1 by the synchronous impedance method.

Armature volts.	Armature amps. per phase.	Field amps.
3600	196.2	71.0
4800	196.2	86.5
6000	196.2	106.0
6300	196.5	112.0
7200	196.0	138.5
7590	195.8	155.0

14. Automatic Voltage Regulation for Generators. It is not practicable nor economical to build alternators (and therefore not wise to specify them) to have a voltage regulation so good that they can carry the ordinary fluctuating central station load without permitting the voltage to vary beyond the limits of good service. Very good voltage regulation increases the cost of the alternator unwarrantably, and is likely to be obtained by sacrifice of other important features, such as efficiency. Also good voltage regulation requires the inductive reactance and the armature reaction of the alternator armature to be low. When short-circuits occur on such alternators, the current runs up to very high values, and the magnetic forces exerted between the parts of the winding become great enough in many cases to twist them out of shape and injure or ruin the insulation. As a precaution against such injuries, the armature reaction and inductive reactance of large alternators is usually made so high that some adjustment of the field excitation is necessary to keep the voltage from fluctuating beyond reasonable bounds as the load changes.

The adjustment of field current may of course be made **by hand,** which would require constant attention by the station operator. This is objectionable not only because it is more expensive than automatic control, but also because it can never be as good as the automatic method. The fluctuations are too rapid to be followed successfully by eye and by hand.

The field current may be controlled **automatically** by means of a "voltage regulator," which is usually of the Tirrill Regulator type. Such regulators usually act to keep the voltage sensibly constant at the generator terminals or bus-bars. But regulators may be constructed to raise the terminal voltage automatically as the load increases, or to **compound the generator.** Compounding is sometimes used, sufficient to compensate the voltage drop on feeders and keep the pressure constant at some point distant from the station.

The Tirrill regulator is a rather complicated device involving the interaction of solenoids, differential magnets, levers and contacts, which it is not in the province of this book to describe, as no new principles are to be learned thereby. Station operatives and specialists who must understand the operation and adjustment of this device, should procure the Bulletins and Instruction Sheets which the General Electric Company publishes concerning it. The principle of operation is simple. The regulator maintains the desired alternator voltage by rapidly opening and closing a shunt circuit across the field-rheostat of the exciter (or direct-current dynamo which supplies the field current for the alternator). The exciter field-rheostat is first turned in until the exciter voltage is greatly reduced and the regulator circuit is then closed. Whenever the alternator voltage is below the value which the regulator is adjusted to maintain, the vibrating contacts come together and short-circuit the field-rheostat of the exciter. The voltage of the exciter and of the generator immediately rise. When it has reached the predetermined value, the regulator contacts are automatically opened and the field current of the exciter must again pass through the rheostat. The resulting reduction in voltage is checked at once by the closing of the regulator contacts, which continue to vibrate in this manner (several times per second) and keep the generator voltage within the desired limits.

15. Excitation for Alternators. An old practice of having a separate exciter for each alternator has been abandoned in plants having several alternators. The excitation is usually furnished by two or more flat-compounded direct-current generators, rated 125 or 250 volts, each connected through its own switching, measuring and controlling equipment to a set of exciter bus-bars. Shunt-wound exciters are preferable where Tirrill regulators are used, also where storage batteries are used. All alternators take field current from these common exciter busses; thus any alternator may be connected to any exciter. The exciter busses usually furnish current also for operating the motors used to open and close oil-switches, adjust the governors of the engines, or control the field rheostats, and sometimes also for lighting the plant.

To insure that the plant shall never be without a source of direct current to build up or maintain the voltage of the alternators, it is usual to keep a storage battery ready to be connected to the exciter busses. It is then possible to drive some of the exciter generators by means of induction motors or synchronous motors supplied with alternating current by the main generators. If there is no storage battery, the exciters are driven by separate steam engines or waterwheels, elaborate precautions often being taken to avoid the possibility of these being stalled. However, it is said that the cost of operating a motor-driven exciter is about one-half the cost of operating an engine-driven exciter.

A Tirrill voltage regulator, by adjusting the voltage of the exciter bus, raises simultaneously the voltage of all generators which are operating in parallel, and thus avoids the troublesome cross-currents between them which may result when their excitations are adjusted independently. When several exciters operate in parallel the regulator is equipped with extra relays and "equalizer rheostats," to keep them adjusted to each other.

16. Load Capacity of an Alternator. The load or output of a direct-current generator is measured in terms of power, and is expressed in watts or kilowatts. The watts or kilowatts output of an **alternating-current** generator increases in direct proportion to the **effective voltage,** to the **effective current,** and to the **power-factor.** We cannot increase the armature current beyond a certain value without exceeding a safe temperature for the armature insulation. To raise the voltage we must increase the speed, the flux, or the number of inductors or turns in series in the armature. After the machine is built, the speed cannot be raised because constant frequency is required. In designing the alternator, the maximum speed is fixed by mechanical limitations (such as stresses due to centrifugal force, balancing of rotor, bearings, and lubrication), or by the fact that there is a most economical speed for the prime mover which cannot be exceeded without prohibitive sacrifice. With a given amount of armature-copper and a given slot-space to put it in, no advantage is gained by increasing the number of turns. Because if we double the turns, the sectional area of the conductor cannot be more than half as great; and for the same maximum I^2R loss as before, this would reduce the maximum allowable current to one-half the former maximum, which would exactly offset the double voltage and leave the capacity unchanged. The flux cannot be increased in a given amount of iron without increasing the flux density. This causes a relatively great increase of core-losses, and is, therefore, limited by the maximum allowable temperature. Moreover, increase of flux requires large increase of field current, and this is limited by heating the field insulation.

We see, therefore, that the current and pressure factors of the power output are limited in the alternator itself, by the fact that they cannot exceed fairly definite limits without injuring the machine. But when the alternator is delivering its maximum amperes and maximum volts, the **real power** or output in **watts** may have any value from the maximum (product volts times amperes = "volt-amperes") at 100 per cent power-factor, to nothing at zero power-factor. As the power-factor of the load depends altogether upon the external circuit and not at all upon the alternator, it is not fair to the alternator to rate its maximum capacity in kilowatts. It is always rated, therefore, in terms of volt-amperes or kilovolt-amperes (kv-a.). The capacity in kilowatts would be the same as the capacity in kilovolt-amperes, if the power-factor of the load were unity. At 50 per cent power-factor, the kilowatt capacity would be reduced just one-half, whereas the kilovolt-ampere capacity would be the same as before. The design of the field winding should be sufficiently liberal to permit, without overheating, a field current which can overcome the demagnetizing effect of reactive lagging components of armature current at any ordinary power-factor, and maintain rated voltage at the terminals.

The **nature of the duty affects the capacity** of an alternator or of any electrical machine. We must distinguish between steady load and fluctuating or intermittent load. It is common practice to rate alternators according to the kilovolt-amperes that they can deliver constantly for an indefinite time without overheating. Ratings for intermittent load, which are higher than the continuous rating, are usually employed for railway and crane motors and the like, but not for alternators.

The amount of kilovolt-ampere capacity of alternators required to supply a given load will depend, therefore, somewhat on the shape of the load-curve and the duration of peak loads. It depends also upon the power-factor of the load; for, in order to supply the same kilowatts with half as great power-factor, it requires twice as many amperes at constant voltage, or twice as many kilovolt-amperes. It is, therefore, profitable to select and adjust the apparatus connected to the station, in such a way as to make the power-factor as high as possible. Induction motors should not be too large for their loads, as a lightly loaded induction motor has a low power-factor. A good proportion of the motor load should be in synchronous motors which may be and should be adjusted to make the line power-factor very high. Service to small customers should be grouped as much as possible on larger transformers, avoiding the use of numerous small transformers. By making the power-factor as high as possible, the size (kilovolt-ampere capacity) and cost of generators required to supply a given amount of effective power is kept as low as possible. The method for determining kilovolt-ampere capacity of generator required to carry a combination of loads is explained fully in Articles 38 and 45. It is necessary and sufficient to know any two of these three factors, for each load; kilowatts, kilovolt-amperes, power-factor. Whether the loads are connected in series, in parallel, or in series-parallel grouping makes no difference in the total kilovolt-amperes, although it may affect the current and e.m.f. in parts of the circuit.

Prob. 40–1. What size alternator would be required to supply loads aggregating 800 kw. steadily at 80 per cent power-factor? What would be the input in horse-power of the prime mover to drive this generator at this load, if the efficiency of the latter at this load is 94 per cent?

Prob. 41–1. Show that where the load on an alternator fluctuates rapidly, while the voltage is kept constant, the rated kv-a. capacity should be approximately equal to the effective or square-root-mean-square value of the kv-a. load-curve.

Prob. 42–1. A shop having its own generating plant of 500 kv-a. rated capacity uses only 300 kw., at 81 per cent power-factor. Another nearby shop, needing more power, offers to purchase 100 kw. at 78 per cent power-factor. Can this generating plant supply both shops together? If not how many kv-a. and kw. can be sold to the second shop at 78 per cent power-factor, without exceeding rated full-load of the plant?

Prob. 43–1. How many kilowatts in incandescent lamps may be added to a 250 kv-a. alternator which is already delivering 160 kv-a. to induction motors at 75 per cent power-factor?

Prob. 44–1. Draw a curve, with power-factor as abscissas and real power output (kw.) as ordinates, for a 400 kv-a. plant operating at its full rated capacity.

Prob. 45–1. The total cost of a 400 kv-a. plant in Massachusetts, using reciprocating steam engines, and including all auxiliaries and incidentals except the building, averages $84.00 per kv-a. capacity. Draw a curve with power-factor as abscissas and cost per kilowatt capacity as ordinates.

17. Short-circuits on Alternators.[*]

When a short circuit occurs close to the terminals of an alternator which is operating at rated voltage and speed, the armature current instantly rises to an effective value of from 10 to 50 times rated full-load current. It then decreases rapidly (in from 0.2 to 2.0 seconds), to a steady value of from 1.5 to 3.0 times rated full-load current. These large initial surges of current produce very severe mechanical stresses within the windings, tending to twist them out of shape and to damage the insulation. The forces between coils or parts of the winding vary as the square of the current flowing in them, therefore, the forces acting at the instant of short circuit are from 100 to 2500 times as great as those acting at rated full-load.

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[*] Much of the data in this article is taken from *General Electric Review*, Feb. 1909, and *Electric Journal*, Nov. 1913.

Obviously, we must either allow a tremendous factor of safety in the mechanical construction of the alternator, or adopt means to limit the amount of current that may flow through a short-circuit. In practice, both methods are used. That is:

(1) The windings are braced securely by clamps, particularly at the ends of the coils where they are not supported by the slots. Fig. 46 shows the end-windings of a turbo-alternator so braced.

(2) Reactance coils are connected in series with the generator leads which go to the switchboard. Fig. 170, "First Course," shows a reactance coil used for this purpose. They usually have cores of concrete and air; iron cores would increase the power loss in the coil (by hysteresis and eddy currents), and the flux would not increase any more than with air core when the current increases on short circuit to values which would oversaturate the iron unless a great quantity of it were used. These coils are usually placed as close as possible to the generator.

(3) Large generators are designed purposely to have a bad voltage regulation, so that the excessive currents on short-circuit shall limit themselves by cutting down the voltage.

The first rush of current on short circuit is very great in relation to the final short-circuit current, because the armature reaction is slow to take effect. During the first few cycles after short circuit, the current is opposed only by the resistance and real or inherent reactance of the armature (the counter e.m.f. induced in the armature conductors by the local flux due to the current they carry). However, as the resistance is small in comparison with the reactance, the short-circuit current lags almost 90° behind the induced e.m.f. and the armature ampere-turns strongly oppose the passage through the armature core of flux due to the main field winding. In the course of a short time, but not instantaneously, the useful flux is greatly reduced by this demagnetizing action — often

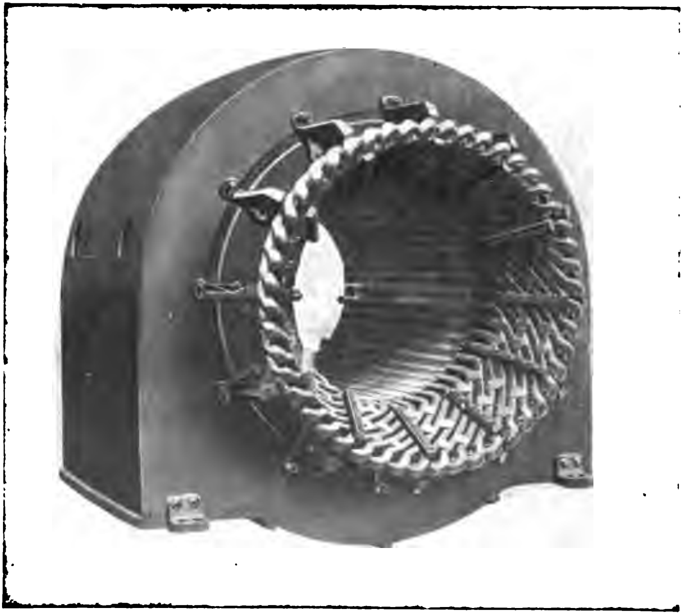

FIG. 46. Armature with end-bells removed showing method of bracing the ends of coils. *Westinghouse Elect. & Mfg. Co.*

to only a few per cent of its normal or full-load value. While the flux in the field cores is being reduced, an e.m.f. is induced in the field coils, acting in a direction to maintain the flux — that is, in the same direction as the normal field current. The transient increase of field current so produced while armature reaction is taking effect, is frequently enough to open the circuit breakers in the field circuit, which helps to relieve the short-circuit.

The resistance of the armature is kept low in order that the efficiency may be high. A common value is 1 per cent, which means that the IR voltage consumed in overcoming resistance, at rated full-load current, is equal to 1 per cent of the terminal voltage at full-load. Consequently, the value and damaging effects of the first rush of current must be limited by increasing the inductance or reactance of the armature winding. The reactance is proportional to:

(*a*) The frequency, directly. (If frequency is doubled, armature reactance is doubled.)

(*b*) The square of the number of conductors per slot on the armature.

(*c*) A factor which depends upon the shape and number of the slots, and their arrangement.

Generators of the largest sizes (in which short-circuit currents are enormously powerful and dangerous) are usually driven by steam turbines, which must be of relatively high speed in order to be economical. Although the frequency is the same as in slow-speed engine-driven alternators, the number of slots is less, and the reactance much less. In an engine-driven alternator the reactance might be 10 per cent (i.e., reactance voltage equal to 10 per cent of terminal voltage, when rated full-load current flows). In a turbo-alternator for the same rated output the reactance voltage might be only one-quarter as large, or 2.5 per cent. If we neglect the (very slight) effect of resistance in determining the short-circuit current, we see that the current in this slow-speed engine-driven alternator will surge up to 10 times normal full-load current at the moment of short-circuit (until the reactance volts equals the generated volts), whereas the current in the turbo-alternator will surge up to 40 times rated current $= \left(\frac{\text{100 per cent of generated voltage}}{\text{2.5 per cent of generated voltage}}\right)$. However, as soon as these excessive currents commence to grow, the generated voltage commences to be reduced by the armature reaction, and the short-circuit current soon settles down to a value which depends upon the synchronous impedance, or upon the voltage regulation of the alternator.

Current-limiting reactances are usually designed to have a reactance of from 3 to 6 per cent (which means a voltage drop across them of from 3 to 6 per cent of terminal voltage when full-load current flows). They do not have much effect upon the voltage regulation of the generating unit, because the regulation depends principally upon the armature reaction or upon the synchronous impedance, which is large compared with these values of reactance.

The short circuit is relieved by an oil-switch, or circuit-breaker, having the contacts immersed in oil. This breaker opened automatically by the action of a relay which is connected in the leads between generator and switchboard. If the relay is set to trip quickly during the first surge of current, great forces are generated at the switch break, often wrecking it. If the relay is set to delay action, perhaps until after the current has fallen, large forces are exerted upon the armature windings and connecting cables, distorting them and often tearing them from their fastenings. The action of switches and relays will be taken up later.

Example 7. What is the impedance (in per cent) of the armature of an alternator which has 6 per cent reactance and 2 per cent resistance?

$$\begin{aligned}\text{Impedance} &= \sqrt{x^2 + R^2} \\ &= \sqrt{40} \\ &\doteq 6.32 \text{ per cent.}\end{aligned}$$

Prob. 46–1. (*a*) What would be the inherent impedance in per cent, for an alternator having 10 per cent reactance and 1 per cent resistance?

(*b*) For an alternator having 2.5 per cent reactance and 1 per cent resistance?

Prob. 47–1. (*a*) What would be the power-factor and angle of phase difference between the induced e.m.f. and the current that flows immediately after a short circuit on the alternator specified in part (*a*) of Problem 46–1?

(*b*) Of the alternator specified in part (*b*) of Problem 46–1?

Prob. 48–1. (*a*) What per cent of rated full-load current would flow immediately after short circuit, in the alternator of part (*a*) Problem 46–1?

(*b*) In the alternator of part (*b*) Problem 46–1? Calculate this current from the per cent of impedance, and compare with the results calculated above in the text when considering only the reactance.

Prob. 49–1. From the power-factor and the per cent of rated current on short circuit, calculate the increase of torque resisting rotation of the alternators of Problem 46–1 (*a*) and (*b*) at the moment of short circuit, in per cent of torque due to non-inductive full-load on the alternator. Note that the speed remains sensibly constant.

SUMMARY OF CHAPTER I

LOADING A GENERATOR:

(a) CAUSES PARTS OF IT TO HEAT UP. The temperature rise limits the load which may be applied.

(b) CHANGES THE EFFICIENCY, first raising it, then lowering it as the load is greatly increased. Extra high efficiency is usually expensive unless the cost of energy is high and the supply limited.

(c) LOWERS THE TERMINAL VOLTAGE because part of the e.m.f. generated is used to overcome the armature resistance and armature reactance and because of armature reaction.

ARMATURE REACTION due to the current flowing in the armature coils of a generator:

(a) DISTORTS THE FLUX distribution of the poles by crowding the magnetic lines to the trailing pole tip.

(b) EITHER WEAKENS OR STRENGTHENS the poles. It may do both in a single-phase machine, alternating at a frequency double that of the e.m.f.

THE SYNCHRONOUS IMPEDANCE of a generator is a term used to denote the combined effect of armature reactance and resistance and armature reaction upon the terminal voltage of a generator. It equals

$$\frac{\text{open-circuit volts, per phase}}{\text{short-circuit amperes, per phase}},$$

both values corresponding to the same field excitation and speed.

THE VOLTAGE REGULATION of a generator may be calculated approximately by the Synchronous Impedance Method. This consists of combining vectorially the terminal voltage at full-load with the reactance drop and the resistance drop of the armature at given power-factor. The result is approximately the terminal voltage at no-load. Then:

$$\text{Regulation (per cent)} = \frac{(\text{no-load volts}) - (\text{full-load volts})}{(\text{full-load volts})} \times 100\%.$$

This method of calculation gives a slightly higher or poorer regulation than is obtained by actually loading the alternator.

THE A.I.E.E. METHOD of calculating the regulation consists of finding graphically the no-load voltage for a field excitation which will produce the required terminal voltage under full-load at given power-factor. The saturation curves at no-load and full-load (current) at zero power-factor are used in this method. The results thus obtained are very close to the values measured on tests.

THE TIRRILL REGULATOR automatically regulates the terminal voltage of a-c. generators by rapidly varying the field strength to make the required increase and decrease of the voltage. Good inherent regulation is undesirable in large machines because the low armature impedance required would not sufficiently limit the value of currents through the machine whenever it may be short-circuited.

THE ALTERNATOR FIELD COILS are excited from busbars fed by direct-current generators. Storage batteries are generally employed as a reserve in case of failure of the d-c. generators.

THE POWER CAPACITY of an alternator depends upon the current it must deliver to supply the power. To keep this current as low as possible for a given amount of power, the power-factor should be high.

LARGE SHORT-CIRCUIT CURRENTS exert tremendous mechanical forces upon the parts of an alternator. Therefore, the construction of all parts is rugged, and the impedance of the armature is high to prevent too large currents. Separate current-limiting reactances are also often connected between terminals and lines.

PROBLEMS ON CHAPTER I

Prob. 50–1. A certain alternator operates steadily at rated full-load with the temperature of its armature constant, and 45° C. above the room temperature, which is 20° C.

(*a*) To what temperature will the armature rise when the room temperature increases to 40° C., and load remains constant?

(*b*) If the actual temperature of the armature must not be allowed to be above 75° C., by what percentage must the current output be reduced below rated value to avoid overheating? Assume that half of the full-load losses are constant, the other half are variable and proportional to the square of the load or current output. The excess of machine-temperature over room temperature is directly proportional to the total losses.

Prob. 51–1. An alternator delivers 100 kw. for 8 hours per day, at 93 per cent efficiency; then 150 kw. for 4 hours at 90 per cent efficiency; then 50 kw. for 12 hours at 89 per cent efficiency. Calculate the all-day efficiency.

Prob. 52–1. The alternator of Problem 51–1 is used for 12-hour service instead of 24-hour service; that is, it operates as stated during the 8-hour and the 4-hour periods, but is shut down completely during the remaining 12 hours. What is the all-day efficiency under these conditions?

Prob. 53–1. An alternator, operating at 70 per cent power-factor, receives 250 horse-power from the engine and delivers 250 kv-a. from its terminals. What is its efficiency?

Prob. 54–1. What would be the efficiency of Alternator No. 13, page 18, when delivering the same (rated full-load) kv-a. as in Table II, but at 80 per cent power-factor instead of 100 per cent? Assume the armature copper-loss to be two-thirds of the total copper-loss at full-load in Table II. Assume also that the full-load field current must be 10 per cent higher at 80 per cent power-factor than at 100 per cent power-factor, to keep the same terminal voltage.

Prob. 55–1. How many more dollars per kv-a. of capacity could you afford to pay for Alternator No. 1 than for Alternator No. 2 in Table II, page 18? Assume the machines to operate at full-load unity power-factor for 12 hours every day. Energy is worth one cent per kw-hour. Capital invested in the machinery must earn at least 12 per cent to pay "fixed charges" in this plant.

Prob. 56-1. The saturation curve (between field current and open-circuit voltage) of a given alternator is a straight line from zero volts up to about 50 per cent above rated voltage. If the voltage regulation of this alternator under given conditions is 20 per cent, by what percentage of its full-load value must the field current be increased or diminished when the load is removed, in order that the voltage shall not change? By what percentage will the I^2R loss in the field windings at zero-load be greater or less than at rated full-load?

Prob. 57-1. The curve between current output and terminal voltage of a given alternator is a straight line. The voltage regulation at unity power-factor is 10 per cent. What is the maximum sudden variation of load (expressed in per cent of rated load) that may occur without changing the terminal voltage more than 2 per cent of its previous value:

(*a*) when operating at full-load;

(*b*) when operating at half-load?

Prob. 58-1. If the load on the alternator in Problem 57-1 fluctuates suddenly over a range of 25 per cent of rated load, what should be the per cent voltage regulation in order that the terminal voltage shall not change more than 2 per cent of rated full-load voltage?

Prob. 59-1. If the saturation curve (between field current and open-circuit volts) for the alternator of Problem 18-1 were practically a straight line from zero volts to 50 per cent above rated voltage, calculate the percentage by which the field current must be increased above its rated-load value, in order to maintain rated terminal voltage with 25 per cent overload, at unity power-factor.

Prob. 60-1. Solve Problem 59-1 on the basis of a load having 80 per cent lagging power-factor.

Prob. 61-1. By what percentage is the I^2R loss and rate of heating in the field greater than at rated full-load, in Problems 60-1 and 61-1?

Prob. 62-1. Draw a vector diagram illustrating a phase relation between current and terminal pressure which would make the voltage regulation zero, or zero-load voltage same as full-load voltage, for the generator of Prob. 18-1.

Calculate the power-factor of the load, for this case.

Prob. 63–1. In Problem 18–1, calculate:

(*a*) the power-factor of the entire circuit including the armature;

(*b*) the total power in the entire circuit;

(*c*) the total I^2R watts lost in the armature;

(*d*) the total power generated minus total I^2R loss. Check this last result against the power output as calculated from terminal volts and amperes, and load power-factor. Does the armature reactance represent any loss of power, or only loss of pressure?

Prob. 64–1. Repeat the calculations of Problem 63–1, with the same alternator operating as specified in Problem 62–1.

Prob. 65–1. Solve Problems 28–1 and 29–1 with respect to a two-phase alternator.

Prob. 66–1. The alternator specified in Problem 28–1 has a power-factor of 80 per cent for the entire circuit including the armature winding. Calculate the values of cross-magnetizing and demagnetizing ampere-turns, on the basis of Fig. 38. Show that the number of demagnetizing ampere-turns is directly proportional to the component of I which lags 90° behind E_i.

Prob. 67–1. Assume that the induced e.m.f. is reduced by the demagnetizing effect of armature current, by an amount directly proportional to the component of I which is 90° behind the induced voltage, and that this amount is 40 per cent at zero power-factor with rated full-load current. Calculate the per cent voltage regulation, when the alternator of Problem 18–1 delivers full-load current lagging 30° behind the induced e.m.f. E_i.

Prob. 68–1. By the synchronous impedance method, calculate the terminal voltage of the alternator specified in Problem 31–1, when delivering current of rated full-load value at zero power-factor, the field excitation being such as would deliver rated full-load terminal volts on open circuit.

Prob. 69–1. By the synchronous impedance method, calculate the open-circuit volts of the alternator specified in Problem 31–1, corresponding to the field excitation which will produce rated full-load voltage at terminals when delivering full-load amperes at zero power-factor.

Prob. 70–1. An ordinary turbo-alternator has a final or "sustained" short-circuit current about 2.5 times rated full-load current when short-circuited under full-load excitation. The resistance drop is 1 per cent. Calculate:

(*a*) synchronous impedance (e.m.f.), and

(*b*) the synchronous reactance (e.m.f.), each as percentage of the terminal e.m.f. at rated load.

Prob. 71–1. The voltage regulation of the turbo-alternator of Problem 70–1 on non-inductive load is found to be approximately 8 per cent. (*a*) Calculate by the synchronous impedance method (Art. 12) its voltage regulation from the per cent of resistance drop and of synchronous reactance drop as calculated in Problem 70–1. Compare with the measured value of 8 per cent. (*b*) Calculate synchronous reactance from measured regulation of 8 per cent, IR being 1 per cent as in Problem 70.

Prob. 72–1. Calculate the inherent reactance in per cent, if the current that flows immediately after short-circuit is ten times rated full-load current of the alternator of Problem 70.

Prob. 73–1. Draw a vector diagram representing values and phase relations of current, terminal e.m.f., resistance e.m.f., reactance e.m.f., and generated e.m.f., at non-inductive rated full-load, for the alternator of Problem 71–1. All voltages are to be expressed in per cent of rated voltage. Calculate:

(*a*) The angle of phase difference between terminal e.m.f. and generated e.m.f.

(*b*) The component of armature current lagging 90° behind generated e.m.f., as a percentage of rated full-load current.

Prob. 74–1. Draw a vector diagram similar to that of Problem 73–1, but representing conditions on short circuit. Calculate:

(*a*) Angle of phase difference between current and generated e.m.f.

(*b*) Component of armature current lagging 90° behind generated e.m.f., as a percentage of rated full-load current.

Note that short-circuit current is 2.5 times full-load current, and the terminal e.m.f. is zero.

Prob. 75–1. Assume that the weakening of useful flux due to armature reaction is directly proportional to the component of armature current which lags 90° behind the generated e.m.f.* (*a*) From the diagrams and results of Problems 73–1 and 74–1, calculate how many times greater this weakening should be on short-circuit than on non-inductive load. (*b*) How many times greater is the synchronous reactance (as calculated from short-circuit data) than the reactance as calculated from 8 per cent regulation and 1 per cent resistance drop on non-inductive full-load (when armature demagnetizing effect is relatively small)?

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* See pages 36 (top) and 39 (middle).

Prob. 76–1. Actual short-circuit measurements by oscillograph on nine alternators, ranging in size from 500 kv-a. to 19,000 kv-a. rated capacity (see Electric Journal, Nov., 1913), showed the current immediately after short-circuit to be between 10 and 26 times rated full-load current. What were the limiting values of inherent reactance in this collection of generators, assuming the resistance drop at full-load to be 1 per cent of rated voltage?

Prob. 77–1. The alternator of Problems 70–1 and 72–1 is operated at $\frac{25}{60}$ of the former speed, keeping the same field excitation. (*a*) By what percentage would the inherent reactance and inherent impedance be increased over their respective former values? (*b*) By what per cent would the synchronous reactance and synchronous impedance be increased over their former values?

Prob. 78–1. (*a*) By what per cent would the initial (effective) value of short-circuit amperes be altered by the change in speed specified in Problem 77–1?

(*b*) By what per cent would the final value of short-circuit current be decreased?

Prob. 79–1. By the synchronous impedance method, calculate the per cent voltage regulation on non-inductive load for the generator specified in Problem 70–1 after making the alteration specified in Problem 77–1.

Prob. 80–1. An external reactance having a drop equal to 5 per cent of terminal voltage at rated full-load is connected in the leads to switchboard from the generator specified in Problem 72–1. The resistance of this current-limiting reactance is entirely negligible. Calculate:

(*a*) Total true reactance and total impedance of armature and coil together, in per cent of rated voltage of alternator.

(*b*) Current flowing through armature and coil immediately after short circuit. Compare results of Problem 72–1.

Prob. 81–1. Calculate the voltage regulation of the alternator combined with its reactance as specified in Problem 80–1, on rated full-load non-inductive. Use the synchronous impedance method of calculation. Draw complete vector diagram to illustrate.

Prob. 82–1. By what percentage is the initial current on short circuit reduced, by connecting a 5 per cent current-limiting reactance in series with the leads from an alternator having 10 per cent inherent reactance?

Prob. 83–1. The inherent reactance of a 25-cycle turboalternator is 5 per cent. What would be the per cent reactance of this same alternator when operated at a frequency of 60 cycles, same voltage?

Prob. 84–1. For some purposes current-limiting reactances placed in each feeder going out from the bus-bars is considered preferable to reactances placed in the generator leads going to the bus-bars. The author of a paper in Proc. A.I.E.E., Feb. 1914, makes the statement that 5000 kv-a. at 0.8 power-factor flowing over a reactance of 3 per cent in a feeder gives a voltage drop of approximately 1.9 per cent. Draw a vector diagram to illustrate the conditions, and verify this result by calculation.

Prob. 85–1. In discussion of the paper referred to in Problem 84–1, an engineer states that his calculations showed, with three-conductor No. 000 feeder subject to short circuit of 60,000 amperes, that the repulsive force between conductors **per running foot** was approximately **one ton.** On this basis, calculate the value of the repulsive force acting between the same conductors when carrying normal full-load current from a three-phase alternator rated 30,000 kv-a. at 11,000 volts.

Prob. 86–1. Oscillograph records show that an oil-switch in average operation cannot be depended upon to open a short circuit in a shorter period of time than 12 cycles on a 25-cycle system. If the short circuit is 60,000 amperes, how many gram-calories of heat energy will be generated in one foot length of No. 000 B. & S. gauge copper feeder before the circuit breaker opens? How many degrees Centigrade temperature rise will this heat produce in the conductor? Assume that no heat is lost during this short time; the specific heat of copper is 0.09846.

Prob. 87–1. Current-limiting reactances are rated in terms of kv-a., the product of current through them and kilovolts drop across them.

(*a*) What would be the rating in kv-a. of one unit of a three-phase 5-per cent reactance set for a 25,000 kv-a. generator?

(*b*) Show that the kv-a. of a reactance varies as the **square** of the current through it.

CHAPTER II

ALTERNATORS IN PARALLEL

It is more economical to produce electric power in large power plants than in small plants (for explanation, see Chapter I). Also, large generators, and engines or turbines, are more efficient and economical than smaller ones. There are numerous central stations with an aggregate rated capacity over 100,000 kv-a., and there are numerous turbo-alternators as large as 25,000 kv-a. rated capacity. It requires several generators (even of the largest size that can be built), therefore, to carry the load of one of the larger stations. Even in a smaller station, the total required capacity is divided into several units, so that only enough capacity need be in operation at any hour of the day to be nearly fully loaded by the power demand at that hour. This results in a higher efficiency than would be obtained with fewer and larger units operating under-loaded during the hours between the peak loads.

Each generator in the power plant delivers its output (through a set of devices for **controlling** and **measuring** the load of the generator and **protecting** the system from trouble) to a set of common conductors called **bus-bars.** Each of the **feeders,** which transmit the power to the centers of distribution or substations from which it is retailed, is connected to these bus-bars through the necessary switches, regulators and meters for controlling and measuring the output of the station and protecting the feeders from damage in case of short-circuits on them. The system of connections for a complete plant having several generators and feeders and with all their auxiliary equipment is too complicated to be introduced here. We shall discuss only the alternating-current features of the interaction between the generators, using for illustration a very much simplified diagram of connections.

18. Alternators in Parallel. Practically all American central stations are committed to the parallel system of distribution at constant voltage. In order to keep a constant voltage while a varying number of generators supply the power, it is necessary to connect them in **parallel** with one another. When so connected, the terminal voltage of all generators **must** be the same (equal to the voltage between the common bus-bars), and the total current delivered through the bus-bars to the load should be equal to the (arithmetical) sum of currents delivered by the armatures of the several generators. However, incorrect adjustments may cause to circulate between the armatures local currents, which never reach the external circuit and therefore contribute nothing to the useful output of the station, although they increase the heating of the armatures and correspondingly reduce the amount of useful current which the alternators can deliver.

We must, therefore, understand clearly the following factors of satisfactory parallel operation of alternators:

(1) How to connect the alternator properly to the system, or "cut it in" as operators often say. Also, how to take it out of service properly.

(2) How to transfer load from one alternator to another in parallel.

(3) How to adjust for minimum armature current and heating, in each alternator, while carrying a given load; or, how to get maximum kilowatt capacity and efficiency in the alternators which are operating.

19. Similarities in Direct-current Parallel Operation. Before we may properly connect two batteries in parallel (as G_1 and G_2 in Fig. 47), we must know that their voltages are nearly equal. Then we connect the positive poles together to one line wire B_1, and the negative poles together to the other line wire B_2. The current taken by any load R connected between B_1 and B_2 will be shared equally by G_1 and G_2, if the internal or generated voltages of the two batteries are equal and their internal resistances are also equal. This is necessary in order that the terminal voltages may be equal, which is essential for a parallel combination. If the open-circuit voltage of G_2 is larger than that of G_1 but the internal resistances are equal, then when they are connected in parallel, G_2 will deliver enough more current than G_1 to make its terminal voltage equal to that of G_1 by reason of the greater internal rI drop.

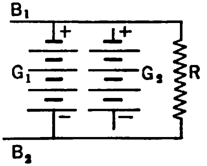

FIG. 47. The e.m.f.'s and the internal resistances of the two batteries G_1 and G_2 must be equal if they are to deliver equal currents when connected in parallel.

If now the external resistance is decreased so as to take 30 amperes more, the batteries will divide this increase equally between them as long as the internal resistances are equal. The terminal voltages were equal before the change, and must be so after the change, hence the increase of rI drop must be the same in both batteries; and since the resistances are equal, the increases of current must also be equal. But if the internal resistance of G_2 is twice as great as that of G_1, the increase of current in G_2 will be only one-half as great as the increase in G_1; that is, G_2 will increase 10 amperes and G_1 will increase 20 amperes. The same reasoning will show that the terminal voltages of G_1 and G_2 cannot be equal unless this is so; the currents will continue to change, and we shall not have equilibrium, until the terminal voltages are equal.

Thus we see that the automatic sharing of changes of load depends upon the relation of internal resistances, whereas the initial distribution of the load is controlled by the relation between the generated or open-circuit voltages. If these voltages were exactly equal, no current would flow in either machine when the external circuit R is opened; both batteries reach zero-load at the same time. But if the voltage of G_2 is larger than that of G_1, then G_2 will force a current through G_1 in a direction opposite to the e.m.f. of the latter, when the external circuit is open; that is, G_2 delivers power and G_1 receives it.

The case of direct-current generators is quite similar, as illustrated in Fig. 48. Suppose G_1 is connected to the bus-bars and delivering current to a load RL. While G_2 is running at constant speed, we adjust its field current (by rheostat R_2) until its open-circuit voltage is equal to the bus-bar voltage or the terminal voltage of the other generator. We then connect the + pole of G_2 to the + bus, and the − pole of G_2 to the − bus. The e.m.f. of G_2 just balances the bus voltage, and G_2 neither delivers nor takes current. If now the induced voltage of G_2 is increased, it causes G_2 to deliver some current to the load. The total load current remains the same as before, therefore the current output of G_1 decreases just as much as that of G_2 increases. These changes cause the terminal voltage of G_2 to fall below the induced voltage, and that of G_1 to rise above its former value, on account of changes in their respective rI drops and armature reactions. The current of G_2 will continue to increase, and that of G_1 to decrease in equal amount, until the terminal voltages of G_1 and G_2 are again equal; then the currents will become constant. The voltage between bus-bars will be increased slightly by this adjustment. If we raise the field excitation of G_2 far enough we

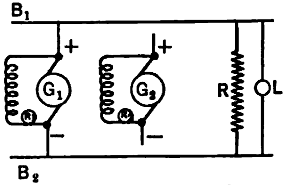

FIG. 48. The e.m.f.'s and the internal resistances of the direct-current generators G_1 and G_2 should be the same in order for them to operate well in parallel. The load of each is controlled by means of its field rheostat which changes the voltage generated.

can make it take all of the load and force current through G_1 in opposition to its induced e.m.f., thus running G_1 as a **motor.**

When the load increases on the system shown in Fig. 48, the sharing of the increase will depend upon the relative voltage regulation of the two generators. Suppose the terminal voltage of G_2 decreases 1 volt per 100 amperes delivered, and that of G_1 decreases 2 volts per 100 amperes. If the external load RL increases by 30 amperes, the current delivered by G_2 will increase by 20 amperes and the current output of G_1 will increase by 10 amperes. This will cause the same decrease of terminal voltage for both machines (namely, 0.2 volt) and will keep the terminal voltages equal.

Variations in the speed of G_1 and G_2 affect the distribution and sharing of load, only in so far as these variations affect the e.m.f. induced in the armatures, and the inherent voltage regulation. The actual value of the speed is immaterial; we control the load by controlling the voltage, and generators of any speed may work together satisfactorily in parallel.

Example 1. A storage battery of 0.03 ohm internal resistance which gives 12 volts on open circuit is connected in parallel with another battery of 0.04 ohm internal resistance which gives 12.2 volts on open circuit. What current is each delivering when the load on the combination is 20 amperes?

Construct Fig. 49.

The current delivered by $B = x$ amp.

The current delivered by $A = 20 - x$ amp.

Voltage across $B = 12.2 - 0.04\,x$.

Voltage across $A = 12 - 0.03\,(20 - x)$.

But the terminal voltage across both batteries must be the same since they are in parallel.

Therefore

$$12 - 0.03\,(20 - x) = 12.2 - 0.04\,x$$
$$12 - 0.6 + 0.03\,x = 12.2 - 0.04\,x$$
$$0.07\,x = 0.8$$
$$x = \frac{0.8}{0.07}$$

Current delivered by $B = 11.4$ amp.

Current delivered by $A = 20 - 11.4$

$= 8.6$ amp.

Prob. 1–2. A storage battery which has an internal resistance of 0.024 ohm and gives 6.00 volts on open circuit is connected in parallel with another battery which has the same internal resistance, but which gives 6.20 volts on open circuit. Draw curves, using as abscissas the total amperes delivered to the external circuit, and as ordinates the following:

(*a*) Amperes in 6.0-volt battery.

(*b*) Amperes in 6.2-volt battery.

(*c*) Terminal voltage. The external current increases from zero to 100 amperes.

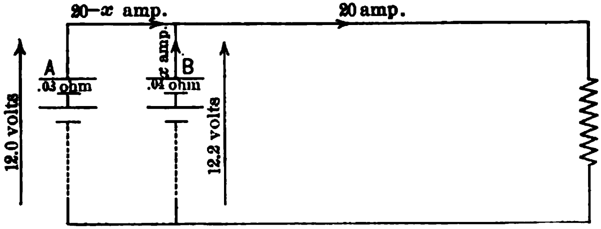

FIG. 49. The batteries *A* and *B* of different e.m.f.'s and different internal resistances are connected in parallel.

Prob. 2–2. A storage battery which has an internal resistance of 0.024 ohm and gives 6.00 volts on open circuit is connected in parallel with another battery which gives the same open-circuit voltage but has an internal resistance of 0.030 ohm. Draw curves, using as abscissas the total amperes delivered to the external circuit, and as ordinates the following:

(*a*) Amperes in 0.024-ohm battery.

(*b*) Amperes in 0.030-ohm battery.

(*c*) Terminal voltage. The external current increases from zero to 100 amperes.

Prob. 3–2. A storage battery which has an internal resistance of 0.024 ohm and gives 6.0 volts on open circuit is connected in parallel with another battery which has an internal resistance of 0.030 ohm and gives 6.2 volts on open circuit. Draw curves, using as abscissas the total amperes delivered to the external circuit, and as ordinates the following:

(*a*) Amperes in the 6.2-volt battery.

(*b*) Amperes in the 6.0-volt battery.

(*c*) Terminal voltage. The external current increases from zero to 100 amperes. Compare these results with those of Problems 1–2 and 2–2.

Prob. 4–2. If the maximum current that may be taken from either battery in Problem 1–2 without injury is 50 amperes, calculate:

(*a*) The terminal voltage and watts output of each battery when operated separately and delivering its maximum current.

(*b*) The total maximum watts output of the two batteries when operated separately.

(*c*) The maximum total watts output when operating in parallel. Compare (*b*) and (*c*), and discuss therefrom the disadvantages of parallel operation without adjustments.

Prob. 5–2. Solve Problem 4 with relation to the batteries specified in Problem 2, and discuss therefrom the disadvantages of paralleling generators having dissimilar characteristics.

Prob. 6–2. Solve Problem 4 with relation to the batteries specified in Problem 3.

20. Synchronizing and Paralleling Alternators. The simplest possible connections for paralleling two single-phase alternators are shown in Fig. 50. Before the main switches S may be closed, connecting the armatures together through the bus-bars B_1, B_2, the following relations should be obtained, at least, approximately:

(1) The induced **e.m.f.'s** of the armatures A_1 and A_2 must be **equal.** As the e.m.f.'s alternate, this should be true **at every instant.**

(2) **Similar terminals** of A_1 and A_2 must be connected to the **same bus-bar.** Similar terminals mean armature terminals which are positive at the same time, and negative at the same time.

Notice that these requirements are like the requirements for paralleling direct-current generators. However, in order that the alternating e.m.f.'s shall be equal at *every* instant we must have:

(1*a*) The frequency of A_1 equal to the frequency of A_2.

(1*b*) The e.m.f. of A_1 in phase with the e.m.f. of A_2.

(1*c*) The wave-form of A_1 the same as the wave-form of A_2.

The apparatus ordinarily used to indicate whether these conditions, except "1c," are fulfilled are a **voltmeter** and some form of "**synchronoscope,**" or synchroscope. The latter is a device to indicate when the alternator is **in synchronism** and **in phase** with the bus-bars to which we desire to connect it. Two alternators are in synchronism with each other when they have the same (constant) frequency. The simplest form of synchroscope is the "synchronizing lamps," *LL*, Fig. 50.

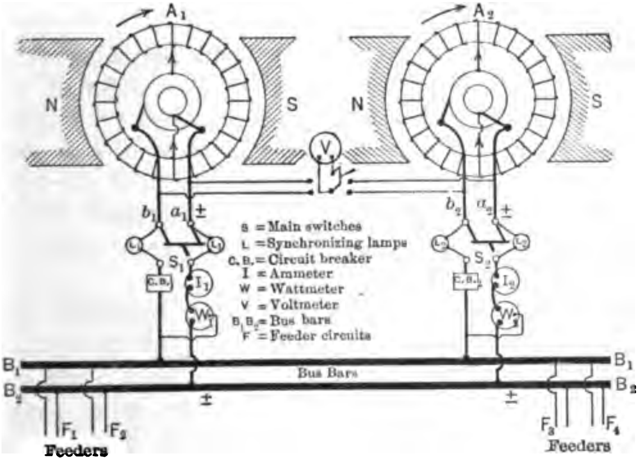

FIG. 50. Simple diagram showing the essential apparatus used in paralleling two alternators.

Consider that S_1 is closed and alternator A_1 is producing an alternating e.m.f. between bus-bars B_1, B_2, but all feeders are disconnected and no current is flowing. In order to connect A_2 in parallel, we first bring up its speed until the frequency is nearly the same as that of A_1 or of the bus-bars. We then adjust the field current of A_2 until its effective voltage is nearly equal to that of A_1. It is customary, and preferable, to use the same voltmeter to measure both voltages, connecting it by means of a voltmeter-switch first to the armature terminals of A_1 or to the bus-bars, and then to the armature terminals of A_2 as indicated in Fig. 50. Although the main switch S_2 is open, the armatures A_1 and A_2 are joined together through the synchronizing lamps which are permanently connected in parallel with the switch blades, as shown. The behavior of these lamps now indicates quite accurately what further adjustments must be made before closing S_2. Thus:

(1) **If the lamps become bright and dark alternately,** it indicates that the frequencies of A_1 and A_2 are different. The number of light-beats per second is equal to the difference between these frequencies. The lamp does not show which alternator is too fast or too slow; the speed of the incoming machine A_2 must be raised or lowered until the light and dark periods are long and follow each other very slowly.

(2) **While the lamps are dark,** when connected as in Fig. 50, they indicate that the resultant voltage of A_1 and A_2 in series is nearly zero, or that the voltages e_1 and e_2 are nearly equal at all instants, and in the **same direction with respect to the bus-bars** or **in opposite directions with respect to each other** in the local circuit between the armatures. If the lamps **remain** dark, we infer that the effective voltages E_1 and E_2 are equal and in phase (with respect to the bus-bars), and the alternators are in exact synchronism. However, it **may** mean that the filament of one of the lamps has broken; therefore, we prefer to adjust the speed of A_2 so that the lamps brighten and darken slowly, and close the switch S_2 at about the middle of a dark period. In this way we have more recent evidence that the lamps are in good operating condition.

(3) **If the lamps stay bright,** it may mean that:

(*a*) E_2 is greater or less than E_1, and the alternators are in synchronism, either in phase, or out of phase.

(*b*) E_2 is greater or less than E_1, and there is great difference between the frequencies. In this case there is a tendency for the light to flicker, which may not be perceptible to the eye if the frequencies differ sufficiently.

(c) The alternators are in synchronism but out of phase; in this case, there may be any relation between the effective values of E_1 and E_2.

In any such event the speed of the incoming alternator, A_2, should be varied until the brightness of the lamps changes slowly; then its voltage E_2 should be raised or lowered until the lamps become quite dark between bright periods. Thus we attain the condition described in (1), and we close the switch S_2 as described in (2). The switch should **never** be closed while the lamps are bright, because the brightness indicates a considerable voltage between points which would be short-circuited by the switch blade.

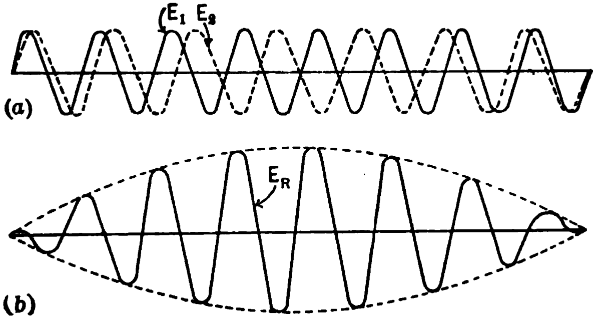

FIG. 51. Wave-form E_2 completes 7 cycles while E_1 completes 8. These in subtractive series produce resultant E_R.

The reason why the synchronizing lamps L brighten and darken alternately when the frequencies of E_1 and E_2 differ, as explained under (1) above, is illustrated in Fig. 51a and 51b. The former shows the two e.m.f. waves, having the same wave-form and same effective value, but differing in frequency; E_2 completes 7 cycles while E_1 is completing 8 cycles. The e.m.f. which lights the lamps is the resultant in the local circuit formed by the armatures A_1 and A_2 in series; and as Fig. 51a is drawn on the assumption that the positive direction of e.m.f. in both armatures is toward the same bus-bar, the local resultant is the vector difference between E_1 and

E_2, and we must reverse E_2 before adding it to E_1. After doing this, we obtain the curve shown as E_R in Fig. 51b, which represents the e.m.f. acting on lamps L_2L_2 in series. In the time during which E_2 completes one less number of cycles than E_1, the e.m.f. acting on the lamps is alternating rapidly but its effective value increases from zero to a maximum and back to zero again (as indicated by the dotted line in Fig. 51b). This causes the lamps to brighten and darken correspondingly; thus, if the frequency of E_1 is 60 cycles per second, and that of E_2 is $\frac{7}{8}$ of 60 or $52\frac{1}{2}$ cycles per second, then the lamps brighten 60 − 52.5 or $7\frac{1}{2}$ times per second.

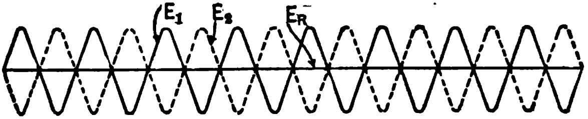

FIG. 52. Wave-forms E_1 and E_2 are now completing the same number of cycles per second. They have a phase difference of 180° and the resulting voltage is zero. The synchronizing lamps are therefore dark.

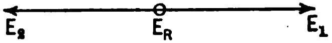

FIG. 53. Vector diagram of conditions in Fig. 52 showing that the resulting voltage is zero.

Now speed up A_2 until its frequency equals that of A_1 (namely, 60 cycles per second), meanwhile keeping E_2 equal to E_1. When this is accomplished, we may find E_2 directly opposed to E_1 at every instant and the resultant E_R nearly zero as shown in Fig. 52. Or we may find E_2 nearly in phase with E_1 in the local circuit between the armatures, and the resultant E_R nearly equal to $(E_2 + E_1)$, as shown in Fig. 54. In the case shown in Fig. 52 the lamps remain dark, as stated under (2) above. In the case of Fig. 54 the lamps remain bright, as stated under (3c) above. We may pass from the condition of Fig. 54 to that of Fig. 52 by speeding up or slowing

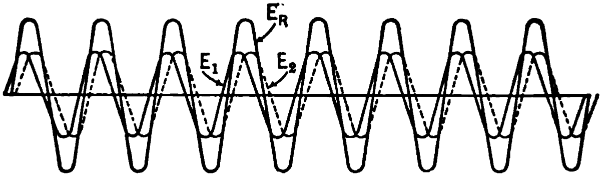

FIG. 54. E_1 and E_2 complete the same number of cycles per second, but E_1 leads E_2 by about 60°. The resultant voltage E_R keeps the synchronizing lamps glowing.

down A_2 very slightly until E_2 comes in opposition to E_1, then changing the speed back again to synchronism.

When A_2 and A_1 are in synchronism and in phase but E_2 is greater than E_1, we have the condition stated under (3a) above and shown in Fig. 56. A resultant e.m.f. E_R acts on the lamps, equal to the arithmetical difference between E_2 and E_1. This e.m.f. E_R has a constant effective value and the same frequency as E_2 and E_1; therefore, the lamps L_2L_2 will glow **steadily.** An incandescent lamp will not emit light when the voltage (E_R) is less than about 40 per cent of the rated voltage of the lamp; this is the reason why we wait until the **middle of a dark period** before closing the main switch S_2, so as to be quite sure that E_R is as small as possible.

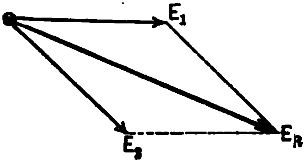

FIG. 55. The vector diagram of the conditions of Fig. 54, showing that E_1 and E_2 have a resultant E_R, which keeps the synchronizing lamps glowing.

When E_2 and E_1 are not in synchronism and are unequal in value, the lamps indicate as explained under (3b) above. Suppose the frequencies are as shown in Fig. 51a. Then E_2 falls behind by 1 cycle while E_1 passes through 8 cycles, or, for each cycle passed through by E_1, E_2 lags $\frac{1}{8}$ cycle further behind E_1. Fig. 57 represents conditions at the instant when the e.m.f. E_L, acting upon the lamps, has its minimum value. Fig. 58 shows conditions at the instant when E_1 has completed one more cycle; Fig. 59 after still

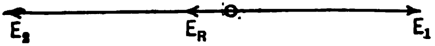

FIG. 56. The voltages E_1 and E_2 complete the same number of cycles per second and have a phase difference of 180°, with respect to each other, but E_2 is greater than E_1.

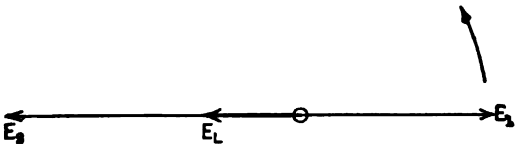

FIG. 57. Vector diagram showing derivation of resultant voltage acting on synchronizing lamps when the e.m.f.'s of the generators are unequal in value and of different frequency. This diagram shows conditions at the time when E_L is minimum.

another cycle of E_1, and so on. Comparing Fig. 57 and 60, we see that the effective value of E_L varies between a minimum equal to $(E_2 - E_1)$ and a maximum equal to $(E_2 + E_1)$, the number of fluctuations per second of this resultant being equal to the difference between the frequencies of E_2 and E_1.

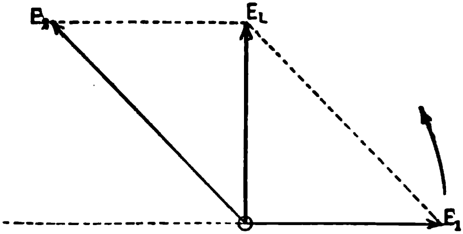

FIG. 58. The voltage across the lamps has this value of E_L when E_1 has completed one cycle more than in Fig. 57, and is $\frac{1}{8}$ cycle further ahead of E_2.

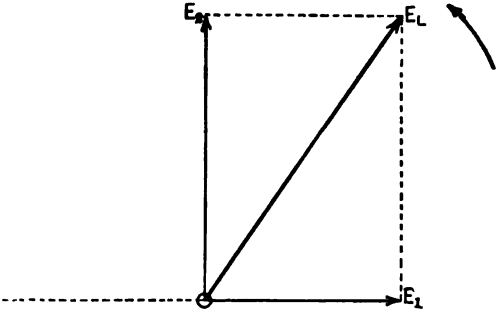

FIG. 59. The voltage E_1 has completed still another cycle and is now another $\frac{1}{8}$ cycle ahead of E_2.

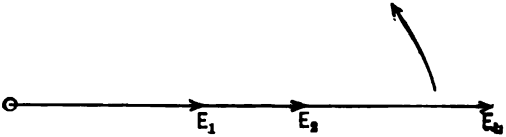

FIG. 60. The voltage wave E_1 has completed two more cycles since Fig. 59, and is $\frac{1}{4}$ cycle further ahead of E_2. This causes E_2 and E_1 to be momentarily in phase and the resulting voltage E_L is a maximum.

Consider now what happens when E_2 and E_1 have the same effective value and are in synchronism and in phase, but have different wave-forms. Thus let the wave of e_2 be extremely peaked, as shown in Fig. 61, and the wave of e_1 be extremely flat, as shown in Fig. 62. The resultant e.m.f. (e_r) in the local circuit between

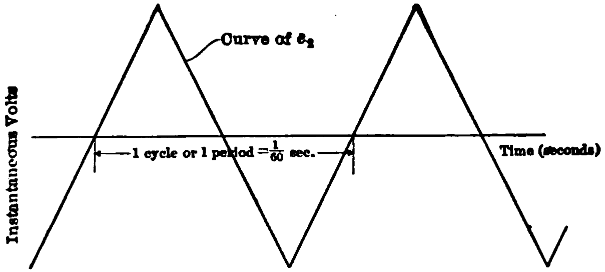

FIG. 61. An extremely peaked wave-form.

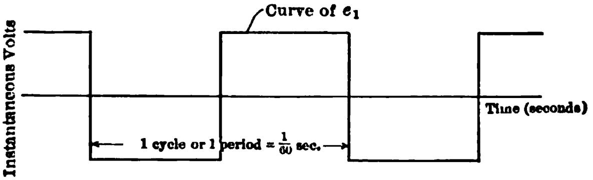

FIG. 62. An extremely flat-top wave-form.

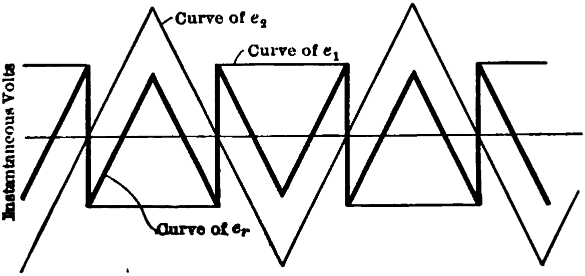

FIG. 63. Curve e_r is the resulting curve when e.m.f. of wave-form in Fig. 61 is joined in series with the e.m.f. of wave-form in Fig. 62. The two e.m.f.'s are in synchronism and have a phase difference of 180°, and have the same effective value. Note that the resultant voltage is not zero, however.

the two armatures, where e_2 and e_1 act in series, must always have an effective value greater than zero, as illustrated in Fig. 63. It would be impossible to find an adjustment of values or phase relation of e_1 and e_2 which would eliminate this resultant e.m.f. e_r as long as the wave-forms of e_1 and e_2 are different.

Example 2. Alternator A_1 in Fig. 50 has a terminal e.m.f. of 110 volts of sine wave-form, and has a frequency of 59 cycles. A_2 has a terminal e.m.f. of 112 volts of sine wave-form, and a frequency of 61 cycles per second. When S_1 is closed and S_2 is open:

(*a*) What is the greatest effective voltage across each lamp, assuming that they are all alike?

(*b*) How many light-beats occur each second?

Solution.

(*a*) The greatest effective voltage across the lamps is always the sum of the effective e.m.f.'s of the two generators.

$$E_1 + E_2 = 110 + 112$$
$$= 222 \text{ volts.}$$

But when S_1 is closed, two lamps (L_2L_2) are in series so that the greatest effective voltage across each lamp is $\frac{220}{2} = 111$ volts.

(*b*) The number of light-beats per second always equals the difference between the frequencies of the generators. In this case it equals 61 − 59 = 2 light-beats per second.

Prob. 7-2. Each alternator shown in Fig. 50 generates an harmonic e.m.f. of 220 volts effective value. The frequencies are 60 and 58 cycles per second. S_1 is closed and S_2 open. The synchronizing lamps are all alike.

(*a*) What is the greatest effective voltage across each lamp L_2?

(*b*) How many of these maxima per minute?

(*c*) What should be the rated voltage of each lamp in order to avoid burning it out prematurely?

Prob. 8-2. Each alternator in Fig. 50 generates an harmonic e.m.f. of 60 cycles frequency. S_1 is closed and S_2 open; all lamps L are alike. The terminal e.m.f. of A_1 is 240 volts and of A_2 is 200 volts, effective values. What is the effective voltage across each synchronizing lamp L_2:

(*a*) When the phase difference between E_1 and E_2 is such that this voltage is greatest?

(*b*) When this voltage is minimum?

Prob. 9-2. The wave-form of e.m.f. in each alternator of Fig. 50 is harmonic, but A_1 generates an effective value of 240 volts at 60 cycles per second, while A_2 generates an effective value of 200 volts at 58 cycles per second. Describe the behavior of each lamp L and calculate maximum and minimum values of the effective voltage across it. Both switches S_1S_2 are open.

Prob. 10–2. One alternator of Fig. 50 generates an e.m.f. wave of the form shown in Fig. 62, and the other generator an e.m.f. as shown in Fig. 61. The voltmeter indicates 220 volts across each alternator on open circuit. The alternators are in synchronism and in phase, as shown in Fig. 63. Calculate:

(*a*) The maximum instantaneous e.m.f. of A_1.

(*b*) Maximum instantaneous e.m.f. of A_2.

(*c*) Effective value of e.m.f. acting in local circuit of armatures, across $L_2L_2L_1L_1$.

Prob. 11–2. Draw the wave of resultant e.m.f. in the local circuit between the alternators of Problem 10–2 for the phase relation which makes the effective value of this resultant the greatest. Calculate this greatest value in effective volts.

21. Synchronizing Currents. Phantom Load. We have gone into details regarding this resultant e.m.f. E_R in the local circuit between armatures, because it determines what happens when the main switch S is closed. In general, we may state that whenever the wave-forms, or the frequencies, or the effective voltages of the two alternators are not exactly the same, or when they are not exactly in phase, there will be a resultant e.m.f. which causes a current to circulate between the armatures as soon as the main switches are closed. As the synchronizing lamps LL are short-circuited by the switch blades, the only reactions limiting the amount of current that flows through the local circuit of the armatures are those due to resistance, inherent reactance and the armature reaction of the armature-currents. As these factors are relatively small, slight differences between E_2 and E_1 produce large values of this circulating current, which is called the **synchronizing current.**

The action of the synchronizing current upon E_2 and E_1 is always such as to reduce E_R; thus, the **synchronizing current limits itself** and tends to reconcile the differences between E_2 and E_1, making the alternators operate together more smoothly. However, the synchronizing current heats the armatures, and a relatively small difference between E_2 and E_1 is sufficient to reduce seriously the capacity of the armatures for delivering useful current to the external load. The synchronizing current is superposed upon any load current which either alternator may be delivering to the external circuit. This subject will be considered in the next article.

While the feeders are disconnected, let us examine the synchronizing current due to a poor adjustment of E_2 and E_1, and the effects which it produces. Consider Fig. 56, where the e.m.f.'s are in synchronism and in phase with respect to the bus-bar voltage and are both of harmonic waveform, but E_2 is greater than E_1. Assume that $E_2 = 2200$ volts, $E_1 = 2000$ volts. Synchronous reactance of each armature = 2.0 ohms; resistance of each armature = 0.2 ohm. Then, assuming that the resistance and reactance in the busbars and in the leads from armatures to switchboard are negligibly small or are included in the above figures we have:

Total resistance in local circuit of armatures $= 0.2 + 0.2 = 0.4$ ohm.

Total (synchronous) reactance, including inherent reactance and armature reaction $= 2.0 + 2.0 = 4.0$ ohms.

Total (synchronous) impedance $= \sqrt{(0.4)^2 + (4.0)^2} = \sqrt{16.16} = 4.02$ ohms.

Then, from Fig. 56, we see that:

$$E_R = E_2 - E_1 = 2200 - 2000 = 200 \text{ volts, effective value.}$$

$$\text{Synchronizing current } (I_S) = \frac{E_R}{\text{total synch. imped.}} = \frac{200}{4.02}$$

$$= 49.8 \text{ amperes (R.M.S. value).}$$

This current lags behind the e.m.f. E_R which produces it, by an angle θ, whose value depends upon the ratio of reactance to resistance in the local circuit. Thus, in Fig. 64,

$$\tan \theta^\circ = \frac{\text{total synchronous reactance of local circuit in armatures}}{\text{total resistance of local circuit in armatures}}$$

$$= \frac{4.0}{0.4} = 10,$$

or

$$\theta = 84^\circ\ 18'.$$

Now, I_S in this case is almost wholly reactive with respect to the e.m.f.'s of both alternators. Thus, if

P_2 = power developed by I_S in flowing through armature which generates E_2

and

P_1 = power developed by I_S in flowing through armature which generates E_1,

then

$$P_2 = I_S \times E_2 \times \cos\theta = 49.8 \times 2200 \times 0.09932$$
$$= +\,10{,}890 \text{ watts}$$

and

$$P_1 = I_S \times E_1 \times \cos(180^\circ - \theta^\circ) = 49.8 \times 2000 \times (-0.09932) = -\,9900 \text{ watts.}$$

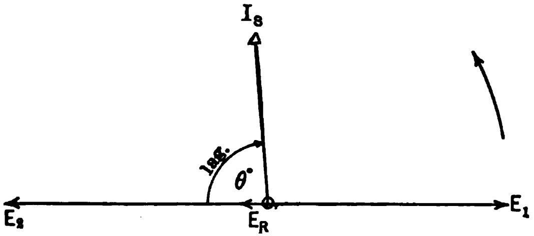

FIG. 64. The synchronizing current I_S must lag behind the voltage E_R by an angle θ whose value depends upon the ratio between reactance and resistance of armature circuits.

Positive power signifies generator action, and negative power signifies motor action. When the induced e.m.f. of any dynamo produces a current (in the same direction as the e.m.f., of course), the magnetic force exerted between this current and the field is such as to oppose the motion which produces the e.m.f. (Lenz's Law). Therefore, it follows that whenever a current flows through the conductors in the direction **opposite** to the e.m.f. which is being induced in them, the force action is also opposite; therefore, it helps to produce the rotation by which the e.m.f. is induced. In other words, we have motor action whenever the current flows in direction opposite to the induced e.m.f., and generator action whenever the current flows in the same direction as the induced e.m.f. These facts underlie the operation of the synchronous motor, as will be seen.

In the present case, we see that 10,890 watts of electrical power are generated in the higher-voltage alternator, A_2, of which 9900 watts go to produce motor action in A_1. The remainder, 10,890 − 9900, or 990 watts, represents the power transformed into heat by the current I_S flowing against the resistance of the two armatures. That is, $I^2R = (49.8)^2 \times 0.4 = 990$ watts (check). The 9900 watts of motor action in the lower-voltage alternator A_1 tends to push it ahead in the direction of rotation. But the instant that the vector E_1 advances with respect to vector E_2 the resultant e.m.f. E_R and current I_S are changed, and in such manner that the power input to A_1 (and motor action produced thereby) are reduced. Therefore, E_1 will advance on E_2 just far enough to reduce the electrical power intake of A_1 to an amount which is sufficient to hold A_1 at synchronous speed; then we shall have equilibrium and I_S will be constant. Thus, in Fig. 65 vector E_1 has advanced in phase about 15° with respect to vector E_2; notice that the **synchronizing current I_S** is thereby **increased in value** (in proportion to E_R, which is increased by the change of phase relation between E_2 and E_1), and thrown **more nearly in phase with E_1**. Even this slight advance of 15° by E_1 changes P_1 from

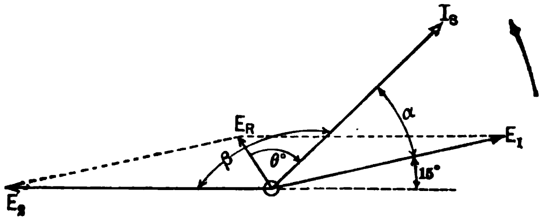

FIG. 65. Voltage E_1 has advanced 15° toward E_2. E_R has therefore become larger than in Fig. 64, also more nearly in phase with E_1. I_S has proportionally increased and is more nearly in phase with E_1.

negative (motor action) to positive (generator action), and changes A_2 from generator to motor. As both alternators are being driven by their engines at exactly the same frequency, the phase displacement will not proceed far enough to produce any motor action, but only far enough to reduce the motor action to a zero value. Hence the phase advance of E_1 due to the motor action of the synchronizing current that flows immediately after the switch is closed under the condition shown by Fig. 64 will be stopped long before it has amounted to 15 electrical degrees.

The most important action of the synchronizing current (I_S, Fig. 64), due to inequality of alternator voltages, is now to be noted. I_S leads the induced e.m.f. E_1 of the lower-voltage alternator by nearly 90°, and it lags behind the induced e.m.f. E_2 of the higher voltage alternator by almost 90°. We have seen (Art. 9) that when the armature current lags 90° behind the induced e.m.f. it exercises a relatively strong demagnetizing influence, and when it leads the induced e.m.f. by 90° it tends to strengthen the field. Hence, we see that I_S will weaken the field of A_2 and hereby reduce E_2, while at the same time it will strengthen the field of A_1 and increase E_1. Therefore, E_2 and E_1 will be made more nearly equal to each other by the armature reactions due to I_S; and as E_2 approaches E_1 in value, E_R is reduced. Here again we see the wonderful self-adjustment of the alternator; the synchronizing current limits itself automatically by bringing about within the machines such actions or reactions as shall make their e.m.f.'s more nearly as they should be.

Prob. 12-2. Before closing the switches in Fig. 50, alternator A_1 is generating 220 volts and A_2 is generating 190 volts. The switches were closed when the synchronizing lamps were at the middle of a dark period, but such a small difference of voltage could not be indicated by the lamps, and was not noticed on the voltmeter as it should have been. Each alternator is rated 100 kv-a. 220 volts, and has a resistance of 1 per cent and inherent reactance of 10 per cent. Calculate for the instant immediately following the switching, with no load on the bus-bars:

(*a*) Amperes flowing in both armatures after switches are closed.

(*b*) Terminal voltage of each armature.

Note that these two terminal voltages should be equal; this furnishes a check on your work. Draw vector diagrams showing how the terminal voltage of each alternator was obtained. The wave-form of each generator is harmonic.

Prob. 13–2. From the data and results of Problem 12, calculate the values of the following quantities the moment after the switch is closed, and before the alternators adjust their phase relations.

(*a*) Electrical power generated in one alternator.

(*b*) Electrical power output from terminals of this alternator, or input to terminals of the other alternator.

(*c*) Electrical power used to overcome induced (counter) e.m.f. of other alternator, or to develop mechanical power in its rotor.

(*d*) I^2R loss, in each armature.

Check your work by comparing (*d*) with the difference between (*a*) and (*b*), or between (*b*) and (*c*).

Prob. 14–2. If each lamp L_2 in Fig. 50 is rated 220 volts and its filament does not become visible until the voltage reaches 90 volts, what is the maximum difference that may exist between the effective values of terminal e.m.f. in the two alternators, while the lamps still appear to be dark during the synchronizing process (S_1 closed, S_2 open)? If the switches are closed when this difference of voltages exists with the lower-voltage machine running light but adjusted to 220 volts, calculate the results called for in Problem 13–2. Conditions, other than voltage, proper for paralleling.

Prob. 15–2. Calculate the results called for in Problem 13–2, after equilibrium has been established. Assume that the prime movers have exactly the same load-speed curve (flat, with speed regulation of 1 per cent).

22. Synchronizing Power. If one alternator happens to be ahead of the other in phase at the moment of closing the main switch (S, Fig. 50), a local synchronizing current will flow which takes electrical power from the machine that leads and delivers it to the machine that lags. Thus, the former tends to be pulled back by generator action, while the latter tends to be pushed ahead by motor action, and the two e.m.f.'s come more nearly into phase. But while this is occurring, the resultant e.m.f. E_R is being reduced, because it is produced by the phase difference between E_2 and E_1, with respect to the bus-bars. And as I_S is directly proportional to E_R we see again that the synchronizing current tends to limit itself, as it brings the alternators more nearly into their proper relation to each other.

For instance, suppose that we close the main switch at a moment when E_1 is 15° in advance of its proper phase relation to E_2, as shown in Fig. 65. The resultant e.m.f. is E_R and the local synchronizing current is I_S. If the values of E_2 and E_1 are known, and the angle E_2OE_1 between them, it is simply a problem in trigonometry to find the value of E_R, also the phase angle between E_R and E_2 or E_1. From the total synchronous reactance and total resistance of the armature circuit we calculate the angle θ. Then, we are able to find the value of the angle between I_S and E_1, or between I_S and E_2. The value of I_S (in amperes) is equal to E_R (in volts) divided by the total synchronous impedance (in ohms) of the circuit between armatures. Then

$$P_1 = E_1 I_S \cos\alpha \quad \text{and} \quad P_2 = E_2 I_S \cos\beta.$$

Notice that P_1 is positive, representing generator action, and that P_2 is negative, representing motor action. This results in pulling vector E_1 back in clockwise direction and pushing vector E_2 ahead in counter-clockwise direction, thus bringing them more nearly into diametrically opposite phase relation, as they should have been before the switch was closed. The total power transformed into heat in the armatures should be

$$I_S^2 (R_1 + R_2) = P_1 + P_2 = (E_1 I_S \cos\alpha + E_2 I_S \cos\beta).$$

The mechanical power P which is interchanged between the machines when they are out of phase and which tends to bring them into phase, is a very important factor in parallel operation of alternators. It is called the **synchronizing power.** Parallel operation is always more satisfactory when the alternators are designed so that a small change of phase relation between E_1 and E_2 produces a large amount of synchronizing power. By comparing Fig. 66 with Fig. 65, we see that this desirable condition is obtained when the reactance of the armatures is large compared with their resistance, because then the angle θ is large, which makes the angle α small and the angle β large for a given phase difference between E_2 and E_1 and a given value of synchronous impedance. Thus, if difficulty is experienced in keeping alternators in parallel, conditions may be improved by inserting reactances in series with the armatures (similar to current-limiting reactances) provided the resistance is not increased in proportion. However, if the reactance is made too large, the amount of the synchronizing current is reduced more than its power-factor is increased and thus the synchronizing power may be reduced. It is possible to determine the conditions under which synchronizing power attains a maximum value for given machines.

Difficulty in parallel operation is more likely to be experienced with alternators having good voltage regulation than with alternators having bad voltage regulation. Good voltage regulation means small armature reaction, small reactance and small synchronous impedance. As shown by Fig. 65 and 66, low reactance means small synchronizing power for a given phase-displacement between E_2 and E_1, or a large amount of phase-displacement to produce enough synchronizing power to hold the alternators in step, which leads to "hunting" of the alternators. Moreover, low

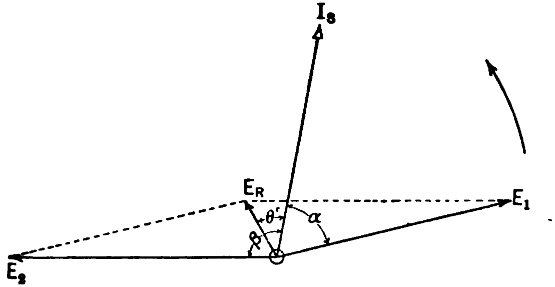

FIG. 66. The larger the angle θ is the larger β will become and the smaller α will be. The angle θ is large when the armature reactance is large in comparison with the armature resistance. Compare Fig. 65.

synchronous impedance means that a slight movement of the armatures (or E_2 and E_1 vectors) with relation to each other, or small values of E_R, will produce large values of I_S. Such movements are likely to result from the inequalities of the force acting upon the piston of a steam or gas engine as the working fluid in the cylinder expands during each stroke, unless heavy flywheels are used. If the voltage regulation of the alternators is good and the reactance correspondingly small, very slight relative movements of the armatures will produce large synchronizing currents (I_S); but under such conditions the synchronizing current represents comparatively little synchronizing power; therefore, greater phase displacements and larger values of I_S are necessary to hold the alternators in step. Hence, we see that the circulating current will be large, and this will reduce the capacity of the alternators for carrying useful load.

If one alternator tends to turn slower than another in parallel with it, the synchronizing power comes into play and prevents this. Suppose that E_1 and E_2 were exactly in phase (as shown by Fig. 64) when the parallel connection was made, but that A_1 has a tendency to run faster than A_2. This would be the case if the synchronizing lamps were brightening and darkening slowly, and the switch were closed in the middle of a dark period. A moment later E_1 would have advanced slightly on E_2, as in Fig. 65. The resultant E_R thereby produced would cause a synchronizing current I_S which would make the fast machine A_1 act as generator and the slow machine A_2 act as a motor. As a result, the fast machine would be slowed down and the slow one speeded up, until they were brought into synchronism. As this condition approaches, the resultant E_R decreases, and the synchronizing current I_S therefore gradually approaches constancy.

Example 10. Two 500-kv-a. alternators each running as a single-phase machine and each having a voltage of 2300 volts were thrown in parallel, when they were at what was believed to be the correct phase relation, that is, the e.m.f.'s were supposed to be 180°

apart. Generator A, however, was 20° ahead of this correct 180° position with regard to generator B. Each generator has a 10 per cent inherent reactance and a 2 per cent resistance. Which generator delivers synchronizing power to the other, and how great is this power?

Draw Fig. 67, and note that if the two e.m.f.'s are not exactly equal and in phase, in the opposite direction to each other, one alternator will force current through the other.

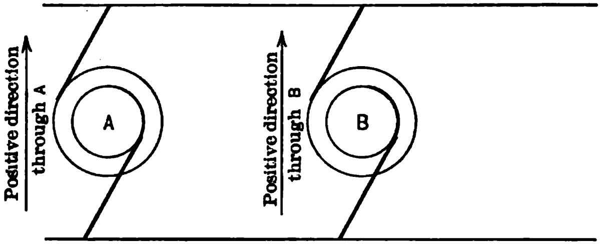

FIG. 67. The two alternators are in parallel, and their e.m.f.'s have the same positive direction with respect to the line.

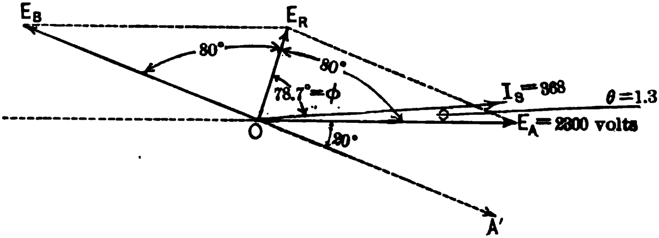

FIG. 68. The synchronizing current I_S is nearly opposite to the voltage E_B. Thus B receives synchronizing power from A in Fig. 67.

Construct Fig. 68, drawing E_B to represent the terminal voltage of generator B. Draw A' at 180° to E_B to represent the e.m.f. of generator A at the position which it should occupy if the generators were in synchronism and in phase. Then draw E_A, leading A' by 20° to represent the terminal voltage of generator A, 20° ahead of its synchronous position. The resultant of these two terminal voltages which tends to send a current circulating through the two machines is E_R. The phase difference between E_A and E_B is 180° − 20° = 160°.

$$\begin{aligned} E_R &= \sqrt{E_A^2 + E_B^2 + 2\,E_A E_B \cos 160^\circ} \\ &= \sqrt{2300^2 + 2300^2 + 2 \times 2300 \times 2300 \times (-\cos 20^\circ)} \\ &= 794 \text{ volts.} \end{aligned}$$

Per cent synchronous impedance of combination = $\sqrt{10^2 + 2^2}$ = 10.2 per cent.

Voltage to overcome impedance at full load = 0.102 × 4600 = 469 volts.

Armature current at full-load, unity power-factor = $\frac{500{,}000}{2300}$ = 217 amperes.

Synchronous impedance of combination = $\frac{469}{217} = 2.16$ ohms.

Therefore the resultant voltage E_R will force through the armatures a synchronizing current of

$$I_S = \frac{794}{2.16} = 368 \text{ amperes.}$$

The phase angle ϕ between E_R and I_S depends upon the relation of the reactance to the resistance of the armature circuit. Thus

$$\tan \phi = \frac{\text{reactance}}{\text{resistance}} = \frac{2 \times 10}{2 \times 2} = 5$$

or

$$\phi = 78.7^\circ.$$

The angle θ, the phase difference between E_A (the voltage across generator A) and I_S (the synchronizing current) $= \frac{160^\circ}{2} - 78.7^\circ = 1.3^\circ$.

The power generated by alternator A equals:

$$\begin{aligned} P_A &= E_A I_S \cos \theta \\ &= 2300 \times 368 \times \cos 1.3^\circ \\ &= 846{,}000 \text{ watts} \\ &= 846 \text{ kw.} \end{aligned}$$

The power used by B tending to synchronize B with A equals

$$\begin{aligned} P_B &= E_B I_S \cos (80^\circ + 78.7^\circ) \\ &= 2300 \times 368 \times \cos 158.7^\circ \\ &= -\,788{,}000 \text{ watts} \\ &= -\,788 \text{ kw.} \end{aligned}$$

The synchronizing power which generator A delivers to generator B is, therefore, 788 kw.

Thus generator A is pulled back by 846 kw. and generator B is pushed ahead by 788 kw. The total I^2R loss is (846 − 788) or 58 kw. The power transferred thru bus-bars is $\left(846 - \frac{58}{2}\right)$ or $\left(788 + \frac{58}{2}\right)$ or 817 kw.

Prob. 16–2. (*a*) If each lamp L_2 in Fig. 50 is rated 220 volts and its filament does not become visible until the voltage reaches 90 volts, what is the maximum phase difference that may exist between the two alternators, each generating 220 volts, while the lamps L_2 still appear to be in their dark period with S_1 closed? If the switch S_2 is closed when this phase difference exists and each alternator is 100 kv-a. 220 volts, 10 per cent reactance and 1 per cent resistance, calculate:

(*b*) Synchronizing current, in effective amperes.

(*c*) Voltage of bus-bars.

(*d*) Power transferred through bus-bars from one alternator to the other.

Prob. 17–2. Repeat calculations of Problem 16–2, on the assumption that each generator has a reactance of 5 per cent and a resistance of 1 per cent. Compare with corresponding results of Problem 16. By what percentage is the I^2R loss due to this synchronizing current increased by the change of reactance? By what percentage is the synchronizing power increased or diminished?

Prob. 18–2. Repeat calculations of Problem 16–2, but on the assumption that each generator has the same impedance but only half as much reactance. Compare with corresponding results of Problems 16–2 and 17–2. By what percentage would the armature I^2R of these alternators be greater than that of the alternators in Problem 12–2, at the same (rated) load in both cases?

23. Distribution of Load on Alternators in Parallel. In practice, an alternator A_1 would be well loaded before A_2 is started and paralleled with it. Thus, in Fig. 69, E_1' represents the terminal e.m.f. of alternator A_1, which is delivering to the external circuit a current I_1' at power-factor equal to ($\cos \alpha$). The output of A_1 is equal to ($E_1'I_1' \cos \alpha$) watts. Now alternator A_2 is manipulated so as to make its open-circuit voltage E_2 equal to E_1', and also in synchronism and in phase with E_1'. The main switch is then closed, but

A_2 neither delivers any current or power, nor takes any. To make A_2 take some of the load from A_1, we let a little more steam, or water, or gas into the prime mover which drives it, as the type may require. This makes the power input greater than the losses in A_2 and the machine speeds up, to absorb the excess as kinetic energy. This causes the vector E_2 to advance on E_1 to a new phase relation represented by E_{22}. The angular advance ϕ produces a resultant e.m.f. E_R in the local circuit of the armatures, and this produces a synchronizing current I_S, lagging θ degrees behind E_R, where I_S is defined completely as usual, by the equations:

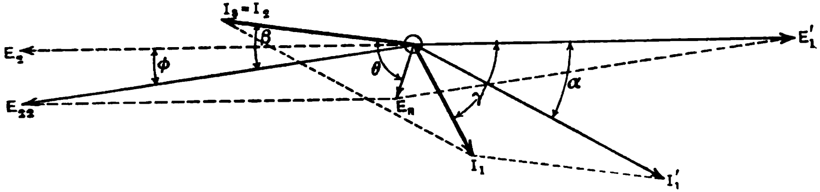

FIG. 69. The polar vector diagram for showing the result of connecting a generator with voltage E_{22} in parallel with a generator of voltage E_1', when the latter is already delivering a current of I_1' at a power-factor of cos α.

$$I_S = \frac{E_R}{Z} = \frac{E_R}{\sqrt{(R_1 + R_2)^2 + (X_1 + X_2)^2}},$$

$$\tan\theta = \frac{\text{total synchronous reactance of } A_1 \text{ and } A_2}{\text{total effective resistance of } A_1 \text{ and } A_2} = \frac{X_1 + X_2}{R_1 + R_2}.$$

Now I_S is the only current flowing through A_2, but in A_1, we have both I_S and I_1' flowing. The total current in A_1 is the vector sum of I_S and I_1', or I_1 in Fig. 69. The phase angle between I_S and E_{22} is less than 90°; therefore A_2 is acting as a generator and is *generating* electric power equal to

$$P_2 = E_{22} I_S \cos\beta.$$

The addition of the synchronizing component (I_S) has changed the total current (I_1) carried by A_1 to less than what it was (I_1') before the adjustment of A_2. Moreover, it has changed the phase angle between E_1' and the current in A_1, to a value (γ) which is greater than what previously was (α). For these two reasons, the total power output from the terminals of A_1 is less than it was before the adjustment of A_2. That is:

$$(E_1'I_1 \cos \gamma) \text{ is less than } (E_1'I_1' \cos \alpha).$$

Thus it appears that letting more power into the engine which drives A_2 has taken some load off A_1 and has put it on A_2. As the load comes on A_2, it produces a resisting torque which opposes the increase of speed that started the readjustment. Therefore the speed will increase only enough to cause A_2 to take on a load sufficient (together with the losses in A_2) to equal the increased input to A_2. When this state has been reached, the increase of speed (started by opening the throttle-valve of the prime mover) is arrested and we have equilibrium, with a steady load and current in each alternator.

The total current output of both alternators ($I_1 \oplus I_2$) should now be the same as the current (I_1') which was delivered by A_1 alone before A_2 was paralleled.

This can be shown by combining I_2 ($= I_S$) vectorially with I_1. We find that the resultant coincides with I_1'. Of course it is necessary to reverse the vector OI_2 before combining it with OI_1, because in Fig. 69 the vectors relating to A_2 have been reversed in order to make them show conditions with relation to the local circuit of the armatures of which the positive directions are opposite, whereas now we desire to know conditions with relation to the bus-bars, the positive direction of which is the same as that through both armatures.

To avoid confusion of lines, Fig. 69 has not been made entirely complete. The production of current in A_2 causes a voltage drop in the armature, which makes the terminal voltage differ from E_{22}, as to both value and phase. The change in the value and in power-factor of the current in A_1 causes a change in the voltage drop within this armature, so that the terminal voltage of A_1 becomes different from E_1' as to both value and phase. The currents will continue to change, and **equilibrium will not be reached, until both the terminal voltages and the frequencies have become absolutely equal.**

Prob. 19–2. Each of the single-phase alternators shown in Fig. 50 is rated 100 kv-a., 220 volts, and each generates a practically harmonic e.m.f. wave. Each has a resistance of 2 per cent and a "sustained" short-circuit current five times normal full-load current, when the field current is such as will produce 220 volts at terminals on full non-inductive load. Calculate:

(*a*) The synchronous impedance, in ohms.
(*b*) The effective resistance of the armature, in ohms.
(*c*) The synchronous reactance, in ohms.

Prob. 20–2. Alternator A_1, Prob. 19, is delivering 125 per cent of its rated full-load kv-a. to an external circuit at 220 volts and 87 per cent power-factor. Alternator A_2 is synchronized perfectly, and connected in parallel to the bus-bars. Draw a vector diagram similar to Fig. 69, showing this condition of affairs. From the resistance and reactance of A_1 and the current flowing in it, locate the vector representing its total induced e.m.f., or excitation voltage on this vector diagram.

Prob. 21–2. Now let the driving force behind A_2 in Problem 20–2 be increased enough to advance the vector of its induced e.m.f. by 10 electrical degrees (that is, $\phi = 10°$ in Fig. 69). Assume that the amount of current delivered to the external load and its phase relation to the induced e.m.f.'s, and the induced voltage of both generators, remain unchanged. Calculate for the instant before any adjustment of phase occurs:

(*a*) The current flowing in A_2.
(*b*) The current flowing in A_1.
(*c*) The power generated in A_2.
(*d*) The power generated by A_1. Draw these current vectors on the vector diagram of Problem 20.

Prob. 22–2. On the vector diagram similar to Fig. 69 representing conditions of Problem 21 add to E_{22} the reactions due to I_2, and thus find the vector representing terminal voltage of generator A_2. Notice that this is not equal to the terminal e.m.f. of A_1, as it was before the redistribution of load. Show how the terminal e.m.f. of A_1 becomes equal to that of A_2.

Prob. 23–2. The two alternators specified in Problem 19 are operating together in parallel each loaded to rated kilovolt-amperes at 87 per cent power-factor, the external load being 200 kv-a. at 220 volts and 87 per cent power-factor. Draw the complete vector diagram showing terminal and induced e.m.f. and armature current in each alternator, and total current delivered from bus-bars. Calculate:

(*a*) The value of generated e.m.f. by synchronous impedance method.

(*b*) The value of the synchronizing component of armature current, in amperes.

24. Governing. Speed Regulation. Load Distribution. We have just seen that in order to increase the load on an alternator which is operating in parallel with others, we must adjust the prime mover which drives this alternator so as to make it push harder. The converse of this is also true; if we desire to decrease the load on this alternator we reduce or throttle the power supplied to drive it, or make the adjustment which would reduce the speed of the alternator if it were operating alone. But as soon as A_2 begins to fall behind A_1, or lag in phase, a resultant e.m.f. and a synchronizing current are produced in the circuit of the armatures. This synchronizing current represents motor action in the lagging machine and generator action in the leading machine (see Art. 22). The resultant of these actions and the load currents already flowing in the armatures is to reduce the generator action or electrical power output of the alternator whose prime mover was throttled down, and to increase correspondingly the generator action and electrical output of the other alternator. The throttled generator will continue to fall behind (as to phase, but not as to speed), and the synchronizing component of current will continue to grow, until the load on this alternator (plus the losses) is reduced to just equal the power received from the prime mover. Then the alternator will cease falling behind and the currents and power outputs of the alternators will be steady.

In most power plants, the prime mover driving each alternator has its own governor, or device to keep the speed "constant" by shutting off automatically the supply of steam- or gas- or water-power as the load on the alternator is reduced, or increasing the power supply as the load on the alternator is increased. This is necessary if the units are ever to be operated singly and without constant attention. A governor is not "stable" or safe unless some increase of speed is necessary to reduce the power supply and some decrease of speed to increase the power supply. Now suppose that the governor controlling the speed of A_2 has a speed regulation of 2 per cent, or allows the speed to rise to 102 per cent of the full-load speed when the load is reduced from rated full-load (of 1000 kv-a., non-inductive) to zero-load. And suppose the governor controlling the speed of

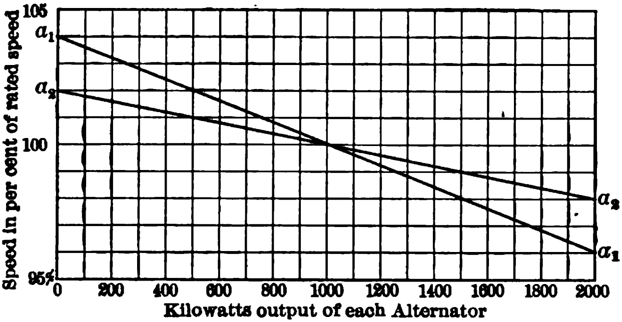

FIG. 70. The relation between output and speed of generators A_1 and A_2.

A_1 has a speed regulation of 4 per cent, and the rated full-load is the same as for A_2.

If the speed of each unit varies in exact proportion to the load, we have conditions as represented in Fig. 70. Thus, at 100 per cent of rated load and speed, A_1 and A_2, each delivers 1000 kw., and the total load is 2000 kw.; at 101 per cent of rated speed, A_1 delivers 750 kw. and A_2 delivers 500 kw., the total load now being (750 + 500 = 1250) kw. Proceeding in this way, we get the data from Fig. 70 by which to plot curves as in Fig. 71, showing the distribution of load between A_1 and A_2 at various values of total load on the bus-bars. From this it appears that **the unit having the better speed regulation (A_2) drops load faster while the**

total load is being reduced, and takes on load faster when the total load is being increased. Two units operating in parallel will keep the load more nearly proportionally divided between them for all values of total load, when their speed regulation is more nearly the same. It is generally found that a closer speed regulation than 4 per cent is undesirable for alternators.

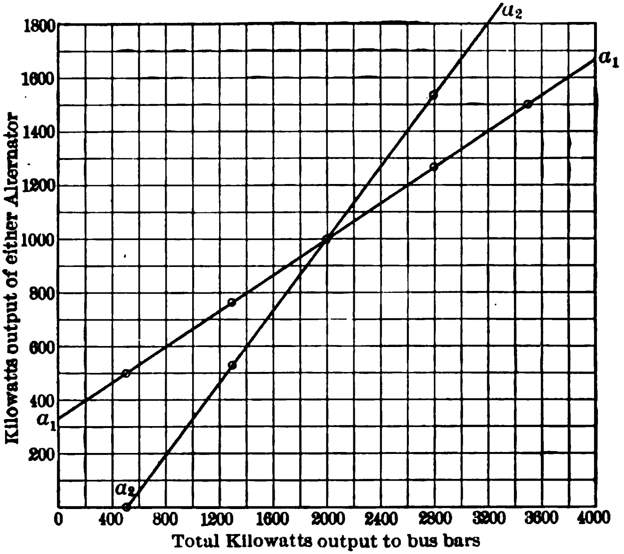

FIG. 71. Relation between total output to bus-bars and the output of each generator. These curves are obtained from those in Fig. 70.

When the switchboard operator desires to make A_2 take a larger share of the load than the automatic action of its governor permits it to take, he adjusts the governor in a way which would make the unit run at a higher speed if it were operating alone. As has been seen, when the alternator attempts to run faster it automatically takes on enough more load to prevent its speed from increasing; its rotor merely forges ahead slightly relative to the other rotor but continues to rotate at practically the same speed as before.

Prob. 24–2. The speed regulation of the prime mover driving alternator A_1 in Problem 23–2 is 1 per cent and that of A_2 is 1½ per cent. Rated full-load of each prime-mover is considered to be 100 kv-a. at 87 per cent power-factor, from the alternator.

(*a*) Draw curves similar to Fig. 70 and 71, showing relation between total kilowatts and kilowatts delivered by each alternator. Use a large scale for speed.

(*b*) At what per cent of rated load kw. is each unit working in Prob. 23?

Prob. 25–2. (*a*) When the total load is reduced to 100 kv-a., still at 87 per cent power-factor, what kw. is each alternator of Problem 24 delivering?

(*b*) If the terminal voltage were kept constant by a regulator, what would be the load component of current in each armature?

(*c*) The regulator used also adjusts the field excitations and generated voltages so that the power-factor of each armature is the same as that of the load. Draw a complete vector diagram for this condition, showing bus-bar e.m.f., current output from each alternator, XI and RI drops in each armature and generated e.m.f. in each armature.

25. Field Excitation Controls Power-Factor but not Load Distribution. It should be clearly understood that the adjustment of the field excitation **does not** control the distribution of load between alternators in parallel. Such adjustment affects mainly the reactive component of current flowing through each armature, by means of the synchronizing current, and merely changes the power-factor of the generators. This is shown in Fig. 72–75, which represent relations in two single-phase generators, or in corresponding phases of two similar polyphase generators. The constants used are as follows:

Volts induced in one phase of each generator (initially) = 2000.

Amperes delivered from one phase of each generator to load = 100.

Phase difference between load current and induced volts, in one phase of each generator equals 30°, or power-factor of entire circuit in each phase = 87 per cent.

Synchronous reactance of one phase of each generator = 2.0 ohms.

Effective resistance corresponding = 0.2 ohm.

Fig. 72 refers to the two alternators together, or to the bus-bars. In Fig. 73 the vectors of A_2 have been reversed to illustrate the phase relations in the local armature circuit formed by this phase of A_2 and A_1. We now increase the induced e.m.f. of A_2 by 10 per cent, making it 2200 volts. As shown in Fig. 74, we have a resultant e.m.f. $E_R = 200$ volts, which produces a nearly reactive synchronizing current (I_S) of almost 50 amperes. The total current flowing in A_2 is now I_2' and in A_1 it is now I_1'.

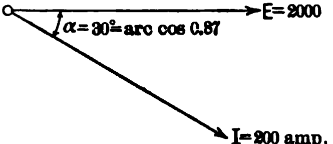

FIG. 72. The vector E represents the induced voltage of each generator with respect to the bus-bars. I represents the current delivered by both generators together.

By this adjustment of the field excitation of A_2, we have reduced the reactive component and also the total value of current in A_1, and have **raised the power-factor of A_1** to practically 100 per cent. On the other hand, we have increased the reactive component and total value of the current in A_2, and have **reduced the power-factor of A_2.** In other words, this adjustment has merely shifted the reactive component of the load current from one alternator to the other; it has not altered the load distribution, because the power component of I_2' is practically the same as that of I_2, and the power

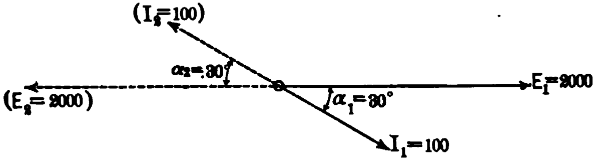

FIG. 73. The currents and voltages of Fig. 72 are here drawn with respect to the circuit between armatures.

component of I_1' is practically the same as that of I_1. In Fig. 75, we have reversed I_2' so as to make the diagram refer

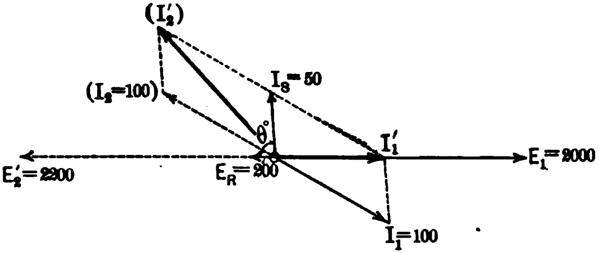

FIG. 74. The e.m.f. of A_2 has been increased to 2200 by field adjustment. The current from generator A_1 becomes I_1' and from generator A_2, I_2'.

again to the bus-bars (as in Fig. 72), instead of to the local armature circuit; it is seen that the resultant of I_1' and I_2', or the total current (I) delivered by both machines to the load, remains exactly the same as it was before the fields were adjusted.

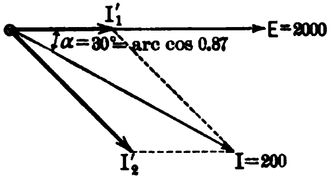

FIG. 75. The vectors of Fig. 74 are here drawn with respect to the bus-bars instead of with respect to the armature circuits. Note that I and E are exactly as in Fig. 72.

26. Control of Frequency and Voltage of Bus-bars. If the bus-bar frequency happens to be low, we must adjust the governors of all the engines correspondingly. Adjustment of any one governor will merely cause this alternator to pick up more load; it would take on the entire load of the station (which would usually be enough to injure it), before the frequency of the bus-bars would be raised appreciably.

If the voltage of the bus-bars is low, we must adjust the field excitation of every alternator correspondingly, unless we desire to shift the reactive component from one alternator to another. We can see from Fig. 74 that further adjustment of the excitation of either alternator would be too much as it would **increase the total reactive current** in both of them, causing the average power-factor of the generators to be lower than that of the load, with consequent reduction of useful capacity.

27. Disconnecting Alternator from Bus-bars. When the kilovolt-ampere load on the bus-bars has decreased so that one of the parallel generating units can be stopped without overloading the alternators that remain, we must consider the best manner of doing this. Any kind of switch is likely to be injured if opened when a heavy current is flowing through it, and particularly when the voltage of the circuit is high. The bad effects seem to increase in some relation to the kilovolt-amperes removed from the circuit by the opening of the switch. They are of all degrees of seriousness, from mere roughening of the contact surfaces and carbonization of the oil (in which the break is usually immersed), to distortion and breakage of the switch parts, throwing-out of oil, and sometimes violent explosion of the switch when opening a short-circuit.

To remove an alternator smoothly from parallel operation, we first decrease the power supply to the engine driving it (by closing the throttle-valve or by adjusting the governor in the direction of a lower speed), until the wattmeter in the circuit of this armature shows that the alternator is delivering practically zero power. If the ammeters now indicate any current through this armature, it must be reactive or quadrature current, and as such it may be reduced to zero value by adjusting the field current of this alternator. When the wattmeter and ammeters all indicate nearly zero, the switch may be opened without injury. Finally, after being disconnected from the electrical circuit, the alternator may be brought to rest by completely shutting off its supply of steam or motive power.

28. The Synchronous Motor. If the supply of steam or other power to the engine is shut off while the alternator is still connected to the bus-bars (on which other alternators are operating in parallel), the unit **will not stop** but will continue to rotate at exactly synchronous speed, taking enough power from the other alternators through the bus-bars to supply all of its losses. The alternating-current generator has in fact simply become a "synchronous motor." Any alternator may be used in this way as a synchronous motor; it is merely a matter of changing its function, and not its construction. However, some synchronous motors intended for use only as "**synchronous condensers**" to correct the power-factor of circuits, have a lighter and cheaper construction than synchronous machines intended to transform mechanical power to electrical power or vice versa.

The action of the synchronous motor may be understood readily from Fig. 65, if we consider that the mechanical driving power to A_2 has been removed. The losses in the A_2 unit compel it to slow down, but instantly this causes it to fall behind A_1 in phase; a resultant e.m.f. E_R is thereby produced in the circuit of the armatures, which causes a current I_S to be delivered to A_2 by the other alternator. This synchronizing current I_S has such phase relation to E_1 and E_2 that it produces strong motor action in A_2, tending to prevent it from slowing down. When the rotor A_2 reaches a position far enough behind the other rotors to cause it to take in enough electrical power to equal its total losses, there is equilibrium between input and losses, and consequently no further tendency for A_2 to fall behind. It therefore continues to rotate at synchronous speed. We might even put a pulley on the shaft of A_2 and make it deliver mechanical power to some other machine. This would cause it to fall back a little further, as shown in Fig. 76, from E_2' to E_2''. In consequence, E_R' would be increased to E_R'', and $I_S \left(= \frac{E}{Z}\right)$ would be increased proportionally from I_2' to I_2''. The mechanical power or motor action developed in A_2 is increased, as it is evident that

$$(E_2'' \times I_2'' \times \text{cosine of angle between } E_2'' \text{ and } I_2'')$$

is greater than

$(E_2' \times I_2' \times$ cosine of angle between E_2' and $I_2')$.

The rotor of A_2 would continue to fall behind until the value of electrical input ($E_2''I_2'' \times$ cos angle between E_2'' and I_2'') has become equal to the output plus the losses, and then it will continue to rotate steadily in synchronism.

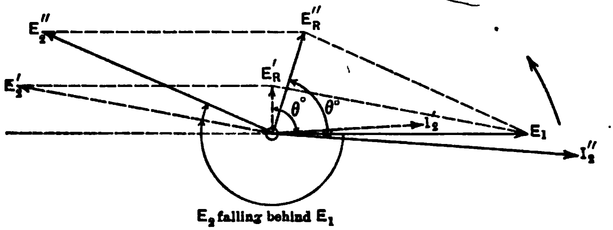

FIG. 76. As more mechanical power is taken from A_2 the voltage E_2 drops back to E_2'' and current increases from I_2' to I_2''.

29. Leading Currents in Parallel Alternators. Synchronous Condenser. When the field of one of the alternators is "over-excited," or is generating a higher e.m.f. than that induced in the other alternator, the synchronizing current is nearly in quadrature with the induced e.m.f. of both alternators. Thus, in Fig. 64 we see that the current I_S which circulates between the alternators whose e.m.f.'s are unequal, lags nearly 90° behind the e.m.f. of the over-excited alternator, and leads the e.m.f. of the under-excited alternator by nearly 90°. We may, therefore, cause an alternator to deliver leading currents by paralleling another alternator with it, and raising the field excitation of the latter. It is usually more convenient to remove the driving power from this second alternator, allowing it to operate as a synchronous motor. By this method, large leading currents can be produced much more cheaply than by ordinary condensers. This so-called "**synchronous condenser**" is now much used for improving the power-factors of generating plants and transmission systems which are required to deliver large amounts of quadrature current.

SUMMARY OF CHAPTER II

ALTERNATORS MAY SAFELY BE CONNECTED IN PARALLEL when:

- (a) Their terminal e.m.f.'s are equal to one another at every instant. This means
  - (1) The frequencies of all are the same.
  - (2) The e.m.f.'s are all in phase.
  - (3) The wave-forms are the same.
- (b) Similar terminals are connected together.

Synchronizing lamps are used to indicate when the conditions in two generators are such that they may safely be connected in parallel.

- (a) If a lamp becomes light and dark alternately, it indicates that the frequencies of the two alternators are different.
- (b) If lamps remain dark, it means that:
  - (1) Either the alternators have the same effective voltage, are in phase, and in synchronism,
  - (2) Or one of the lamp filaments is broken. Raise speed of one machine slightly to test this.
- (c) If the lamps remain light, it means that:
  - (1) Either the e.m.f.'s have different effective values, but are in phase and in synchronism.
  - (2) The e.m.f.'s have greatly different frequencies.
  - (3) Or the alternators are in synchronism but out of phase.

A SYNCHRONIZING CURRENT flows between two generators connected in parallel, if their e.m.f.'s do not fulfill the above conditions for parallel operation. This synchronizing current tends to bring the two e.m.f.'s into the proper relation.

THE SYNCHRONIZING POWER is the power transferred from one machine to the other by means of the synchronizing current. It is given by the faster generator to the slower generator and by means of its motor action increases the speed of the slower or lagging generator. A high reactance in the armature circuit, even though it produces a poor regulation, is desirable because it increases this synchronizing power by raising the power-factor, and reduces hunting.

WHEN A SPARE ALTERNATOR IS SWITCHED IN, its e.m.f. is equal to the e.m.f. of the bus-bars and in synchronism with it. The speed of the prime mover is then increased, which increases the speed of the spare alternator and advances its phase relation and causes a synchronizing current to flow through the armatures of all the machines on the bus-bars. The resultant of this synchronizing current and the current which each generator is delivering will be smaller than the original currents delivered by the several generators, the phase difference between the resulting current and the e.m.f.'s of the generators will be greater and therefore the power-factors will be less. Accordingly, these generators are delivering less power to the bus-bars than before the spare generator was thrown in. The synchronizing current is the only current delivered by the spare generator. If no change is made in the current drawn from the bus-bars, the vector sum of the currents in the individual armatures remains the same as before the spare generator was added, the decrease in power from the original generators being equal to the power delivered by the spare generator.

AN AUTOMATIC GOVERNOR ON THE PRIME MOVER decreases the power supply when the load on a generator connected to it falls off and the speed increases. This causes the speed increase to be very small, although there must be some increase in order to operate the governor. The better the regulation of a prime mover, the greater the share it will take when the total load is increased, and the smaller the share it will take when the load is decreased. Hence prime movers of parallel generators should have nearly the same speed regulation.

IN ORDER TO CAUSE A CERTAIN ALTERNATOR TO TAKE A CERTAIN SHARE OF THE LOAD, the governor on the prime mover is adjusted by hand in a way which would cause it to run at a higher speed if it were operating alone. The alternator forges ahead and slightly advances the phase of its e.m.f. which causes it to take a larger share of the load. **INCREASING THE FIELD OF A GENERATOR TO RAISE ITS VOLTAGE WILL NOT CAUSE THE GENERATOR TO TAKE A GREATER SHARE OF THE LOAD,** it merely changes the phase relation between the currents and e.m.f.'s of the several alternators.

TO RAISE THE VOLTAGE OF THE BUS-BARS, the field strength of all the generators must be increased.

TO RAISE THE FREQUENCY OF THE BUS-BARS, the speed of all the alternators must be increased.

TO DISCONNECT AN ALTERNATOR FROM THE BUS-BARS, decrease the power from the prime mover until the wattmeter shows that the generator is delivering practically no power. Then adjust the field current until the ammeter shows no current through the armature of the alternator; open switches and shut off all power from the prime mover.

AN ALTERNATOR WILL CONTINUE TO ROTATE AS A SYNCHRONOUS MOTOR if, when running as a generator in parallel with other generators, the power is shut off from its prime mover and it remains connected to the bus-bars.

OVER-EXCITING THE FIELDS OF A SYNCHRONOUS MOTOR causes it to take currents which are leading with respect to the e.m.f. of the line and thus improves the power-factor of a line supplying lagging loads. When so used the machine is called a SYNCHRONOUS CONDENSER.

PROBLEMS ON CHAPTER II

NOTE. In some of the following problems, graphical solutions are much more convenient than trigonometric or algebraic solutions. The graphical solution is permissible if the drawing be made to such large scale that any desired quantity may be scaled off with engineering accuracy.

Prob. 26-2. A direct-current shunt generator rated 100 kw., 220 volts is operated in parallel with another rated 200 kw., 220 volts. The voltage regulation of both generators is 4 per cent. Draw curves using amperes in external circuit as abscissas, and as ordinates the following:

(*a*) Amperes in 100-kw. generator.
(*b*) Amperes in 200-kw. generator.
(*c*) Terminal voltage.

The external load varies from 300 kw. at 220 volts, to zero-load. Assume that the voltage drop is exactly proportional to armature current in each generator, and that they share the load properly at full load.

Prob. 27-2. The 100-kw. d-c. generator of Problem 26 operates in parallel with a 200-kw. 220-volt generator having a 2 per cent voltage regulation. Draw curves similar to those for Problem 26. Carry the total load to 450 kw.

Prob. 28-2. If the two d-c. generators are adjusted to take their respective rated full-loads when operating in parallel, at rated voltage, what would be their respective open-circuit or zero-load voltages when cut apart?

(*a*) In Problem 26?

(*b*) In Problem 27?

Prob. 29–2. While the generators of Problem 27 are operating in parallel at 220 volts, both adjusted so as to be fully loaded, a sudden overload comes on the bus-bars, increasing the external current to 200 per cent of normal. (*a*) What per cent overload does each generator assume? (*b*) By what percentage is the rate of heat development (I^2R) in its armature increased above normal rate? Assume curve between terminal volts and armature current to be a straight line for each generator.

Prob. 30–2. What difference between the voltages of A_1 and A_2 (expressed in per cent of their rated voltage) is sufficient to make the synchronizing current alone of Fig. 50 equal to rated full-load current, the e.m.f.'s being exactly in phase? Each alternator is rated 100 kv-a. 220 volts, and has a resistance of 1 per cent and a reactance of 10 per cent.

Prob. 31–2. When this difference (specified in Problem 30–2) exists between the effective values of E_1 and E_2, what is the I^2R loss and the power developed in each armature by the synchronizing current at the moment of closing the switches, expressed as percentage of the values of these quantities at rated full-load (non-inductive)? The machines are otherwise not loaded and the average of their voltages is equal to the normal induced voltage.

Prob. 32–2. An alternator rated 500 kv-a. is operating in parallel with another rated 1000 kv-a. Switchboard instruments show that the former is carrying 400 kw. load at 95 per cent power-factor lagging, while the latter is carrying 600 kw. at 65 per cent power-factor lagging. Each prime-mover has 5 per cent speed regulation.

(*a*) Why is this not the best operating condition?

(*b*) What adjustments should be made?

(*c*) After these adjustments have been made, what should be the load in kilowatts on each generator, and its power-factor in per cent?

(*d*) What is the power-factor of the load on bus-bars, and its total value in kilovolt-amperes?

Prob. 33–2. In order to distribute the load in proportion to the rated capacities of the alternators in Prob. 32, and to keep the bus-bar frequency at 60 cycles per second, by what amount must the mean turning effort of each prime mover be increased or diminished, expressed in terms of (*a*) pound-feet, (*b*) per cent of the former value.

Note. The 500 kv-a. machine has 8 poles and the 1000 kv-a. machine has 10 poles.

Prob. 34-2. The bus-bar pressure in Prob. 32 is 2300 volts, three-phase, as rated. Each (delta-connected) alternator has 2 per cent resistance, and synchronous reactance 40 per cent. Draw two complete vector diagrams illustrating conditions before readjustment, one with reference to the common bus-bar voltage, the other with reference to the local circuit between the two armatures. Calculate therefrom the following quantities:

(*a*) Total kilovolt-amperes delivered by each alternator.
(*b*) Amperes per terminal delivered by each alternator.
(*c*) Voltage that would be delivered between terminals of each alternator if disconnected from the bus-bars.

Prob. 35-2. Repeat the diagrams and solutions of Prob. 34, for the conditions existing after readjusting the alternators so that they both operate at the same power-factor as the total load on bus-bars, and divide the total power (kw.) in proportion to their rated capacities. Bus-bar voltage kept at 2300.

Prob. 36-2. Two parallel alternators having constants as in Prob. 34, together carry the same total load as in Prob. 32. However, by manipulation of field currents the larger machine is made to deliver its full rated load of 1000 kv-a. to the bus-bars at 2300 volts. Neglecting the effect of the internal power losses in the alternators upon the distribution of load between them and upon the speed of the engines, calculate:

(*a*) Power factor of each alternator, in per cent.
(*b*) Increase or decrease of useful flux or total induced e.m.f. of each alternator, as per cent of corresponding value from Prob. 34.

Prob. 37-2. Two alternators having constants as in Prob. 34, operate as in Prob. 32 before the readjustment was made. Draw a vector diagram to show the effects of altering the steam supply to both prime movers by such amount that the same total kilowatts is equally divided, and the frequency is not altered. Neglect the power lost in the alternators. The fields are readjusted so as to give each alternator the same power factor as the total load, and the same bus voltage of 2300. Calculate: (*a*) Change of phase angle between induced e.m.f.'s, from Prob. 35. (*b*) Change in amount of total induced e.m.f. of each alternator, from Prob. 35.

Compare these results.

Prob. 38–2. A three-phase, 6600-volt alternator rated 500 kv-a. is parallel with another rated 1000 kv-a., 6600 volts, 60 cycles. The inherent speed regulation of the engine driving the first alternator is 5 per cent and of the second alternator 4 per cent, based on 85 per cent power-factor at full-load of generator.

(*a*) What is the greatest combined load in kilovolt-amperes that these alternators can deliver without overloading either more than 25 per cent beyond its rated capacity?

(*b*) What would be the relation between the power-factor of each alternator and the bus-bar power-factor, under this condition?

Prob. 39–2. What are the kilovolt-amperes, kilowatts and power-factor of the total load delivered from the bus-bars, if each alternator in Prob. 38 is delivering its rated full-load kilovolt-amperes, but the 500 kv-a. machine operates at 90 per cent power-factor while the 1000 kv-a. machine operates at 70 per cent power-factor?

Prob. 40–2. How many more kilowatts could be taken from the bus-bars at the same voltage and power-factor without exceeding the rated capacity of either machine, if the field excitation and throttles of the alternators in Prob. 39 were readjusted so as to make the best use of each machine?

Prob. 41–2. Two alternators rated 2300 volts, three-phase, 60 cycles have full-load capacities of 500 and 1000 kv-a. respectively. The first alternator gives a sustained short-circuit current equal to two times rated load current, with full field excitation,* while the second gives four times full-load current. In both alternators the ratio of synchronous reactance to resistance is 10 : 1. If these two alternators are thrown into parallel when the terminal voltages are each equal to 2300 but 10 electrical degrees out of phase, calculate:

(*a*) Resultant voltage in local circuit.
(*b*) Synchronizing current in amperes.
(*c*) Synchronizing power in watts.
(*d*) Synchronizing torque in per cent of normal full-load torque of the smaller alternator.

Both alternators are star-connected.

Prob. 42–2. Repeat the calculations of Prob. 41, considering that the alternators are paralleled when only two electrical degrees out of phase.

(*a*) Is the synchronizing power directly proportional to the phase-difference between the e.m.f.'s?

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* Full field excitation here means that which would give rated voltage at no-load.

Prob. 43-2. Both alternators specified in Prob. 38 are delivering their rated kilovolt-amperes to the bus-bars, at rated voltage and frequency, and 85 per cent power-factor. Assuming that the characteristics of each governor are such that the speed changes by a uniform amount for each kilowatt of load added or dropped by the machine, as expressed by the per cent speed regulation, calculate:

(*a*) Frequency when total load on bus-bars is reduced 50 per cent.

(*b*) Kilowatts delivered by smaller alternator under this condition.

(*c*) Kilowatts delivered by the larger alternator under this condition.

Prob. 44-2. (*a*) At what value of total load, in kilowatts, will one of the alternators of Prob. 43 cease to deliver power?

(*b*) Which alternator will cease to deliver power?

(*c*) What will be the frequency under this condition?

Prob. 45-2. If the supply of driving power to the under-loaded machine in Prob. 44 be discontinued, and the total loss in iron and armature copper and in friction be equal to 5 per cent of its rated full-load (non-inductive) output, calculate:

(*a*) What kilowatts will be taken by the synchronous motor now operating from the remaining generator?

(*b*) What will be the frequency?

Prob. 46-2. Solve Prob. 41 on the assumption that the ratio of synchronous reactance to resistance in both alternators is only 2 : 1. Compare corresponding results of these two problems, and discuss therefrom the relation between synchronous reactance and resistance, and synchronizing power, other things being equal.

Prob. 47-2. Solve Prob. 46 on the assumption that a 10 per cent reactance has been inserted in each of the cables leading from the armature of the larger alternator to the bus-bars. Compare the synchronizing current and synchronizing power with corresponding results of Prob. 46 and discuss this feature of current-limiting reactances.

Prob. 48-2. The two alternators specified in Prob. 38 are adjusted so that each is delivering 75 per cent of its rated kv-a., at 85 per cent power-factor, rated voltage and frequency. Assuming the same change of speed per kilowatt of load as specified in Prob. 38, calculate:

(*a*) What maximum total kilowatts can be taken from the bus-bars without overheating either armature?

(*b*) What will be the frequency at this load?

Prob. 49–2. (*a*) At what value of total load (kilowatts) will the two alternators, after being adjusted as in Prob. 48 and then left to themselves, divide the load (kilowatts) equally between them?

(*b*) What will be the frequency under this condition?

Prob. 50–2. When the total bus-bar load on the two alternators of Prob. 48 is such that the frequency has become equal to 61.5 cycles per second, calculate:

(*a*) Kilowatts delivered by the 500 kv-a. alternator.
(*b*) Kilowatts delivered by the 1000 kv-a. alternator.
(*c*) Total kilowatts output from bus-bars.

Prob. 51–2. A three-phase, star-connected alternator, rated 100 kv-a., 2300 volts, 60 cycles, is delivering 125 per cent of its rated kv-a. at rated voltage and 80 per cent power-factor to the bus-bars. It has a resistance of 4 per cent and synchronous reactance of 40 per cent.

Another alternator identically similar has its open-circuit e.m.f. brought up to exactly the proper voltage, frequency, and phase relation, and the parallel connection is completed. Assuming that the open-circuit voltage, or total induced voltage, or excitation voltage of the incoming alternator is raised in five steps, each ten per cent of the initial value, while the kilowatt output from bus-bars remains unaltered, calculate for each step:*

(*a*) Resultant e.m.f. in local circuit.
(*b*) Synchronizing current in amperes.
(*c*) Kilowatts which tend to be shifted from one alternator to the other by means of the synchronizing current.
(*d*) Terminal voltage.

Draw curves showing change of each of these quantities, all with respect to excitation volts as abscissas.

Prob. 52–2. If both prime movers in Prob. 32 have inherent speed regulation of 5 per cent, calculate the actual values of the following quantities when the total load has increased to 1500 kv-a. at the same power-factor as in Prob. 32, and the excitation voltages of both alternators are adjusted to give rated terminal voltage at load power-factor:

(*a*) Current per terminal from each alternator to bus-bars.
(*b*) Kilovolt-ampere output of each alternator.
(*c*) Power-factor of each alternator.
(*d*) Frequency.

Prob. 53–2. To what values of kilowatts, power-factor, and kilovolt-amperes should each alternator of Prob. 51 be adjusted, in order that the total losses shall be reduced to minimum, while the amperes and power-factor of the total load on bus-bars remain constant?

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* Do not consider change of phase relation of armature e.m.f.'s due to the shifting of load.

Prob. 54–2. When the alternators are paralleled as synchronized in Prob. 51, what will be the values of the following quantities if the excitation voltage and the steam supply of each alternator is adjusted so the total power and the reactive load are equally divided at rated terminal voltage:

(*a*) Kilowatt output of each alternator?

(*b*) Kilovolt-ampere output of each alternator?

(*c*) Power-factor of each alternator?

(*d*) Per cent by which the excitation voltage of each alternator must be increased or diminished?

Prob. 55–2. Draw a complete vector diagram showing relations between current and excitation voltage in both machines, terminal voltage, and total current delivered from bus-bars, for the phase relation of armatures which gives maximum synchronizing power, as nearly as you can determine it for the alternators specified in Prob. 51. Calculate the maximum synchronizing power in kilowatts. Note that power taken in by a machine represents "synchronizing power" only when it exerts motor action, and not when it merely supplies I^2R loss.

CHAPTER III

TRANSFORMERS

FUNDAMENTAL PRINCIPLES. EFFICIENCY AND REGULATION

A TRANSFORMER changes alternating current at any given voltage into alternating current of the same frequency but at some other voltage. It is to be distinguished from a **converter**, or a **rectifier**, which takes in alternating current and delivers direct current, or vice versa.

Were it not for the transformer, there probably would not be any large systems for generating and distributing power by alternating-current electricity such as we now have. It was the development of an efficient transformer which influenced American engineers to use alternating current instead of direct current where large quantities of power were to be transmitted over long distances.

The reasons for this statement have been indicated in the First Course, Chapter I. By means of the transformer, we may transmit electric power at high voltages which permit the wires to be small and correspondingly long, without prohibitive expense for copper or excessive I^2R loss on the lines. At any point along the line where we need some power, we simply tap on a transformer — the simplest, most rugged and durable and most efficient electrical apparatus that has ever been devised, requiring less space and attention, and less investment per kilowatt capacity, than any other electrical machine. By suitable winding of the transformer, we obtain alternating current at low pressure, of any value best suited to the lamps, motors, heaters, or other devices that are to be operated.

30. Structure of the Transformer. The purposes for which transformers are used in systems for distributing electrical power have been explained in the First Course. There also will be found pictures illustrating the external appearance of some types of transformers, and diagrams showing how and where they are connected into the system. Before proceeding to examine in some detail the exact theory of these connections, the proper methods of operation and the operating characteristics of various types of transformers, all of which are to be treated in this and the following chapter, it would be well to familiarize ourselves with the structure and parts of the transformer.

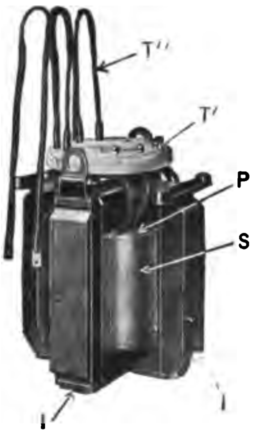

FIG. 77. The core and coils of a small transformer removed from the case. The coils S and P are linked together by the core I. *The Western Electric Co.*

Figure 77 shows the essential interior parts of a small (10 kv-a.) transformer such as is used to supply power to light a building or a group of buildings. One or more coils S of heavy wire and one or more coils P containing a much larger number of turns of smaller wire surround or are surrounded by a core I made of laminations or sheets of special steel. The terminals of the heavy wire coils are brought out at T'', and the terminals of the light wire coils at T'. These two sets of coils and the iron core which links them together are the essential parts of the transformer. Fig. 78 is a diagram of a top view of a transformer of this type. The coils marked "primary" are connected together and to the source of power, while the coils marked "secondary" are connected together and to the load. The alternating-current power is transferred from primary to secondary by means of magnetic flux in the iron core, which is shown dark in Fig. 78.

31. Methods of Cooling Transformers. The losses which occur in transformers are very small, usually not more than from 2 to 4 per cent of the input. These losses always result in heating the transformer. When the transformer is in operation, the currents in the primary and secondary coils cause a loss of power, I^2R, due to resistance. The flux in the iron core also causes a loss of power due to hysteresis and eddy currents. All of these losses appear as heat, and the temperature of the transformer will rise above that of the surroundings by an amount sufficient to cause the heat to flow away as fast as it is generated within the transformer. The core and coils are generally immersed in mineral oil or petroleum within a casing of sheet iron or cast iron. The purpose of the oil is not only to improve the insulation between the coils, but also to carry the heat from the hot inner parts of the coils and core to the cooler outside surface of the casing, by means of the circulation which is set up in the oil. The coils are separated from one another and from the cores by spaces called oil ducts (see Fig. 78) which permit the oil to circulate around the innermost parts where the heat is generated, thus tending to keep the temperature more even throughout the transformer, or to avoid "hot spots."

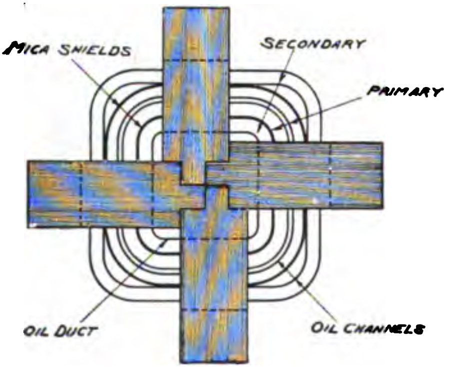

FIG. 78. The diagram of the top view of a transformer of the shell type shown in Fig. 77. *The General Electric Co.*

The power capacity and the losses of a well-designed transformer increase approximately in proportion to the volume of its core and coils, or to the cube of its dimensions, other things being equal. But the exterior surface from which heat is naturally dissipated increases only in proportion to the square of its dimensions. That is, if we double all dimensions of the transformer, the amount of iron in the core and of copper in the coils will be approximately 2^3 or 8 times as great. If these materials were used to just as good advantage as before, the kilovolt-ampere capacity would be also 8 times as great; and assuming the efficiency to be approximately the same, the losses would be 8 times as large as before. But the exterior surface of the transformer is increased only 2^2 or 4 times when the dimensions are doubled. Therefore, we must lose 8 times as much heat as before, from 4 times as much surface, or twice as much heat per square foot of surface, per minute. This requires a temperature rise approximately twice as great. Obviously, transformers of the larger sizes must be provided with additional cooling surfaces or some means of carrying away heat faster from the natural cooling surfaces, in order to avoid excessive temperatures.

Some large transformers, particularly older ones, are of the air-blast type. They are placed over a chamber from which a blower forces a strong current of air through numerous ventilating ducts that are left between the coils and the core. Such transformers are not immersed in oil, and therefore have the following disadvantages. First: if a fault occurs in the insulation, the resulting arc is not suppressed (as by the oil in the oil-filled type) but is rather fanned into flame by the air currents. Second: although the air used is cleaned, yet, in spite of this precaution, the transformer often becomes coated with dirt, which is bad for the insulation. Third: if anything occurs to stop the blowers, the temperature rises very rapidly. Fourth: air-blast transformers are more bulky for a given kilovolt-ampere capacity than water-cooled or oil-cooled types, and are not satisfactory for pressures much above 35,000 volts.

Auxiliary surfaces for natural air-cooling may be furnished by nests of vertical pipes connected into the transformer case at top and bottom as shown in Fig. 79. These pipes *C* are spread far enough apart to permit natural convection currents of air to circulate freely. The case and pipes are filled with oil, which serves to carry the heat from the coils and core where it is generated, out to these cooling surfaces.

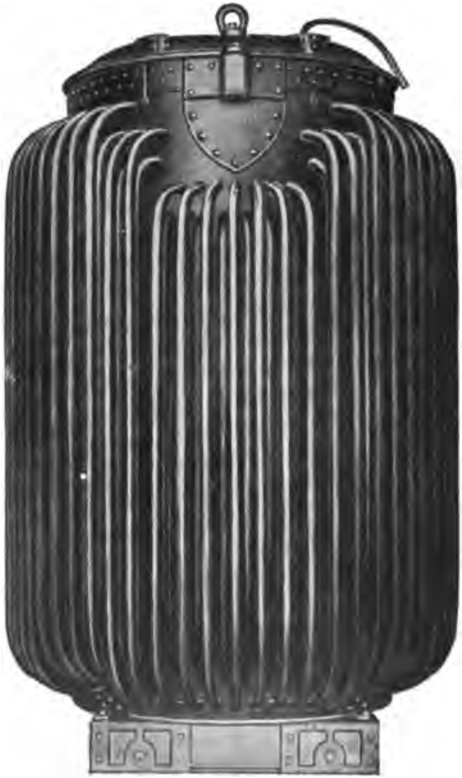

FIG. 79. Additional cooling surface is furnished by the pipes. The oil which carries the heat from the core and coils circulates through the pipes and is cooled. *General Electric Co.*

Auxiliary surface for artificial cooling by cold water is often used in the oil-insulated water-cooled (O. I. W. C.) type of transformer. One of these is shown removed from its case in Fig. 80. The space between transformer and case, and the ducts between coils and cores are all filled with oil which circulates naturally between the heated surfaces and the cool surfaces. The heated oil rises to the top of the case. The cooling coils (*C*), carrying a current of cold water, absorb the heat. The cooled oil flows down along the outside of the case to the bottom, and then repeats the process. The cooling coils should be of seamless tubing, as the slightest leakage of water into the transformer will weaken the insulation seriously. A great many of the larger transformers are of the O. I. W. C. type.

The largest transformers are cooled by a forced circulation of the oil with which the transformer is filled. By this method very much greater amounts of heat may be carried away from the surfaces of coils and cores than by any natural circulation of oil, the oil currents being more rapid and far-reaching. The oil may be cooled either inside or outside the transformer case, by water or by air currents.

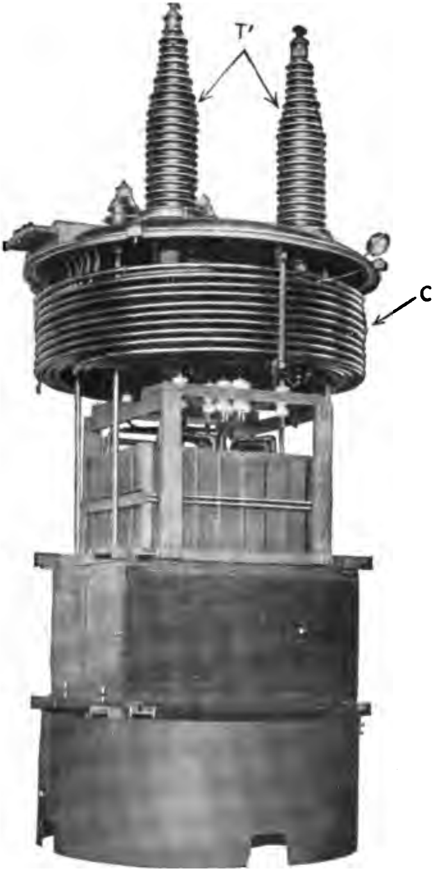

FIG. 80. Interior of an oil-insulated water-cooled transformer, showing the copper cooling coils, *C*. These coils carry water which cools the oil. *General Electric Co.*

It is difficult to bring out the terminals of a coil for very high tension work through the metal case of the transformer without danger of breaking down their insulation or of serious leakage of current between terminals. Some terminals are filled with oil and are correspondingly bulky and out of proportion, as shown at *T′* in Fig. 80. The "condenser type" of terminal is much more compact for the same operating voltage and same factor of safety, and represents a more recent and intelligent design. (See Trans. A. I. E. E., Vol. XXVIII, page 209.)

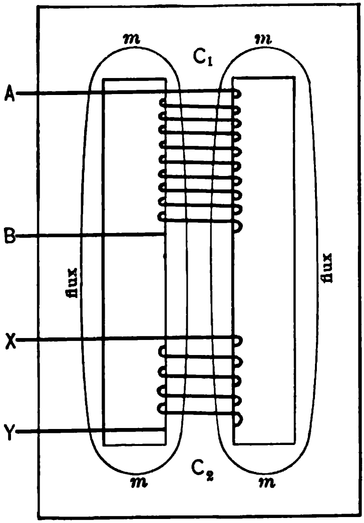

FIG. 81. Diagram of a transformer. An alternating current in the primary coil *AB* sets up an alternating flux in the iron core C_1C_2. This flux induces an alternating voltage in the secondary coil *XY*.

32. How the Transformer Changes the Voltage. The essential principle upon which the transformer operates has been explained in the First Course, Chapter V. Two coils (or two groups of coils) are wound upon the same iron core. One of these coils, *AB* in Fig. 81, is connected to a generator or a transmission line which delivers alternating current to it. This current produces an alternating magnetic flux *mm* in the iron core C_1C_2. The variation of the flux from zero to maximum value, then to zero again, first in one direction and then in the opposite direction, through the magnetic circuit which is common to the two coils, induces an alternating e.m.f. in the other coil *XY*. If an electrical circuit be completed through coil *XY* by connecting a lamp, a motor, or any conductor externally between the terminals, this induced e.m.f. will produce a current and deliver power.

Thus, although the two coils are electrically insulated from each other, power taken in by one coil is transmitted to the other coil by means of the magnetic flux which links with both of them. Of course, as stated above, there are losses in the transformers — due to hysteresis and eddy currents in the iron core and to I^2R in both coils — so that the power given out by the coil XY is slightly less than the power taken in by the coil AB. By providing a proper number of turns and suitable insulation in each coil, the **primary** (AB) may take in power at high voltage and the **secondary** (XY) may deliver this power at low voltage. Due to the use of high voltage, the amount saved annually on the cost of transmission line and of line losses, is much greater than the cost of fixed charges and losses of the transformers, which then become necessary to change the voltage to values suitable for generation and utilization by practicable apparatus.

33. Ratio of a Transformer. The coil AB produces a flux that links with the coil XY and induces an e.m.f. in both AB and XY. In fact, it even produces e.m.f.'s in the iron core C_1C_2 which cause the so-called "eddy currents" in the iron, which are neglected in this article.

Let E = Effective value of the e.m.f. induced in a single turn of both AB and XY by the flux threading both coils.

N_P = Number of turns in primary coil AB.

N_S = Number of turns in secondary coil XY.

Since the e.m.f.'s in all the turns of each coil are in phase with one another,

EN_P = Total induced voltage in primary coil AB.

EN_S = Total induced voltage in secondary coil XY.

The ratio of the induced e.m.f. in the primary to the induced e.m.f. in the secondary can then be expressed by the following equation:

$$\frac{\text{Induced e.m.f. in Primary}}{\text{Induced e.m.f. in Secondary}} = \frac{EN_P}{EN_S} = \frac{N_P}{N_S},$$

$$\frac{N_P}{N_S} = \frac{\text{Number of turns in Primary}}{\text{Number of turns in Secondary}}.$$

This ratio $\frac{N_P}{N_S}$ is called the **Ratio of a Transformer,** and is equal to the ratio of the induced voltage in the primary to the induced voltage in the secondary, when no flux lines exist which do not thread both coils.

Example 1. There are 1200 turns in the primary coil of a transformer and 120 turns in the secondary coil. A flux of 800,000 lines threading both coils is made to alternate according to the sine law, at the rate of 60 cycles a second.

(*a*) What is the effective voltage induced in each turn of both coils?

(*b*) What is the total effective induced voltage in the primary coils?

(*c*) In the secondary coil?

(*d*) What is the ratio of the induced voltage in the primary coil to the induced voltage in the secondary coil?

(*e*) What is the "ratio of the transformer"?

(*a*) The flux makes 4 changes between 0 and maximum value during one cycle. See Art. 53, First Course.

$$\text{Av. } E \text{ (induced in each turn)} = \frac{800{,}000 \times 60 \times 4}{10^8} = 1.92 \text{ volts.}$$

$$\text{Effective } E = 1.11 \times \text{av. } E \text{ (for a sine wave-form).}$$
$$= 1.11 \times 1.92 = 2.13 \text{ volts.}$$

(*b*) E (induced in primary coil) $= 2.13 \times 1200 = 2560$ volts.

(*c*) E (induced in secondary coil) $= 2.13 \times 120 = 256$ volts.

(*d*) $$\frac{\text{Induced } E \text{ in Primary}}{\text{Induced } E \text{ in Secondary}} = \frac{2560}{256} = 10.$$

(*e*) Ratio of the Transformer $= \frac{1200}{120} = 10.$

Prob. 1-3. Answer the five parts of Example 1 if the flux has a frequency of 40 cycles per second.

Prob. 2-3. The primary coil of a certain transformer has 1470 turns.

(*a*) How many magnetic lines must thread the primary coil and the secondary coil if the effective induced voltage in the primary is 2460 volts?

(*b*) How many turns must the secondary have in order to have the induced voltage in it equal to 123 volts?

Assume that the magnetic flux is made to vary according to the sine law at a frequency of 25 cycles.

Prob. 3–3. In a transformer with a ratio of 5, how many turns must there be in the primary and in the secondary coils, if the induced voltage in the secondary is to be 236 volts? The flux has a value of 885,000 lines and varies according to the sine law at a frequency of 60 cycles.

Prob. 4–3. How many turns would have to be used in the coils of the transformer in Prob. 3, if the frequency were 25 cycles?

34. Phase Relations between Flux and Induced E.M.F. The value of the induced e.m.f. **at any instant** in either coil is directly proportional to the number of turns in that coil, and to the rate at which lines of flux are appearing or disappearing within each turn **at that instant.** Thus the induced e.m.f. has zero value when the flux reaches its maximum value (and is constant for an instant before it begins to decrease), and the e.m.f. has its maximum value when the flux is zero (and is changing at its greatest rate). In other words, the induced e.m.f. is 90° out of phase with the alternating flux.

The direction of the induced e.m.f. at each instant must be such that the current which it would produce in the circuit would have a magnetizing action opposed to the change of flux which generates the e.m.f. This is in accord with Lenz' law as stated in Chapter V of the First Course. If this were not so, it would not be possible to attain a settled condition in the circuit, and we should have a perpetual source of energy; because as soon as current and power were delivered from the secondary coil XY, the flux would be increased by the magnetic action of the current. This would induce larger e.m.f. which would produce more current, and so on to infinity.

Thus, in Fig. 82, the curve ϕ represents the variations of the flux (linking both coils) produced by the magnetizing current i_m flowing in the primary coil, and in phase with it. Then the curve e'_s represents variations of the e.m.f. which is thereby induced in the secondary coil and e'_p represents the e.m.f. induced in the primary coil. In the magnetic circuit, we consider as positive the direction of flux produced by a current which flows in a positive direction through the electric circuit. In Fig. 82, the positive direction of e.m.f. e or of current i_m is from A to B and from X to Y, within the coil in either case. Notice that if any current is produced by the induced voltages e'_s or e'_p, it would oppose the change of the flux ϕ, and therefore of the current producing ϕ. Thus such a current would be increasing while ϕ is decreasing and it would be positive while ϕ is negative. Notice also that both induced e.m.f.'s e'_s and e'_p **lag 90°**

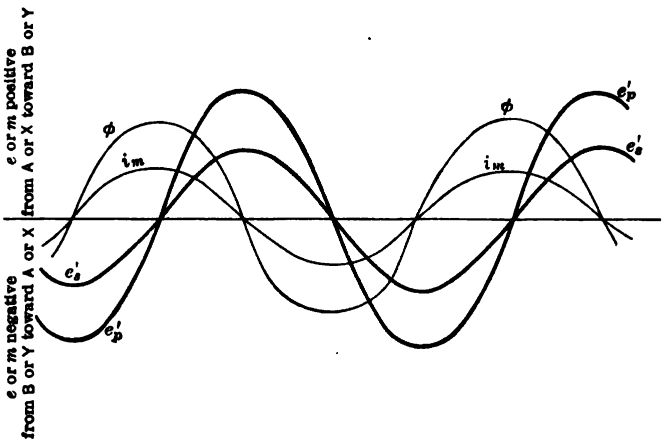

Fig. 82. The magnetizing current, i_m, produces a flux of the sine wave-form, ϕ. This flux sets up an e.m.f. e'_p of sine wave-form in the primary coils and an e.m.f. e'_s of sine wave-form in the secondary coils. Note that e'_p and e'_s are in phase with each other but lag 90° behind the flux ϕ and the magnetizing current i_m.

behind the magnetizing current i_m. This fact that the induced e.m.f. always lags 90° behind the current and flux which produces it is also shown in Art. 54, First Course.

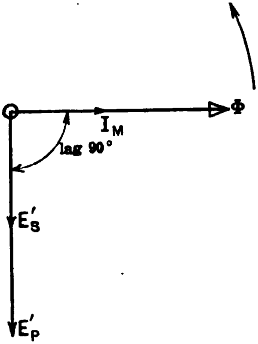

Fig. 83. The vector diagram for the sine curves of Fig. 82. The e.m.f.'s E'_p and E'_s, induced in the primary and secondary, lag 90° behind the magnetizing current I_M and the flux ϕ.

Assuming that all of these quantities vary with time according to the sine law (an assumption which will be discussed later) we may represent their values and relations by vectors as in Fig. 83. The lengths of the vectors represent effective values of the corresponding quantities. No current is flowing in the secondary coil, as yet, but a current I_M is flowing in the primary coil which sets up the flux. In order to force the magnetizing current I_M through the primary coil, we must apply an alternating e.m.f. to the primary coil which is large enough to overcome not only the IR drop, $I_M R_P$, caused by current I_M flowing against the resistance R_P of the primary coil, but also the e.m.f. E'_P which is induced in the turns of the primary coil when the current I_M flows. Now, any good transformer is designed so that the coil resistances R_P and R_S are made relatively small, to keep the losses low and the efficiency high, and I_M is made small to keep the power-factor high. The resistance drop in the primary is therefore very small in comparison with the induced (counter) e.m.f. E'_p in the primary. For most practical calculations the e.m.f. that must be impressed on the primary coil may be assumed to be opposite and equal to the induced voltage E'_p, as represented by E_p in Fig. 84. This would be true not only for effective values, but also for instantaneous values. Therefore, if the wave-form of e.m.f. between the mains and impressed on the primary is harmonic, the wave-form of the induced e.m.f. must be very nearly harmonic. It follows also that the flux must vary harmonically, because the induced e.m.f. is at every instant proportional to the rate of change of the flux, and the rate of change of an harmonic quantity itself varies harmonically.

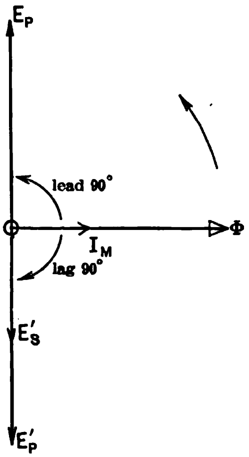

FIG. 84. The voltage E_p must be impressed on the primary coil in order to overcome the induced voltage E'_p. (Resistance drop of the coils is neglected in this diagram.)

35. How the Transformer Adjusts Itself to Changes of Load. Now suppose we complete the circuit of *XY* externally in Fig. 81 and allow E'_s, the voltage induced in the secondary coil, to produce a current I_s, lagging $\theta°$ behind E'_s, as represented in Fig. 85. The magnetic effect of this load current I_s tends to change the flux ϕ. But the slightest alteration of ϕ, of course makes corresponding change in the induced e.m.f. E'_p in the primary. Now E'_p very nearly balanced the impressed voltage E_p before I_s began to flow, and the resultant voltage in the coil was almost zero (being just sufficient to force the current I_M through the coil against its resistance R_p). Thus the change in the induced voltage E'_p will cause the resultant voltage in the coil to attain an appreciable value. This resultant will cause more current to flow from the mains into the primary. The additional current (represented by I'_p) is called the "**load component" of primary current,** and it will automatically increase in value until it develops a **magnetic action exactly equal and opposite to that of I_s.** When this occurs, the flux ϕ is restored to its original or zero-load value, and there no longer exists any unbalanced e.m.f. in the primary. In other words, equilibrium is restored and the current is steady again. The total current taken by the primary coil *AB* from the supply line would be the vector sum of the load component of primary current I'_p and the magnetizing current I_M.

In order that I'_p may have a magnetic effect exactly equal and opposite to that of I_s, the following relations must exist:

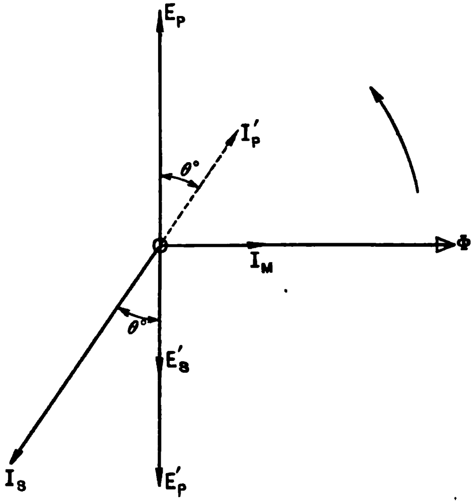

FIG. 85. When a current I_s is allowed to flow in the secondary coils, a current I'_p, at 180° to I_s, will flow in the primary coil.

(*a*) The phase of I'_p must be exactly opposite to that of the load current I_s.

(*b*) The ampere-turns in the primary due to the current I_p must be exactly equal to ampere-turns in the secondary due to the load current I_s.

As N_p equals number of turns in the primary and N_s the number of turns in the secondary, this relation is expressed mathematically as follows:

$$N_pI'_p = -N_sI_s.$$

Ignoring the directions, which are expressed by angles in the vector diagram, we find the following numerical relation to exist between these currents:

$$\frac{\text{Load component of current in primary } (I'_p)}{\text{Load current in secondary } (I_s)} = \frac{N_s}{N_p}.$$

Now, the magnetizing current I_M never exceeds 10 per cent of the load current I'_p in any well-designed transformer, and in large transformers is considerably less than 1 per cent. (See Table A.) Also, I_s is the only current flowing in the secondary. Therefore the statement made above for **load currents** is also practically true for total currents. That is, we may say without serious error for all but the very smallest and poorest transformers, that the total primary and secondary currents at any load **near or above full load** are inversely proportional to the number of turns in the primary and secondary coils. As the load approaches zero, this simple statement departs further from the truth. When I_s equals zero and the total primary current reduces to I_M, the above simple statement is, of course, absolutely wrong.

36. Exciting Current. Magnetizing Current. Hysteresis Current. We have assumed the flux ϕ in Fig. 83, 84, 85, to be maintained entirely by a **magnetizing current** I_M which is in phase with ϕ. If the core or the magnetic circuit of the transformer were entirely composed of air or wood or some other non-conducting "non-magnetic" substance, this assumption would be correct. But when we use iron in the magnetic circuit, a certain amount of energy must be taken from the mains by the primary coil during each cycle of flux, to be transformed into heat and lost by "molecular magnetic friction" or hysteresis, and by eddy currents, in the mass of the iron. We notice that the vector I_M in Fig. 85 is in quadrature (90°) with the vector E_p (terminal e.m.f. of primary), hence it represents zero power intake from the mains. In order to supply the power lost in the iron core during successive cycles, the primary coil must take a certain amount of "**magnetic power component**" or "**hysteresis component**" of current (I_H), in phase with E_p, as shown in Fig. 86. For convenience, this hysteresis component is made to represent the losses due to eddy currents as well as hysteresis in the core. That is,

$$I_H = \frac{P_c}{E_p}$$

$$= \frac{\text{Watts lost in core due to hysteresis and eddy currents}}{\text{Terminal volts of primary coil}}.$$

The total current which is necessary simply to maintain the flux is called the "**exciting current,**" and it is the vector sum of I_M and I_H. It is the zero-load current of the transformer,—the (measureable) current which actually flows in the primary when the secondary circuit is open. In Fig. 86 and 89 it is represented by the vector I_E. As ϕ and I_M are in quadrature with E_p and I_H, we see that:

Value of exciting current or zero-load current

$$I_E = \sqrt{I_H^2 + I_M^2}.$$

Power-factor of exciting current or zero-load current

$$= \frac{I_H}{I_E} = \frac{I_H}{\sqrt{I_H^2 + I_M^2}}.$$

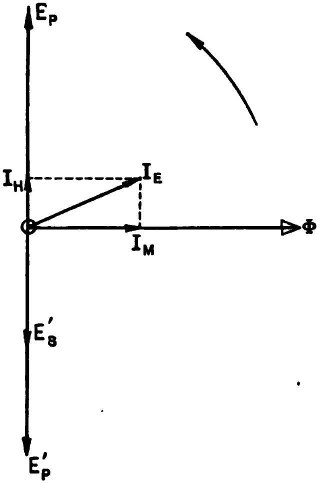

FIG. 86. The hysteresis current I_H, leading I_M by 90°, must flow in the primary coil in order to supply the hysteresis and eddy-current loss in the core. The exciting current I_E is the vector sum of the magnetizing current I_M and the core-loss current I_H.

In transformers of good design, proportion and materials, and of good workmanship, operating at rated voltage and frequency and sine wave-form of e.m.f., the exciting current is between 1 and 10 per cent of the rated full-load current depending on the size (kilovolt-ampere) and the voltage. The ratio between I_H and I_M is usually such that the power-factor of the exciting current I_E is from 10 to 50 per cent.

When the magnetic circuit contains no iron, or when iron is used in such manner that the maximum flux density B_m is not carried above the "knee" of the saturation curve of the steel (K in Fig. 87) the exciting current (I_E) has a sine wave-form, if the impressed e.m.f. has a sine wave-form. This is because the exciting ampere-turns, N_pI_E, and therefore the instantaneous values of I_E, are at every instant exactly proportional to flux density B. This fact is shown by the curve between B and N_pI_E being practically straight between the points O and K. But if B_m is carried beyond the knee of the saturation curve, as is usually the case with silicon steels used in transformers, the wave-form of the exciting current is not truly sinusoidal even when that of the impressed e.m.f. is harmonic. The reason for this is indicated in Fig. 88, for which we are indebted to the *Electric Journal*, September, 1914. If we follow the changes in flux as it proceeds through a cycle, we see that:

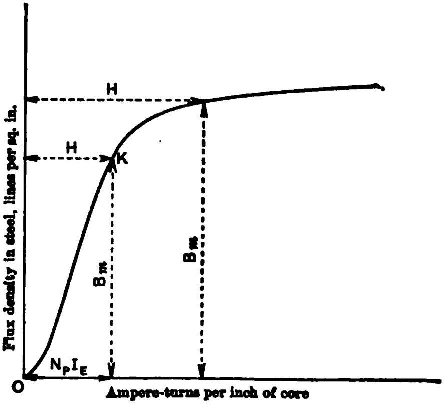

FIG. 87. The saturation curve for steel. The flux is practically proportional to the exciting current until the saturation point K is reached.

(*a*) When $B = 0$, $I_E = OA$.
(*b*) When B increases to BD, I_E has increased to OB.

(*c*) When B increases to CE, I_E has increased to OC.
(*d*) When B decreases to AF, I_E has decreased to OA again.

And so on. In this way the curve for exciting current required to produce a sine curve of flux and induced e.m.f. has been deduced, and it is plainly a non-sine wave-form. In vector diagrams we represent the exciting current by the **equivalent harmonic** or one which has the same R.M.S. (root-mean-square) value.

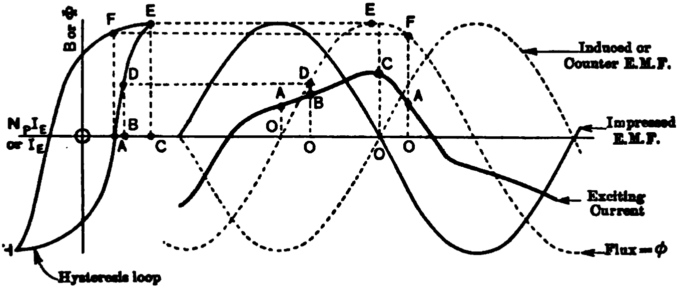

FIG. 88. The curve of exciting current is shown by the heavy line. It is not a sine curve, even though the impressed e.m.f. is a sine curve, particularly when the magnetization is carried beyond the saturation point.

Example 2. The following data are given in the *General Electric Review*, July, 1910, for a 400,000-volt transformer built for testing high-tension insulators: Rated capacity, 250 kv-a. Frequency, 60 cycles. Primary voltage, 1150 or 2300 (for parallel or series connection of two primary coils). Secondary voltage, 400,000. Core loss, 8400 watts (constant). Copper loss, 1270 watts at full load. Exciting current, 7.18 per cent (of rated full-load current). Efficiency at full-load, 96.3 per cent. Resistance drop, 0.51 per cent. Reactance drop, 5.4 per cent.

Compute: —
(*a*) The core loss current.
(*b*) The exciting current.
(*c*) The power-factor of the exciting current.

From this data we derive the following:

(*a*) $I_H = \dfrac{8400 \text{ watts}}{2300 \text{ volts}} = 3.65$ amperes.

(b) $I_E = 0.0718 \times \frac{250{,}000 \text{ watts}}{2300 \text{ volts}} = 7.8$ amperes = zero-load current.

(c) Power-factor of exciting current $= \frac{3.65}{7.8} = 0.468 = 46.8\%$.

The maximum value of the flux ϕ is nearly constant at all loads if the line voltage and frequency are constant, as is usual. The drop of voltage within the coils is a very small percentage of the applied voltage, in consequence of which the counter e.m.f., and the flux which induces it, are almost as nearly constant as the line voltage. It follows that at all loads the exciting current of a constant-voltage transformer is constant in value and in phase relation to the impressed voltage E_p. Thus in Fig. 89, if the load I_S is doubled, the current I'_P also is doubled, but the exciting current I_E remains the same in value and position. Therefore the primary current I_P (vector sum of I'_P and I_E) can never be exactly propor-

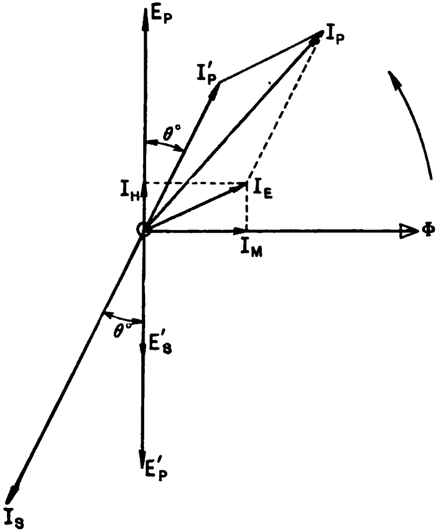

FIG. 89. The primary load-current I'_P is directly proportional to the secondary load-current I_S. The total primary current I_P, which is the resultant of I'_P and the constant exciting current I_E, is not quite proportional to the secondary current I_S, but, for all but light loads, may be so taken.

tional to the current output of the transformer I_S, and the power-factor of the primary current will usually differ slightly from the power-factor of the secondary current. But as I_E is very small compared with I'_P at all loads except very light loads, we may assume for most practical calculations that the amperes input to the primary is directly proportional to the amperes output from the secondary, and that the power-factor of the input to the primary is equal to the power-factor of the output from the secondary.

Prob. 5–3. A transformer rated 5 kv-a, 60 cycles, 2300 to 110 or 220 volts, takes a current of 0.10 ampere at 0.40 power-factor from 2300-volt mains when the secondary circuit is open. Calculate: (*a*) Exciting current as per cent of rated full-load current. (*b*) Magnetizing current in amperes and in per cent. (*c*) Core loss current in amperes and per cent.

Prob. 6–3. (*a*) At zero load with secondary open, the transformer of Prob. 5 takes how many watts from the line? (*b*) If the resistances of primary and secondary coils are such that the primary copper loss is equal to the secondary copper loss at rated full-load, the total copper loss being then equal to the core loss, calculate the copper loss in watts at zero load.

Prob. 7–3. A transformer rated 10 kv-a., 2200/110–220 volts, 60 cycles, takes 100 watts from a 2200-volt, 60-cycle line, when its secondary is open. (*a*) If the magnetizing component is 90 per cent of the exciting current, what is the zero-load power-factor of the transformer?

(*b*) What is the exciting current for this transformer in amperes and in per cent of rated full-load current?

Prob. 8–3. If the transformer of Prob. 5 and 6 delivers full-load current at 80 per cent power-factor from the secondary, calculate: (*a*) Amperes load component of primary current. (*b*) Total amperes input to primary including exciting current.

37. Relation Between Flux, Voltage and Frequency in the Transformer. The maximum density B_m attained by the flux in the iron core during each cycle is of great importance, as it affects all operating characteristics of the transformer. The value of B_m depends upon the voltage and the frequency applied to a coil, the number of turns in the coil, and the sectional area of the core. The equations representing these relations are fundamental in all calculations of transformers.

Let Φ = maximum total number of lines of flux through core, linking with both primary and secondary coils. This flux alternates in direction and varies harmonically.

N_P = number of turns in primary coil.

N_S = number of turns in secondary coil.

f = frequency = number of cycles of flux or of e.m.f. per second.

As we have seen in Art. 33, ϕ lines of flux appear or disappear within each turn of either coil every quarter period or $\frac{1}{4}$ of $\frac{1}{f}$ seconds, $= \frac{1}{4f}$ seconds.

Average rate of cutting flux by each turn of either coil

$$= \frac{\Phi}{\frac{1}{4f}} = 4f\phi \text{ lines per second.}$$

Therefore,

$$\text{Average e.m.f. induced in each turn} = \frac{\phi \times 4f}{10^8} \text{ volts.}$$

As the flux varies harmonically, the induced e.m.f. also varies harmonically. (See Appendix.) The form-factor of an harmonic e.m.f. (or ratio of effective to average value) is 1.11. Therefore,

Effective value of e.m.f. induced in each turn

$$= \frac{1.11 \times \phi \times 4f}{10^8} \text{ volts.}$$

Effective voltage induced in entire primary coil

$$= \frac{4.44\,\phi f N_P}{10^8} = E'_P.$$

Effective voltage induced in entire secondary coil

$$= \frac{4.44\,\phi f N_S}{10^8} = E'_S.$$

The amount of the core losses and the magnetizing current depend upon the maximum density of flux B_m: If A represents the area of core section at right angles to the flux, we have,

$$\Phi = B_m A.$$

Thus,

$$E'_P = \frac{4.44\,B_m A f N_P}{10^8}, \quad \text{or} \quad B_m = \frac{10^8\,E'_P}{4.44\,A f N_P},$$

and

$$E'_S = \frac{4.44\,B_m A f N_S}{10^8}, \quad \text{or} \quad B_m = \frac{10^8\,E'_S}{4.44\,A f N_S}.$$

From these equations we see that:

(1) Changing the voltage, changes B_m in exact proportion, all other things remaining equal.

(2) Changing the frequency, while keeping the voltage constant, causes B_m to change in inverse proportion to f.

(3) Changing the frequency and voltage in proportion to each other keeps B_m unchanged.

(4) Changing the number of turns in a coil, while keeping the voltage across this coil constant, causes B_m to vary in inverse proportion to N.

(5) Changing the number of turns in a coil and the voltage across it in proportion to each other leaves B_m unchanged.

(6) Changing the cross-sectional area of the iron core, at constant voltage and frequency, will change B_m in inverse proportion if the number of turns in the coil remains fixed. If N is also changed, in inverse proportion to A, then B_m remains fixed.

The highest voltage at which we can operate a given transformer depends not only upon the insulation,* but also upon B_m. If B_m is to be carried much beyond the "knee of the saturation curve," the exciting current or zero-load current must be much increased, the power losses in copper and in iron are much increased, and the temperature rises unless the current output is reduced. If we desire to build a transformer for higher voltage at a given frequency, we must increase the area of the core (and volume of iron), or the number of turns, or both, so as to conform to the above equations while keeping B_m within a reasonable or economical limit (see Table III). We must also use more or better insulation. Of course we may increase the kilovolt-ampere capacity at any given voltage and frequency by increasing the current and enlarging the size of wire in both coils enough to keep the I^2R loss relatively low and the efficiency high.

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* Standardization Rules of A.I.E.E. require the insulation of a transformer to stand test for 60 seconds at voltages of from 2 to 5 times the operating voltage, depending on the type of transformer. See § 247 to 257, Standardization Rules, A.I.E.E.

Transformers suitable for a given voltage must have more iron (larger value of A in equation) when designed for a low frequency than when designed for a high frequency. Or, we may keep B_m within bounds by changing N in inverse proportion to f, keeping the same core. But in this case, since the coils must carry the same current to deliver the same kilovolt-amperes at the given voltage, the I^2R loss will be changed approximately in proportion to the number of turns or in inverse proportion to the frequency. To avoid overheating and reduction of efficiency, both cross-section and length of wire would have to be doubled at the same time the frequency was reduced to half to maintain the same capacity; and this entails a larger core in order to obtain the necessary winding space.

Prob. 9–3. If a 60-cycle transformer be designed for 1 volt per turn, what must be the total maximum flux?

Prob. 10–3. If a transformer core has a cross sectional area of 100 sq. in. and the primary coil has 10,000 turns, what effective value of line voltage at 60 cycles will give a maximum density of 60,000 lines per sq. in. in the core?

Prob. 11–3. (*a*) How many flux-turns, or interlinkages between turns and maximum cyclic value of flux, are necessary in a 25-cycle transformer for a 2300-volt line?

(*b*) What must be the area of the core of this transformer if we use 2000 turns in the 2300-volt coil, and B_m is not permitted to exceed 50,000 lines per sq. in.?

Prob. 12–3. What maximum flux density will be attained in a core having 20 sq. in. sectional area, surrounded by a coil of 3000 turns connected to a 6600-volt 60-cycle line? Sine wave of e.m.f.

Prob. 13–3. A transformer primary consists of two equal coils which are designed to be connected in series to a 2200-volt line.

(*a*) If they are connected in parallel instead of in series, by what percentage will the flux density be increased or decreased from its normal value?

(*b*) By what percentage will the secondary voltage be increased or decreased from its normal value?

38. Losses and Efficiency of the Transformer. The losses of power in the transformer are classified as **copper losses** and **core losses.** The copper losses consist of $I_S^2R_S$ in the secondary coils and $I_P^2R_P$ in the primary coils. The core losses are due to hysteresis and eddy-currents in the iron core of the transformer. The total losses amount to only a small percentage of the power transformed when the output is anywhere near full-load. Efficiencies at full-load, usually calculated on basis of unity power-factor load, unless otherwise stated, range between 95 per cent for very small transformers (½ kv-a.), and 98.5 or 99.0 per cent for very large transformers (5000 to 15,000 kv-a.).

The methods for calculating efficiency that were explained in Art. 3, Chapter I, may be applied also to transformers. The statements made there concerning relation between fixed losses and variable losses at the load where maximum efficiency occurs, apply more accurately to transformers than to generators. To these should be added the statement that a low load-factor generally produces a low all-day efficiency.

Load-factor has various meanings in popular usage (see any Electrical Handbook), but the one sanctioned by "Standardization Rules" of the A.I.E.E. is as follows: "The load-factor of a machine, plant, or system is the ratio of the average power to the maximum power during a certain (specified) period of time." According to this definition, when the load on a generator is perfectly steady or **constant,** the **load-factor** is **unity:** and whenever the load-factor is less than unity, it is implied that the load is unsteady or varying. Lower load-factors correspond either to wider range of fluctuation or to more prolonged periods of light load. The **daily** load-factor and the **monthly** load-factor are significant mainly in relation to the efficiency of the plant, which has some effect upon the cost of a kilowatt-hour. But this cost depends principally upon the **annual** load-factor. In most central stations, the highest load in the entire year lasts only three or four hours per day for about two months, during December and January. To carry this supreme peak, machinery must be bought which stands idle or operates lightly loaded the remainder of the year. Fixed charges must be paid and earned for this investment, and these charges are usually levied against all the kilowatt-hours sold, making the cost of each kilowatt-hour much larger than it would be if the load were steady. For, if the load were steady, we could deliver the same number of kilowatt-hours with less kilowatts capacity and therefore less first-cost.

A load-factor of 35 or 40 per cent is exceptionally good for a central station. **High load factors** are obtained by connecting as many **different types** of **power-consuming devices** as possible, to the station, and getting as many **different customers** as possible.

The core loss of a given transformer is nearly constant at all loads, provided the applied voltage and frequency are constant. Investigation has disclosed that the power lost due to hysteresis depends upon the chemical composition of the iron used, the mechanical and heat treatment to which the iron has been subjected, the frequency of reversals, the maximum value of flux attained during the cycle, and the wave-form of induced e.m.f. or of flux. All of this is expressed in the equation

$$P_H = WK_H fB_m^{1.6},$$

where

P_H = watts lost due to hysteresis.

f = frequency of induced e.m.f., cycles per second.

B_m = maximum value of flux density, lines per square inch. The exponent of B_m varies with the composition and treatment of the steel; 1.6 is its *average* value.

W = pounds weight of iron acted on by the alternating flux.

K_H = a constant for any given kind of iron, but different for various kinds of iron, depending upon the chemical composition, treatment, hardness, etc.

Values of P_H reported in *Trans. A.I.E.E.*, Vol. XXVIII, page 465, range between 1.0 and 2.0 watts per pound for ordinary steels containing no silicon, and from 0.54 to 0.82 watts per pound for "silicon steels," containing from 3 to 4 per cent of silicon. These values were measured while the flux varied harmonically, and B_m was 64,500 lines per square inch. The frequency was 60 cycles per second.

The loss due to eddy currents is an I^2R loss caused by currents in the mass of the iron. These are produced by e.m.f.'s which are induced in the iron just as e.m.f. is induced in primary and secondary coils by the alternating flux. When alternating flux traverses a solid block or bar of iron the eddy current loss is very large, often heating the iron in a few minutes so that it is too hot to touch. By cutting up the iron into sheets parallel to the direction of the flux, and varnishing, oxidizing or otherwise insulating the sheets so that current cannot pass from one to another, the paths which the eddy currents must traverse are made longer. This reduces the watts loss in proportion to the square of the thickness of the laminations. The eddy-current loss may be represented by the equation:

$$P_E = WK_E\, f^2B_m^2t^2,$$

where

P_E = watts lost due to eddy currents.
f = frequency, cycles per second.
B_m = maximum flux density in core, lines per square inch.
t = thickness of laminations of core, inches.
W = weight of iron acted on by alternating flux, pounds.
K_E = a constant for any kind of iron, but different for various grades of iron, inversely proportional to their specific electrical resistance.

Tests show that P_E ranges between 0.36 and 0.70 watts per pound for ordinary annealed sheet steel, and between 0.12 and 0.18 watts per pound for special "silicon-steels" used in transformers (containing 3 to 4 per cent of silicon). These values are on the basis of a frequency of 60 cycles, a maximum flux density B_m of 64,500 lines per square inch, and a value of 14 mils (0.014 inch) for the thickness (t).

From the above equation it is seen that we can easily reduce the eddy current loss by making the laminations thinner, whatever may be the values of B_m, f, K_E and W. The limit is reached commercially when $t = 14$ mils, however, because it is not practicable to handle thinner sheets than this in manufacturing operations. Thus, if the sheets were twice this thickness, or 28 mils, the loss in silicon steels would be between $\frac{t_1^2}{t^2} \times 0.12$, or 0.48, and $\frac{0.028^2}{0.014^2} \times 0.18$, or 0.72 watt per pound at 60 cycles and 64,500 lines per square inch. To reduce the eddy-current watts to the previous values while using iron 28 mils thick, we should reduce B_m to 32,250 lines per in.^{2}. In ordinary practice the laminations are thin enough to reduce the eddy-current loss to a value not exceeding 25 per cent of the hysteresis loss.

The only operating conditions which affect the hysteresis loss, P_H, and the eddy current loss, P_E, and therefore the core loss which is their sum, are B_m and f. The relation between B_m, f, and the induced voltages in the coils, has been explained in Art. 37. The induced e.m.f. in the primary will be very nearly equal to the applied e.m.f. because the resistance and reactance of a well-designed transformer cause only a small percentage voltage drop. If the frequency and effective value of the line e.m.f. are constant, therefore, B_m is nearly constant. If the wave-form also is constant, as is usual, the core losses remain the same at all loads. The total watts input to the transformer at zero-load, with the secondary circuit open, is practically equal to the core loss. This input really includes a slight amount of I^2R loss due to the exciting current: but as the exciting current is less than 10 per cent of the full-load current, this I^2R loss is less than (0.10 × 0.10) or 1 per cent of the full-load copper loss in the primary, or about one-half of one per cent of the total full-load copper loss. This is entirely negligible in comparison with the core losses. Therefore we say that the watts input with open secondary are equal to the core losses. Connections for measuring core losses are shown in Fig. 90.

The cores of modern transformers are usually made of "silicon steel," which contains from 3 to 4 per cent of silicon alloyed with the iron. Although the permeability is reduced by this addition of silicon, the hysteresis loss and eddy-current loss corresponding to given values of B_m and f are both greatly reduced. Another practical advantage of silicon steel is that it is "non-aging." It is found that the value of K_H in the formula for hysteresis loss increases quite remarkably in ordinary steels if they are maintained hot (80° to 100° C.) for long periods of time, say six months. This increase may finally amount to several hundred per cent. As the aging produces higher losses and further heating, the condition aggravates itself and causes overheating and a very serious increase in operating expense. Although silicon steel is harder to work with and is more expensive than ordinary steel, the cost of the transformer is not much increased because higher magnetic densities and less quantity of iron may be used without causing excessive core losses.

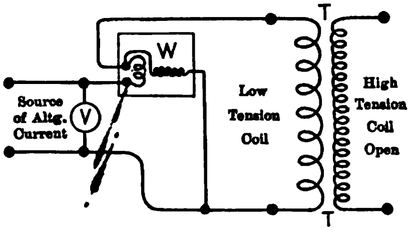

FIG. 90. Connections for measuring the core loss of a transformer. Normal voltage, indicated by voltmeter V, is impressed on the low-tension terminals and the power taken is indicated by the wattmeter W. The high-tension terminals are open. (If the wattmeter is not "compensated," the power consumed by its pressure-coil must be calculated (from voltage and coil-resistance) and subtracted from the wattmeter reading connected in this way.)

The copper loss in transformers is about equally divided between primary and secondary coils. If $N_P = 10\,N_S$, then $I_P = \frac{1}{10}\,I_S$ and $I_P^2 = 0.01\,I_S^2$. Therefore, to make $R_P I_P^2 =$

$R_S I_S^2$, we must have $R_P = 100\,R_S$. Then, $100\,R_S \times 0.01\,I_S^2 = R_S I_S^2$. In a test of 5 different samples of 15 kv-a. distributing transformers, designed to step-down the voltage from 2400 to 240 $\left(\text{or, } \frac{E_P}{E_S} = \frac{10}{1}\right)$, it was found that the ratio $\frac{R_P}{R_S}$ ranged between 72 and 108. The distribution of copper loss will vary correspondingly, of course. The value of copper loss is usually **calculated** from the rated-load current in primary and secondary, and the coil resistances. It may be **measured,** however, by short-circuiting the low-tension coil of the transformer through an ammeter and applying just enough volts to the high-tension coil to produce rated full-load current. The watts input measured under this condition are practically equal to the full-load total I^2R loss, because the flux required to produce this current in the short-circuited transformer is such a small percentage of the flux at rated voltage that the core losses are negligible. The connections for making this test are shown in Fig. 91. For accurate work the wattage reading (W) may be corrected to allow for the core losses (which may be measured on open-circuit with the same voltage applied) and for the power lost in the pressure coil of the wattmeter.

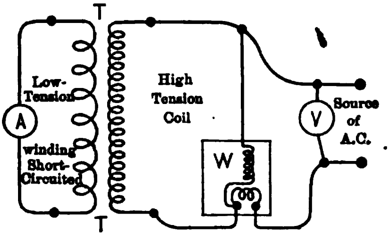

FIG. 91. Connections for measuring the copper loss in a transformer. Sufficient voltage is impressed on the high-tension terminals to cause full-load current to flow through the short-circuited low-tension coils. The copper loss will be indicated on the wattmeter W.

The guaranteed values for losses, efficiencies at various loads, voltage regulation at various power-factors, and exciting current, for two lines of distributing transformers manufactured by a well-known large company, are given in Table A. The "Type SA" transformers have a higher core loss and lower efficiency than "Type S" transformers. Higher flux densities and less material

TABLE A

PERFORMANCES OF "TYPE S" SINGLE-PHASE TRANSFORMERS

2200 Volts High Tension on 2200-Volt Circuit — 60 Cycles

Kv-a.	Watts loss.		Per cent efficiency.				Per cent regulation.				* Exciting current.
	Iron.	Copper.	Full load.	¾ Load	½ Load	¼ Load	100% P. F.	90% P. F.	80% P. F.	60% P. F.	
½	15	13	94.7	94.4	93.2	88.7	2.62	3.21	3.28	3.16	8.0
1	20	24	95.8	95.7	95.1	92.0	2.42	3.03	3.12	3.04	5.5
1½	25	35	96.0	96.0	95.5	92.7	2.36	2.96	3.07	3.00	4.0
2	30	42	96.5	96.5	96.2	93.8	2.12	2.76	2.88	2.86	3.6
2½	33	51	96.8	96.8	96.5	94.5	2.08	2.71	2.83	2.83	3.3
3	34	64	96.8	97.0	96.8	95.2	2.16	2.79	2.91	2.88	3.0
4	40	75	97.2	97.3	97.1	95.7	1.90	2.77	3.00	3.12	2.5
5	45	93	97.3	97.5	97.3	96.1	1.90	2.76	2.99	3.11	2.3
7½	62	125	97.6	97.7	97.6	96.4	1.70	2.60	2.84	3.00	2.2
10	80	148	97.8	97.9	97.7	96.5	1.51	2.42	2.68	2.89	1.9
15	105	212	97.9	98.0	97.9	97.0	1.44	2.36	2.63	2.85	1.6
20	131	268	98.0	98.1	98.0	97.1	1.39	2.51	2.87	3.21	1.5
25	147	319	98.2	98.3	98.2	97.4	1.33	2.45	2.82	3.17	1.3
30	163	374	98.2	98.4	98.3	97.6	1.32	2.45	2.82	3.16	1.2
37½	197	433	98.3	98.4	98.4	97.7	1.20	2.34	2.72	3.09	1.2
50	240	550	98.4	98.6	98.5	97.9	1.15	2.29	2.68	3.07	1.0

* In per cent of full-load current.

PERFORMANCES OF "TYPE SA" SINGLE-PHASE TRANSFORMERS

2200 Volts — 60 Cycles

Kv-a.	Watts loss.		Per cent efficiency.				Per cent regulation.			
	Iron.	Copper.	Full load.	¾ load.	½ load.	¼ load.	100% P. F.	90% P. F.	80% P. F.	60% P. F.
½	21	13	93.6	93.0	91.1	85.1	2.61	3.00	2.98	2.76
1	28	24	95.0	94.8	93.6	89.5	2.41	2.82	2.82	2.64
1½	35	35	95.5	95.4	94.5	91.0	2.34	2.76	2.76	2.60
2	41	42	96.0	95.9	95.1	92.0	2.11	2.55	2.58	2.46
2½	45	51	96.3	96.2	95.6	92.8	2.05	2.49	2.53	2.42
3	47	64	96.4	96.4	95.9	93.6	2.14	2.57	2.60	2.48
4	55	75	96.8	96.9	96.4	94.4	1.89	2.35	2.40	2.33
5	63	93	97.0	97.0	96.7	94.8	1.87	2.33	2.39	2.32
7½	90	125	97.2	97.2	96.9	95.0	1.68	2.16	2.23	2.20
10	115	148	97.4	97.4	97.0	95.3	1.50	2.21	2.38	2.49
15	148	212	97.6	97.7	97.4	95.9	1.43	2.15	2.33	2.46
20	181	268	97.8	97.8	97.6	96.2	1.36	2.09	2.27	2.41
25	206	319	98.1	98.1	97.8	96.5	1.31	2.24	2.52	2.76
30	225	374	98.0	98.1	97.9	96.8	1.28	2.21	2.50	2.75
37½	278	433	98.1	98.2	98.0	96.9	1.19	2.13	2.43	2.69
50	338	550	98.3	98.3	98.1	97.1	1.14	2.20	2.56	2.89

are used in the former than in the latter, resulting in a cheaper transformer. The normal operating temperature is also higher and the overload capacity correspondingly less.

Example 3. How many dollars less per year does it cost to operate a 50 kv-a. Type S transformer (Table A) than a 50 kv-a. "Type SA" transformer? How many dollars more can we afford to pay for the "S" than for the "SA"?

With the same load in both cases, as a basis of comparison, the total annual watt-hours of copper loss will be the same for both types of transformers. But the difference of 98 watts in the core loss (338 − 240) amounts to 854 kilowatt-hours in one year (365 × 24 × 0.098), because the distributing transformer usually remains permanently connected in readiness to serve, and the core loss is continuous every hour in the year. If energy costs 2 cents per kw-hr. delivered at the transformer, it costs $17.08 per year less to operate the "S" transformer. If each dollar invested is required to earn $0.13 per year to cover fixed charges, we could afford to pay $\frac{17.08}{0.13}$ or $131.38 more for the "Type S" than for the "Type SA" transformer. If the extra cost of the S transformer were less than $131, we should gain by choosing it in preference to the "SA" transformer; but if the extra cost were more than $131, we should prefer the "SA" type. If (as is often the case) the copper losses are higher in transformers which have lower core losses, the advantage of low core losses is correspondingly reduced.

This treatment of the cost of transformer losses and effect of distribution of losses upon the value of the transformer, is by no means complete. For complete treatment, see *Trans. A.I.E.E.*, Vol. XXX, page 2181.

Let us calculate the commercial efficiency of the 50 kv-a. "Type S" transformer, at full-load non-inductive.

Output in watts

$$= (\text{kv-a.} \times 1000) \times (\text{power-factor}) = 50{,}000 \times 1.0.$$

Total loss in watts

$$= \text{core loss} + \text{copper loss} \qquad = 240 + 550 = 790.$$

$$\text{Efficiency} = \frac{\text{output}}{\text{output} + \text{total loss}} = \frac{50{,}000}{50{,}790} \qquad = 0.984 = 98.4\%.$$

If the transformer operates at rated full-load kv-a. but at 80 per cent power-factor, the core loss remains approximately the same, and the copper loss likewise, but the power output is reduced. Thus,

Efficiency at full-load, 80 per cent power-factor

$$= \frac{50{,}000 \times 0.8}{40{,}000 + 240 + 550} = 98 \text{ per cent.}$$

When the output or load of the transformer is reduced, the copper loss decreases as the square of the current, but the core loss remains approximately the same if the line voltage is constant, as usual. Thus, at quarter load or one-quarter of full-load current, and 50 per cent power-factor, we would have:

Core loss

$= 240$ watts.

Copper loss

$= (\frac{1}{4})^2 \times 550$ watts $= 34.4$ watts.

Output

$= \frac{1}{4}$ of 50,000 volt-amperes × 0.5 (power-factor) $= 6250$ watts.

Efficiency

$$= \frac{6250}{6250 + 240 + 34} = 95.8 \text{ per cent.}$$

Comparing this with the value of 97.9 per cent for quarter load non-inductive which appears in Table A, we perceive again the sacrifice in efficiency due to low power-factor.

Prob. 14–3. The hysteresis loss in a given sample of transformer steel is 0.60 watt per pound at 60 cycles and a maximum density of 64,500 lines per square inch for a harmonically varying flux.

(*a*) What would be the watts per pound at 25 cycles frequency?
(*b*) At 133 cycles frequency? (Same B_m in all cases.)
(*c*) What is the flux density in lines per square centimeter?

Prob. 15–3. What would be the watts lost per pound due to hysteresis in the steel of Prob. 14, if the maximum flux density at 60 cycles were

(*a*) Half as great?
(*b*) Twice as great?
(*c*) 45,000 lines per sq. in.?

Prob. 16–3. How many watts would be lost in hysteresis in a core weighing 120 lbs. made of the same steel as in Prob. 14, but operated at 25 cycles with a flux density of 75,000 lines per square inch?

Prob. 17–3. The eddy-current loss in a given sample of transformer steel is 0.15 watt per pound when worked at 60 cycles with a maximum flux density of 64,500 lines per square inch, the thickness of laminations being 14 mils. If B_m remain the same, but the steel is worked at 25 cycles, how many watts are lost per pound? By what percentage must the voltage across the exciting coil be changed when f is thus reduced, in order not to change B_m?

Prob. 18–3. What would be the eddy-current loss per pound of the same steel as in Prob. 17, worked at the same frequency (60 cycles) and the same density, but used in sheets of No. 26 gauge, having a thickness of 18.7 mils?

Prob. 19–3. If the steel of Prob. 17, in laminations 18.7 mils thick, were used at 25 cycles frequency but the same maximum density (64,500 lines per square inch), what would be the watts lost per pound in eddy currents?

Prob. 20–3. What would be the watts lost per pound in eddy currents for the steel of Prob. 17, if used in a 25-cycle transformer at 96,750 lines per square inch, the thickness being 18.7 mils?

Prob. 21–3. If Prob. 14 and 17 both refer to the same sample of silicon-steel, what per cent of the total core loss is due to hysteresis, and what per cent is due to eddy currents? ($f = 25$.)

(*a*) When $B_m = 64{,}500$ lines per sq. inch?

(*b*) When $B_m = 96{,}750$ lines per sq. inch?

39. Station or Power Transformers. Distributing Transformers. It is not economical or practicable to build alternators capable of generating pressures as high as are necessary for most economical distribution of electric power. It is usual, therefore, to wind the generator for a moderately high pressure and to use large "step-up" transformers within the station to raise the voltage to a value best suited to the transmission line. Thus we now have several transmission systems operating at 140,000 volts, although the generated voltage rarely exceeds 6600 to 13,000 volts. These large transformers are called **"station transformers"** or **"power transformers."** Similar ones are also used as "step-down" transformers in substations, where they reduce the very high pressure of the transmission line to a moderate pressure, suitable for distribution through the streets of a town, or for operating a large converter or alternating-current motor. The terms **step-up** and **step-down** indicate merely whether the power output of the transformer is at a higher or a lower pressure than the input; the same transformer may be used either way, provided we do not allow the voltage across any coil or the flux density in the iron to be higher than that for which it was designed.

Thus, to supply alternating current for lights and motors over a town or area of say two miles radius, we would have a **transformer substation** containing a station transformer large enough to supply the maximum total kilovolt-ampere demand of this area. The transformer would be wound and insulated so as to be able to take in this much power from the 140,000-volt transmission line or feeder, and to deliver it at about 2200 volts to the local distributing lines or mains. Individual consumers, or groups of adjacent consumers, take power from these distributing mains through "**distributing transformers,**" which take in power at the pressure of the distributing mains, usually about 2200 volts, and give it out at 110, 220, 440, or 550 volts, as may be necessary to suit the consumers' apparatus. Distributing transformers are therefore designed to change relatively small amounts of power, rarely over 50 kv-a., from moderate voltage to low voltage. Station transformers are designed to change large amounts of power, 100 to 10,000 kv-a., either from moderate pressure to a very high pressure within the generating station, or from very high pressure to moderate pressure in the substation.

The reasons for using two stages in this step-down transformation from high-tension feeder to consumer, are as follows:

(*a*) It is impracticable to make high-tension transformers in sizes smaller than about 100 or 200 kv-a.

(*b*) The insulation for very high pressures is so expensive and so difficult to build that the cost per kilovolt-ampere capacity is much greater than when moderate pressures are used. This is true of overhead transmission or distributing lines and underground cables, as well as of transformers.

(*c*) In most cities or thickly populated districts, there are laws or ordinances limiting the voltage that may be used. In this connection it should be noted that 1000 or 2000 volts would be just as quickly fatal as 100,000 volts to people who might come in contact with the wires, but the risk of breaking down the insulation and of producing dangerous disturbances in adjacent low-tension circuits would be much greater if very high pressures were used on the distributing mains.

(*d*) A large space is required between line wires on high-tension circuits, which not only takes more room than is available in well-populated districts, but requires higher, heavier and more expensive poles. It is found impracticable to use underground cables for pressures higher than about 25,000 volts.

Station transformers may differ from distributing transformers also in the relative proportion of fixed losses and variable losses. The peak load of the various consumers connected to a substation is less likely to be excessive than the peak load of the consumers connected to a distributing transformer, because the number and diversity of the power-consuming apparatus connected to the substation are greater than the number and diversity of that connected to a distributing transformer. Therefore the load-curve for a station transformer is likely to be smoother than the load-curve for a distributing transformer; or, the load-factor of the former is likely to be higher than the load-factor of the latter. The transformer, in either case, should be designed or selected so that the relation between its fixed losses and its variable losses produces maximum efficiency at the load under which the transformer operates most of the time, or so that its all-day efficiency is as great as possible. The reasons for these statements will be understood by careful study of Art. 4 on efficiency of the alternator, and by solution of the following problems.

Prob. 22–3. A paper by J. D. Ross in *Trans. A.I.E.E.*, April, 1912, gives valuable and complete detailed data on efficiencies of all parts of a 13,000 kv-a. hydro-electric generating plant and transmission system. The generating station contains 9 transformers each rated 1500 kv-a., stepping up the generator pressure of 2300 volts to the transmission pressure of 60,000 volts. These transformers were all in circuit continuously. Careful computations from actual readings and records of calibrated meters yielded the following results:

Total constant loss (iron loss) in 9 transformers in one year = 926,000 kw-hr.

Total variable loss (copper loss) in same transformers in same year = 200,000 kw-hr.

Total input to same transformers in same year = 28,648,000 kw-hr.

Calculate:

(*a*) All-day efficiency for the average day.

(*b*) Average kw. iron loss, per transformer.

(*c*) Average kw. copper loss, per transformer.

(*d*) Average kw. input, per transformer.

(*e*) Average input as per cent of rated load (1500 kw. at unity power-factor).

(*f*) Average constant loss as per cent of total loss.

(*g*) Average variable loss as per cent of total loss.

Prob. 23–3. In the Seattle Municipal Light and Power System referred to in Prob. 22–3, the main substation in Seattle contains eight (8) transformers, each of 1500 kv-a. capacity (or 1500 kw. at unity power-factor) stepping down from the 60,000-volt transmission pressure to 15,000 volts for distribution to smaller substations. The data on the transformers in the main substation are as follows:

Total constant (iron) loss in 8 transformers in one year = 692,000 kw-hr.

Total variable (copper) loss in same transformers in same year = 217,500 kw-hr.

Total input to same transformers in same year = 27,144,700 kw-hr.

Calculate same items as in Prob. 22–3, and compare corresponding values. On this basis, discuss relation between all-day efficiency, relative proportion of constant and variable losses, and average load as percentage of rated load of the transformer. These step-down transformers are of exactly the same construction as the step-up transformers in the generating station (Prob. 22–3).

Prob. 24–3. Current from the main substation of Prob. 23–3 is distributed at 15,000 volts to two smaller substations and to about twelve mills and factories which use large amounts of power. There are 30 transformers connected to the 15,000-volt lines stepping down to 2400 volts. They range in size from 750 kv-a. to 50 kv-a., and have a combined capacity of 6250 kv-a. The data on these transformers, altogether, is as follows:

Total constant (core) losses in one year = 694,000 kw-hr.
Total variable (copper) losses in one year = 81,100 kw-hr.
Total input in same year = 11,493,500 kw-hr.

Calculate same items as in Prob. 22–3, and compare corresponding results of Prob. 22, 23 and 24. Continue the discussion along same lines as in Prob. 23–3.

Prob. 25–3. Power is distributed from the smaller substations of Prob. 24 by means of seventeen 2400-volt mains. Connected to these mains are 1082 distributing transformers, ranging in size from 2½ kv-a. to 50 kv-a., and with an aggregate full-load capacity of 9268½ kv-a. The data on these transformers for the same year are as follows:

Total constant (core) loss in one year = 960,000 kw-hr.
Total variable (copper) loss in same year = 431,000 kw-hr.
Total input in same year = 12,478,300 kw-hr.

Calculate same items as in Prob. 22, and continue the comparison and discussion along the lines specified in Prob. 23 and 24.

40. Effect of Operating Transformers at Wrong Voltage. If we raise by a given amount the effective value of the e.m.f. impressed upon the primary of a transformer, a relatively greater change in the exciting current is produced. The amount and importance of this effect may be judged by the following extract from the standard specifications for transformers used by the U. S. government (see Circular No. 22, U. S. Bureau of Standards).

"The exciting current shall in no case exceed 10 per cent of the full-load current, and for transformers of 10 kw. or larger, shall not exceed 8 per cent of full-load current. With an applied voltage 10 per cent above normal, the exciting current shall not exceed 20 per cent of the full-load current."

In other words, it is probable that a 10 per cent increase of (effective) voltage may more than double the exciting current, so that this limit must be set by a specification. Raising the voltage increases both the core-loss current I_H and the magnetizing current I_M, therefore the exciting current I_E is increased faster than either of its components. If I_M increases faster than I_H as the voltage is raised the power-factor of the exciting current becomes less. Fig. 87 shows how great an increase of I_M is necessary when the voltage is raised to a value which carries B_m above the knee of the saturation curve of the core material. Fig. 92 shows how rapidly I_E increases when the pressure is increased more than a few per cent above rated voltage. In fact, the overvoltage may be sufficient to cause the exciting current to be larger than rated full-load current, so that the transformer would be overheated without load.

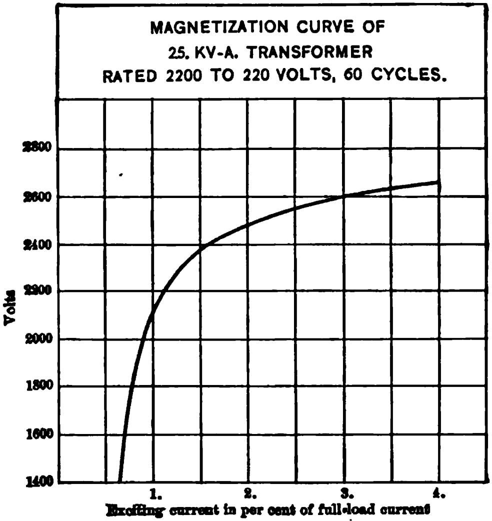

FIG. 92. When 2100 volts are impressed in the primary coils the exciting current is 1 per cent of the full-load current. When the impressed voltage is raised (less than 20 per cent) to 2500 volts, the exciting current is doubled, becoming 2 per cent of the full-load current. *From Proc. N.E.L.A., 1909, Vol. 1.*

Raising the voltage increases the range through which the flux density in the iron core varies during each cycle. In fact, if the frequency is constant, the maximum flux density, B_m, attained during each cycle increases in direct proportion to the voltage. This causes an increase of the magnetizing current I_M which is more than proportional to the value of B_m or to the voltage, on account of decrease in permeability of the iron due to increase in saturation. The increase of B_m also causes a sharp increase in the amount of core losses and therefore also in the core-loss current I_H, since

$$I_H = \frac{\text{Total core loss}}{E_p}.$$

The effect of voltage on core loss is illustrated by Table I, which is adapted from Taylor's excellent book "Transformer Practise." From this table we see that if we operate an ordinary 60-cycle transformer at 10 per cent overvoltage, the total core loss is about 23 per cent higher than it would be in the same transformer operated at its rated voltage. Then

$$\frac{\text{Core loss}}{\text{Voltage}} = \frac{1.23 \text{ times normal core loss}}{1.10 \text{ times normal voltage}} = \frac{1.23}{1.10}$$

or 1.12 times normal core-loss current.

In other words, a 10 per cent increase of voltage above normal increases the hysteresis component of the exciting current 12 per cent above normal. This relation is not the same for all transformers, but depends upon the quality of iron used, and how near to the knee of the magnetization curve it is being worked at normal voltage, as well as upon the relation between hysteresis and eddy-current losses.

For the effect of voltage changes upon the magnetizing current I_M, consider Table II which is adapted from Bulletin U. S. Bureau of Standards, Vol. 5, No. 4, on "The Testing of Transformer Steel." It shows for a particular sample of silicon steel, such as is used in transformers, the relation between magnetizing ampere-turns per inch length of transformer core and the flux density produced in the core (expressed in lines per square inch). Table III is taken from "American Handbook for Electrical Engineers," and shows flux densities usually employed in transformers.

TABLE I

VARIATION OF CORE LOSS IN A 60-CYCLE TRANSFORMER WITH VARYING VOLTAGE

Per cent of rated voltage.	Per cent of core loss at rated voltage.	Per cent of rated voltage.	Per cent of core loss at rated voltage.
Per cent	Per cent	Per cent	Per cent
80.8	66.0	104.0	109.0
84.0	71.0	105.0	111.0
88.0	77.0	110.0	123.0
92.0	84.0	115.0	137.0
96.0	92.0	119.0	149.0
100.0	100.0		

TABLE II

Lines of flux per square inch.	Ampere-turns $N_p I_M$ per inch length of iron core.	Lines of flux per square inch.	Ampere-turns $N_p I_M$ per inch length of iron core.
20,000	0.710	50,000	1.96
25,000	0.840	55,000	2.28
30,000	1.02	60,000	2.69
35,000	1.21	65,000	3.20
40,000	1.42	70,000	3.92
45,000	1.66	75,000	4.80

From Bull. U. S. Bureau of Standards, Vol. 5, No. 4, Magnetization Curves for Silicon Steel used in Transformers. Relation between B_{max} and amperes (effective value) of magnetizing component of the exciting current.

TABLE III

USUAL VALUES OF B_m IN PRACTISE

Size of transformer.	Kind of steel.	Lines per square inch.	
		25 Cycle.	60 Cycle.
Small........	Ordinary transformer sheet...	50,000	40,000
Small.........	Silicon steel................	70,000	60,000
Large........	Ordinary transformer sheet...	75,000	65,000
Large.........	Silicon steel................	90,000	75,000

From American Elec. Engrs. Handbook. Wiley & Sons, page 1619.

Thus we see that for small (distributing) transformers having high-grade (silicon) steel, a density of 60,000 lines per square inch is usual for 60-cycle circuits. From Table II we see that approximately 2.69 ampere-turns are necessary per inch. Now let us operate this transformer at 10 per cent above its rated voltage. The flux density will increase in proportion to the voltage (see Art. 37), and will become 66,000 lines per square inch (1.10 × 60,000). From Table II we see that this will require the magnetizing ampere-turns ($N_P I_M$) to be increased from 2.69 to about 3.35 ampere-turns per inch. That is, the total magnetizing ampere-turns must be increased in the ratio $\left(\frac{3.35}{2.69} = \frac{1.25}{1.00}\right)$. As the number of turns is fixed, this means that the magnetizing current I_M must be increased by 25 per cent over its value at rated voltage.

For example, let us take a distributing transformer which is rated 15 kv-a., 2400 to 240 or 120 volts. By test the core loss is 108 watts, and the copper loss calculated from full-load current and measured resistance of coils is 224 watts. This transformer takes an exciting current of 0.175 ampere from 2400-volt mains. Let us assume that it has such steel and operates normally at such part of its magnetization curve that the data of Tables I, II and III will apply. Then the core-loss current I_H is $\frac{\text{108 watts core loss}}{\text{2400 volts applied}}$ or 0.045 ampere. I_M is $\sqrt{0.175^2 - 0.045^2}$ or 0.169 ampere. Assuming that the transformer conforms to Tables I, II and III, when operated at normal voltage, we find that operation at 10 per cent overvoltage (or at 2640 volts high-tension) increases I_H to (1.12 × 0.045) or 0.0505 ampere and I_M to (1.25 × 0.169) or 0.2106 ampere. The new value of exciting current is therefore $I_E = \sqrt{0.0505^2 + 0.2106^2}$ or 0.2107 ampere. The power-factor of the zero-load current is now $\frac{0.0505}{0.2107}$ or 24 per cent. At normal voltage it was $\frac{0.045}{0.175}$ or 25.7 per cent.

The exciting current is usually so small that ordinary variations of voltage such as are due to line drop would not cause it to become an important factor in the operation of the transformer. But if the coils of a transformer, or the phases of a group of transformers, are incorrectly connected, the transformer may be compelled to operate with a maximum flux density as much as two times normal, which would enormously increase both the core losses and the exciting current. In fact, the transformer might burn itself out while unloaded (with the secondary circuit open). This condition would so soon become apparent, however, by the destruction of the transformer, if not otherwise, that it need not be considered as an operating condition. For usual variations of voltage, the effect upon cost of operation and load capacity of the transformer are more important than upon the exciting current.

When operating this transformer at 10 per cent overvoltage, the core loss will be increased about 23 per cent according to Table I, or to 123 per cent of the core loss at normal voltage. The test core loss was 108 watts. Core loss at 10 per cent overvoltage would equal $1.23 \times 108 = 133$ watts. Now the total losses in continuous operation cannot be allowed to exceed the total losses (core loss plus copper loss) at rated full-load, without injuring the transformer or shortening its life. Copper loss and core loss at full load, normal voltage are stated as $224 + 108 = 332$ watts. Therefore the copper losses must be reduced to $332 - 133 = 199$ watts at 10 per cent overvoltage. This requires that the currents in both primary and secondary be reduced to $\frac{\sqrt{199}}{\sqrt{224}}$ or 0.942 of the values which they have, respectively, at rated load and rated voltage. The current output from secondary to load would have to be reduced to something less than 94 per cent, on account of the slight relative increase of primary current due to the larger exciting current at overvoltage. But if the current output of the transformer is 0.94 times normal and the voltage is 1.10 times normal, the total kv-a. output is 1.034 times normal. That is, we can deliver 3.4 per cent more power than the transformer is rated for (at the same power-factor) by raising the voltage 10 per cent, without injuring the transformer. However, the trick will not bear repetition; for if we were to raise the voltage 20 per cent or 30 per cent above normal, we should find that the current output must be decreased by a greater percentage than the voltage is increased, and the power capacity of the transformer at large overvoltages would be less than its rated kv-a.

Now consider the increased cost of operating the transformer above rated voltage. We may look at the matter from several view-points, as indicated in the following problems. Perhaps it is simplest to consider the difference in core losses only. When we operate at normal voltage, the core loss is 108 watts, and when we operate at 110 per cent of normal voltage it is 133 watts, an increase of 25 watts. This loss continues as long as the transformer is connected to the mains, and in readiness to serve, which is usually 24 hours per day for a distributing transformer. The overvoltage therefore increases the daily energy loss by 24 × 25 or 600 watt-hours, which in one year amounts to $\frac{600 \times 365}{1000} = 219$ kw-hr. If energy costs 2 cents per kw-hr. delivered at the transformer, this represents an extra operating cost of 0.02 × 219 or $4.38 per year for this one 15 kv-a. transformer. This additional operating expense may or may not be justified by a greater power capacity of the transformer due to the overvoltage, depending upon how nearly saturated the core is when operating at rated voltage, and upon the grade of iron used. Competition in transformer manufacture is so keen, and the materials are operated normally so close to the economical limits of densities, that it does not pay to operate at more than a few per cent overvoltage.

Prob. 26–3. The 15 kv-a. transformer referred to in the text above delivers a varying load equivalent to 5 hours full load and 19 hours zero load. Assuming that "full load" means the **same current output** as at rated load and voltage, calculate:

(*a*) Kilovolt-ampere output at 10 per cent overvoltage.

(*b*) Core and copper losses in watts at 10 per cent overvoltage, "full-load."

(*c*) Total energy lost in 24 hours in kilowatt-hours.

(*d*) Excess of this energy lost at overvoltage, over energy lost at normal voltage.

(*e*) Annual cost of this excess loss, at 2 cents per kw-hr.

Prob. 27–3. Repeat solution of Prob. 26 on the assumption that "full load" means the same total kilovolt-ampere output as rated on nameplate of the transformer (15 kv-a.).

Prob. 28–3. Repeat solution of Prob. 26, on the assumption that "full load" means the load that gives same total watts loss as at rated load and voltage (332 watts).

Prob. 29–3. Calculate the commercial efficiency and the all-day efficiency of the transformer operated as in Prob. 26.

Prob. 30–3. Calculate the commercial efficiency and the all-day efficiency of the transformer operated in Prob. 27.

Prob. 31–3. Calculate the commercial efficiency and the all-day efficiency of the transformer operated in Prob. 28.

Prob. 32–3. A distributing transformer rated 15 kv-a. when operated at rated voltage has a maximum flux density of 60,000 lines per square inch in its silicon-steel core, which has the characteristics indicated in Tables I and II. If it be operated at a pressure 10 per cent lower than its rated voltage, calculate:

(*a*) Percentage decrease in magnetizing current.

(*b*) Percentage decrease in core losses.

(*c*) Percentage decrease in core-loss current.

Prob. 33–3. At rated full-load the transformer of Prob. 32 has an efficiency of 98.0 per cent and its copper loss and core loss are in the ratio 3/2. The exciting current is 5 per cent of rated load current. Calculate:

(*a*) Normal core-loss current.

(*b*) Normal magnetizing current.

(*c*) Exciting current at 10 per cent below rated voltage.

Prob. 34–3. (*a*) By what percentage may the current output of the transformer in Prob. 32 and 33 be increased when operating at 10 per cent below rated voltage, without exceeding the total loss and temperature rise corresponding to rated load and voltage?

(*b*) What would be the largest allowable kilovolt-ampere output of this transformer at 10 per cent below rated voltage?

Prob. 35–3. (*a*) If transformers like that specified in Prob. 32 and 33 cost approximately $3.60 per kv-a. of capacity, by how many dollars must the investment be increased or decreased in order to furnish exactly the same kilovolt-ampere capacity when operating the transformers 10 per cent below rated voltage as when operating exactly at rated voltage?

(*b*) What is the total yearly cost of this difference of investment, allowing 6 per cent interest, 5 per cent depreciation, and 2 per cent taxes and insurance?

Prob. 36–3. The transformer of Prob. 32 and 33 operates 5 hours at "full load" and 19 hours at zero load every day, on the average. (*a*) By how many dollars are the total annual energy losses at 10 per cent below rated voltage greater or less than the total annual losses at rated voltage, energy being worth 2 cents per kw-hr. (*b*) By how many dollars are the total annual energy sales handled through this transformer, increased or diminished by the change of voltage? "Full load" is the load which gives same total watts loss as rated load (15 kv-a.) at rated voltage. From the results of Prob. 35 and 36 together, discuss the total economic gain or loss due to operating transformers at less than rated voltage.

41. Leakage Reactance of Transformer. Voltage Regulation. It has been shown that the direction of e.m.f. induced in the secondary coils is such that the currents in the secondary circuit produced by this e.m.f. oppose the magnetizing force of the primary coils. This counter magneto-motive force in the magnetic circuit due to the secondary ampere-turns N_sI_s cannot reduce appreciably the total quantity of flux threading the primary coils, because enough more current will automatically flow in the primary to generate sufficient flux and counter e.m.f. to nearly equal the line voltage. But the magnetic opposition of N_sI_s ampere-turns results in forcing some of the primary flux out of the core, so that it completes its circuit without linking with the secondary turns. Similarly, as soon as the load current flows, the secondary coil will form some local flux-lines around itself, which do not link with the primary. Thus, Fig. 93 is the same as Fig. 81, except that it represents conditions when current flows in the secondary. The exciting current maintains the mutual flux *mm* linking with both coils, whether there is a load current or not. But when the load currents I_s and I'_p flow, a local flux (l_p) linking with the primary turns and a local flux (l_s) linking with the secondary turns are formed. These are called "**leakage fluxes**" because they have leaked away from the core into the air, apart from one another. If they had remained in the core through all turns of both coils, they would have been a part of **the mutual flux** (m).

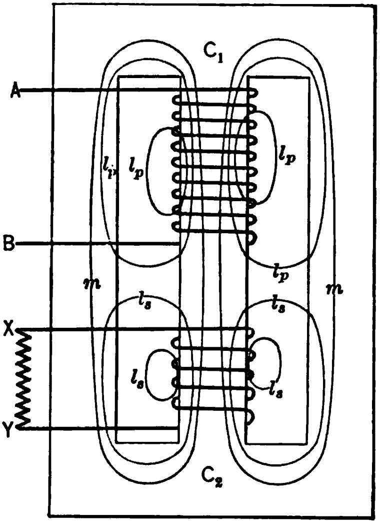

FIG. 93. When the transformer of Fig. 81 is loaded, a "leakage flux" l_s and l_p exists which does not thread the two coils but leaks into the air spaces. The mutual flux m, however, still threads the two coils.

The primary leakage flux (l_p, Fig. 93) is in phase with and proportional to the primary current I_P which produces it. The secondary leakage flux l_s is in phase with and proportional to the secondary current I_S which produces it. The flux l_p induces in the primary coil an e.m.f. X_PI_P, lagging 90° behind l_p and the primary current I_P. The flux l_s induces in the secondary coil an e.m.f. X_SI_S lagging 90° behind l_s and the secondary current I_S. These induced e.m.f.'s due to the reactance representing the leakage flux (called the "leakage reactance"), together with the voltage drops I_PR_P and I_SR_S due to resistance in the primary and secondary coils, cause the performance of the **actual transformer** to be somewhat different from that of the **ideal transformer** whose vector diagram was shown in Fig. 89.

Thus, in the actual transformer having resistance and (leakage) reactance associated with each coil, the ratio between terminal e.m.f. of primary and secondary is not exactly equal to the ratio of turns. The induced counter e.m.f. E'_p must change as the load (I_S and I'_P) increases, on account of voltage drops (I_PR_P and I_PX_P), due to resistance and reactance of the primary. The induced e.m.f. in the secondary E'_S is always equal to the product of induced e.m.f. in the primary, times the ratio of turns $\left(E'_S = E'_P \times \frac{N_S}{N_P}\right)$. But the terminal e.m.f. E_S of the secondary must differ from E'_S on account of the voltage drops (R_SI_S and X_SI_S) in the secondary coils. Therefore, the total change of secondary terminal e.m.f. E_S, due to increase of load I_S, depends very much upon the e.m.f. reactions caused by the leakage fluxes, or upon the leakage reactance of the transformer. This change in terminal voltage, due to change in load, is called "voltage regulation."

The inherent voltage regulation of the transformer will be improved by any method of design or construction which reduces the leakage reactance or tends to hold all flux in the iron core and to prevent local fluxes from forming around individual coils or turns. The best and usual method for accomplishing this improvement is to avoid bunching all of either the primary or the secondary turns in a single coil, as has been done in Fig. 93. Thus, the primary turns may be divided into five similar coils which are interleaved with five or more coils among which the secondary turns are divided, as shown diagrammatically in Fig. 94. This arrangement is much superior to that of Fig. 93, there being very little chance for flux to pass through one primary coil without linking also with the closely adjacent secondary coils. Even if some flux does leak away from the iron core, it must have interlinked at least a part of the secondary with a part of the primary. In other words, it is a part of the mutual flux and does not contribute to the voltage drop or difference between primary and secondary voltages, and therefore it cannot be counted in the leakage reactance. Fig. 95 is a photograph of a group of primary and secondary coils thus interleaved and assembled, all ready to have the laminated core built up around them. These coils are for a 500-kv-a. 66,000-volt transformer.

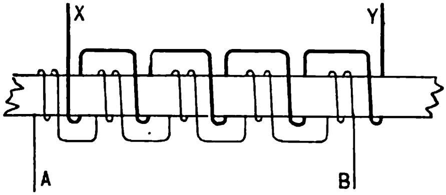

FIG. 94. The leakage flux is reduced by interleaving the primary coils AB and the secondary coils XY.

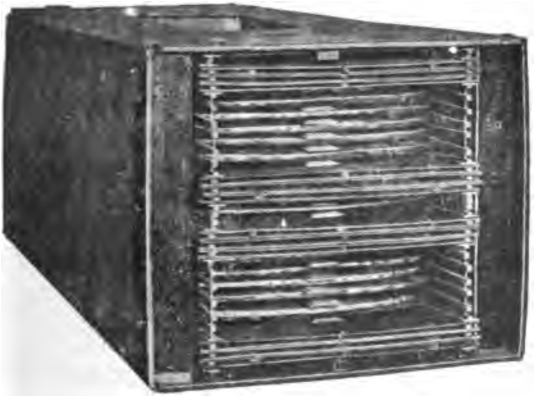

FIG. 95. A photograph of a group of primary and secondary coils interleaved as in the diagram of Fig. 94. *The General Electric Co.*

42. Practical Vector Diagram of the Transformer. The following facts have been developed in the foregoing articles, and the practical vector diagram must represent them all, as shown in Fig. 96 and 97:

(*a*) The primary coil carries an exciting current (I_E) which is approximately the same at any load as it is at zero load (or with secondary circuit open). This current is usually small as compared with full-load current (from 1 to 10 per cent), and it lags nearly 90° behind the e.m.f. E_p impressed on the primary. This current produces or maintains the mutual flux.

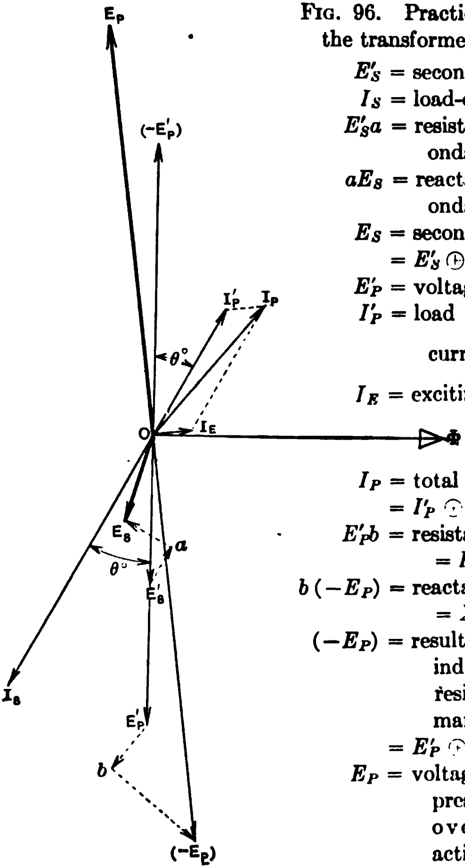

Fig. 96. Practical vector diagram for the transformer.

E'_S = secondary induced voltage.
I_S = load-current in secondary.
E'_Sa = resistance reaction in secondary.
aE_S = reactance reaction in secondary.
E_S = secondary terminal voltage.
$= E'_S \oplus E'_Sa \oplus aE_S$.
E'_P = voltage induced in primary.
I'_P = load component of primary current $= \frac{N_S}{N_P} \times I_S$.
I_E = exciting current.
I_P = total current in primary.
$= I'_P \oplus I_E$.
E'_Pb = resistance reaction in primary $= R_PI_P$.
$b\,(-E_P)$ = reactance reaction in primary $= X_PI_P$.
$(-E_P)$ = result of combined primary induced voltage, primary resistance reaction, and primary reactance reaction.
$= E'_P \oplus E'_Pb \oplus b\,(-E_P)$.
E_P = voltage which must be impressed upon primary to overcome the opposing action of $-E_P$.

(*b*) The mutual flux (*m* in Fig. 81 and 93, and ϕ in the vector diagram), which passes through both primary and

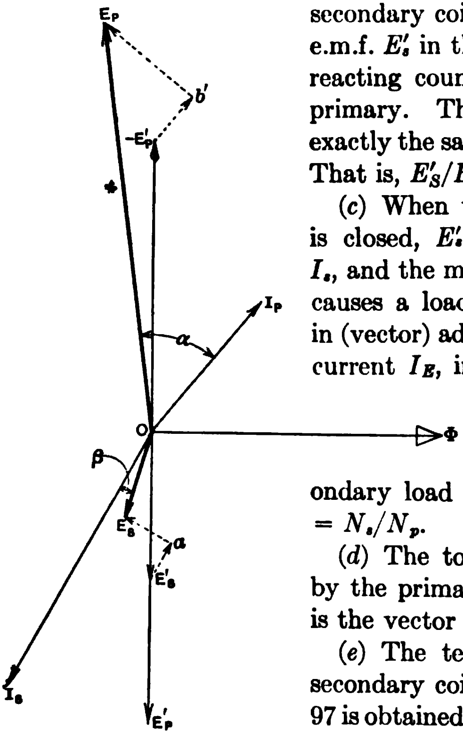

FIG. 97. A simplified practical vector diagram for the transformer, corresponding to Fig. 96. In this figure the voltages required to overcome the various primary reactions are shown rather than the reactions themselves. Thus $b'E_P$ represents the voltage required to overcome the primary reactance, and $-E'_Pb'$, the voltage to overcome the primary resistance, etc.

secondary coils, induces the acting e.m.f. E'_s in the secondary and the reacting counter e.m.f. E'_p in the primary. These e.m.f.'s are in exactly the same ratio as the turns. That is, $E'_S/E'_P = N_s/N_p$.

(c) When the secondary circuit is closed, E'_s produces a current I_s, and the magnetic action of this causes a load current I'_p to flow, in (vector) addition to the exciting current I_E, in the primary. The load current in primary, I'_p, is opposite in phase to the secondary load current I_s, and $I'_p/I_s = N_s/N_p$.

(d) The total current I_p taken by the primary side from the line is the vector sum of I_E and I'_p.

(e) The terminal e.m.f. of the secondary coils OE_s in Fig. 96 and 97 is obtained by adding vectorially the resistance reaction E'_sa and the reactance e.m.f. aE_s, due to leakage flux around the secondary coils, to the total e.m.f. OE'_s which is induced in the secondary. The resistance reaction E'_sa is numerically equal to R_sI_s and is opposite to I_s in phase. The leakage reactance e.m.f. aE_s is numerically equal to X_sI_s and lags 90° behind I_s.

(f) The terminal e.m.f. of the primary, OE_p in Fig. 96 and 97, is the e.m.f. which must be impressed upon it in order to produce E_s volts at the terminals of the secondary, which is delivering I_s amperes to a load. E_p must be equal and opposite to the (vector) sum of the induced counter e.m.f. E'_p and the reacting e.m.f.'s due to the resistance and the leakage flux in the primary coils. In Fig. 96 we have added (vectorially) the resistance reaction E'_pb and the e.m.f. $b(-E_p)$ induced by primary leakage flux, to the primary induced counter e.m.f. OE'_p, to get the total e.m.f. $O(-E_p)$ which the line e.m.f. must overcome or balance. The line e.m.f. OE_p must be **equal and opposite** to this total reacting e.m.f. $O(-E_p)$. The resistance reaction E'_pb in the primary is equal in value to R_pI_p and opposite in phase to I_p. The e.m.f. $b(-E_p)$ induced by the primary leakage flux is equal in value to X_pI_p and lags 90° behind I_p.

Fig. 97 is exactly the same as Fig. 96, but simplified. I_E and I'_s have been left out, showing only the total currents in primary and secondary. The construction for deriving OE_s from OE'_s is exactly the same as in Fig. 96. The vector $O(-E'_p)$ represents the component of primary impressed e.m.f. that is required to **overcome** the induced counter e.m.f. OE'_p. To it are added the component of impressed e.m.f. $-E'_pb'$ required to **overcome** the resistance reaction and the component of impressed e.m.f. $b'E_p$ required to **overcome** the e.m.f. induced by primary leakage flux. These latter two components are exact counterparts or opposites of the corresponding reactions shown in Fig. 96. The student may take his choice between Fig. 96 and 97 on the basis of simplicity and clearness.

The power-factor of the load is $\cos \beta$ and the power-factor of the whole transformer (or of the primary current I_p) is $\cos \alpha$.

43. Voltage Regulation Depends on Power-Factor. When the load is non-inductive, or when the current I_s delivered by the secondary is nearly in phase with the secondary terminal e.m.f. E_s, the variations of E_s due to changes of I_s are principally due to the resistances of the coils and not so much to the leakage reactances. But when the load current I_s leads or lags nearly 90° with respect to E_s (or, when the load power-factor is low) the variations of E_s due to changes of I_s are principally caused by leakage reactance, and not so much by coil resistance.

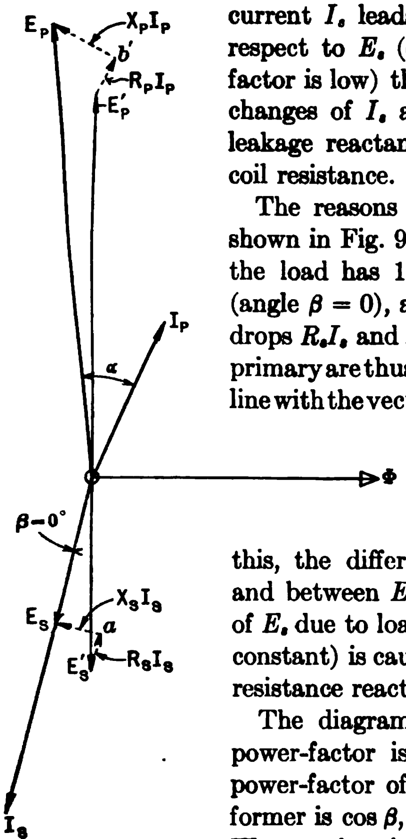

FIG. 98. The difference between E_S and $\left(E_P \times \frac{N_S}{N_P}\right)$, i.e., the regulation, is due mostly to the resistance drop $E'_S a$ because the power-factor of the load is unity.

The reasons for these statements are shown in Fig. 98, 99 and 100. In Fig. 98 the load has 100 per cent power-factor (angle $\beta = 0$), and we see that resistance drops $R_s I_s$ and $R_p I_p$ in the secondary and primary are thus brought into almost direct line with the vectors of induced e.m.f., while the reactance drops $X_s I_s$ and $X_p I_p$ are almost perpendicular to the induced e.m.f.'s. On account of this, the difference between E_p and E'_p and between E'_s and E_s, or the variation of E_s due to load (while E_p is maintained constant) is caused almost entirely by the resistance reactions $R_s I_s$ and $R_p I_p$.

The diagram for lagging load of low power-factor is shown in Fig. 99. The power-factor of the output of the transformer is cos β, and of the input is cos α. We see that in this case the reactance drops $X_s I_s$ and $X_p I_p$ are almost directly in line with the induced and terminal voltages, while the resistance drops $R_s I_s$ and $R_p I_p$ are almost perpendicular thereto. Consequently the variation of E_s due to change of I_s is principally caused by leakage reactance.

The diagram for leading load of low power-factor is shown in Fig. 100. The same remarks apply here as to the diagram for lagging load. Notice in this case, however, that the leakage reactances tend to make E'_p, larger than E_p and E_s larger than E'_s. That is, if we keep the line voltage E_p

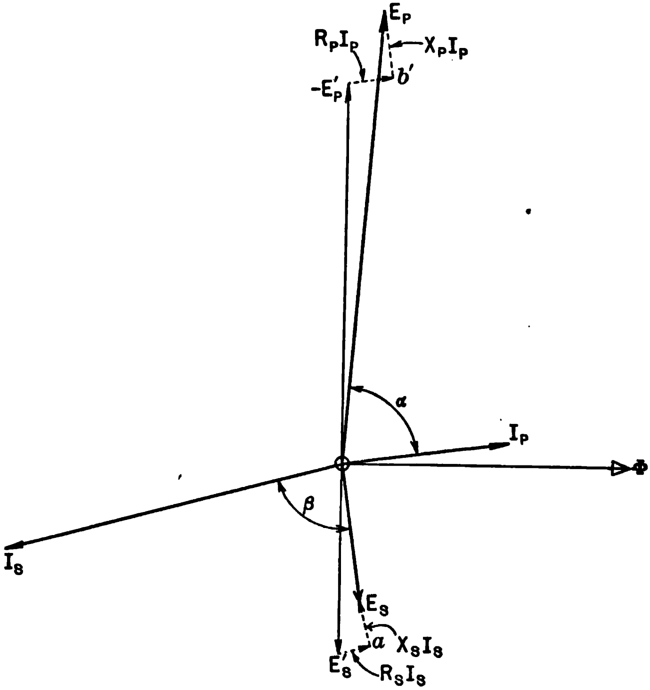

FIG. 99. The difference between E_S and $\left(E_P \times \frac{N_S}{N_P}\right)$, i.e., the regulation, is due mostly to the reactance drops X_SI_S and X_PI_P, because the power-factor of the load is nearly zero.

constant, as usual, the secondary terminal voltage E_s tends to rise as the leading load I_s increases, on account of leakage reactance.

44. Importance of Regulation of the Transformer. At this point we must distinguish between "constant-potential transformers" and "constant-current transformers." A constant-potential transformer is one whose primary takes power from constant-voltage mains and whose secondary delivers power at as nearly constant voltage as is practicable. A constant-current transformer is one whose primary takes power from constant-voltage mains, and whose secondary delivers power at as nearly constant current as possible, the terminal e.m.f. of the secondary being automatically maintained at a value which is as nearly as possible proportional to the impedance of the load circuit. By far the larger number of transformers are of the constant-potential type, including station or power transformers and distributing transformers.

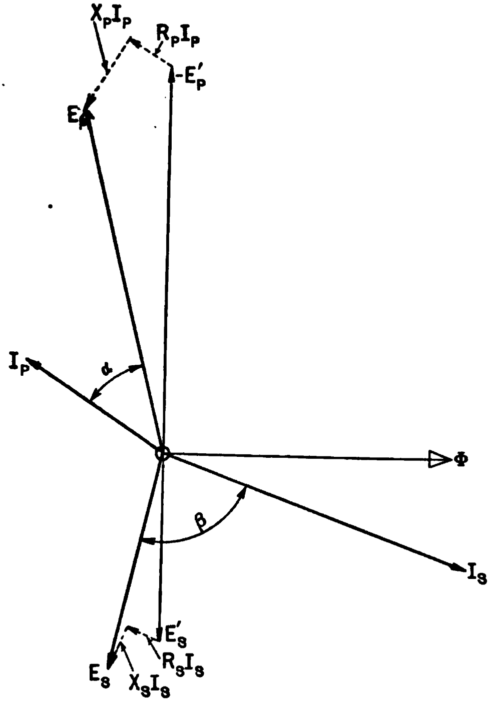

FIG. 100. The secondary terminal voltage E_S may actually be greater than the secondary induced e.m.f. E'_S if the load has a leading power-factor.

The official definition of "regulation" for constant-voltage transformers is given in Art. 284, Standardization Rules of A.I.E.E., Dec. 1914.

"In constant-potential transformers the regulation is the difference between the no-load and rated-load values of the secondary terminal voltage at the specified power-factor (with constant primary impressed terminal voltage), expressed in per cent of the rated-load secondary voltage, the primary voltage being adjusted to such a value that the apparatus delivers rated output at rated secondary voltage."

If the resistances and leakage reactances of the primary and secondary coils of the transformer are relatively large, the voltage regulation is high, or poor. That is, the transformer itself contributes to the voltage drop or fluctuation of voltage due to load currents, which is undesirable. For this reason it is usual for the purchaser of transformers to specify that their regulation shall not exceed certain values as maximum limits, or to select and purchase those transformers which have the lower or better regulation. Some purchasers have already regretted doing this, and are now demanding transformers with poorer regulation, particularly in the large sizes which are used in connection with systems having a large amount of generating capacity. The reason is simple. Good regulation is obtained by making the coil resistances small and by designing and arranging the coils so that the leakage flux and leakage reactance are small. In consequence, the impedance of the transformer is small. If the secondary becomes short-circuited a very large current will therefore flow.

In fact, if the impedance is 2 per cent (that is, if the voltage drop consumed by impedance with rated full-load current flowing is 2 per cent of rated voltage), a current equal to 50 times $\left(\frac{\text{100 per cent of rated voltage}}{\text{2 per cent of rated voltage}}\right)$ rated full-load current will flow when the secondary is short-circuited. This presumes that the generating plant and the transmission line between generators and transformers are large enough to maintain full normal voltage while delivering this excessive current. A current 50 times normal produces magnetic or mechanical forces between the coils and internal parts of the transformer, which are 50^2 or 2500 times as large as those which exist at rated full-load. As in the case of generators (see Art. 17), this possibility of excessive forces requires that we shall either design the transformer with a very great factor of safety and liberality of mechanical strength, or design it so that the impedance of the transformer shall be high and the voltage regulation correspondingly poor, thus limiting the short-circuit current to lower values.

Frequently a number of transformers are interconnected in parallel, as will be explained later. If the percentage impedances of the transformers, when so connected, are not exactly equal, the transformer with the larger percentage impedance (indicated by a poorer or higher regulation) will "shirk" or "lie down" — that is, it will not take its proper share of the load; and the transformer with the lower percentage impedance, or better voltage regulation, is likely to overload itself and burn out. The moral is, that transformers should not have their secondaries tied together in parallel unless their percentages of impedance are nearly equal, or unless their voltage regulations at the same power-factor are nearly equal.

Improvement of the regulation of a transformer can be accomplished only at the sacrifice of other characteristics, or by an increase in the amount of active material in the transformer, or by improvements in the quality of this material or of the workmanship. In other words, better regulation costs more money in one way or another. For instance, one way to improve regulation is to reduce the coil resistances. But this requires a larger cross-sectional area and greater weight and cost of copper. Another method is to reduce leakage by using a core of lower magnetic reluctance. But this requires a larger cross-sectional area of core and weight and cost of iron. In view of these facts, the wisdom of requiring very close voltage regulation in the transformer (or in the generator or transmission line either, for that matter) may reasonably be questioned, particularly since it is possible to compensate voltage changes at the end of the line by an automatic "feeder voltage regulator" (see Art. 63). Usual values of regulation for distributing transformers are from 1 to 2 per cent for sizes of 10 kw. and larger, sometimes as large as 4 per cent for smaller sizes and low power-factors of load.

45. Impedance of the Transformer. Equivalent Resistance and Reactance. If we knew the exact values of the resistance R_s and the reactance X_s of the secondary coils, and of the resistance R_p and reactance X_p of the primary coils of any given transformer, we could by means of Fig. 98, 99 or 100 find its voltage regulation in per cent. We would adjust the line voltage E_p to a value which, after all the voltage drops were vectorially subtracted as shown, would produce rated volts E_s at the terminals of the secondary while the transformer is delivering rated full-load amperes, I_s, or rated full-load kilovolt-amperes, $\left(\frac{E_sI_s}{1000}\right)$. Now, when the load is entirely removed by opening the secondary circuit, the voltage drop in both primary and secondary is reduced to zero (if we neglect the drop in primary due to exciting current, which is relatively insignificant). Thus if the line voltage E_p be maintained constant (see definition of regulation) the induced counter e.m.f. in the primary will become equal to E_p at zero load, and the secondary terminal e.m.f. will change to a value of $\left(E_p \times \frac{N_s}{N_p}\right)$. Therefore,

$$\text{Regulation in per cent} = \frac{\left(E_p \times \frac{N_s}{N_p}\right) - E_s}{E_s} \times 100 \text{ per cent.}$$

However, this method cannot be applied as indicated in Fig. 98, 99 and 100 and the equation above, for the reason that there is no practical way to measure X_p and X_s separately. We can measure R_p and R_s separately by the ammeter-voltmeter method or the Wheatstone bridge (see Timbie's "Elements of Electricity, Chapter V). We can also calculate the **combined** effect of X_p and X_s from readings taken during a short-circuit test of transformer as in Fig. 91.

The most practical solution of the difficulty is to use what are known as the **equivalent reactance** and the **equivalent resistance** of the transformer.

Consider the resistance R_s and reactance X_s of the secondary to be reduced to zero, and enough extra resistance R' and reactance X' to be added in the primary circuit to cause the secondary terminal e.m.f. to have the same value E_s as in the actual transformer at full load. Then the total resistance of the primary $(R_p + R')$ is known as the **equivalent primary resistance,** and the total reactance of the primary $(X_p + X')$ is known as the **equivalent primary reactance.** To find the relation between R' and R_s, and between X' and X_s, let us consider Fig. 101, which is quite similar to Fig. 98 or 99 having but a few additional vectors. In this demonstration, as in all calculations of regulation, we neglect entirely the exciting current and the very small reactions or drops due to it, considering only the currents due to load.

In Fig. 101 we have the following values and relations shown:

E'_s = induced e.m.f. in secondary, due to mutual flux.

E'_p = induced (counter) e.m.f. in primary, due to mutual flux.

$$E'_p = E'_s \times \frac{N_p}{N_s}.$$

I_s = load current (or total current) in secondary.

I_p = load current (assumed to be total current) in primary.

$I_p = \frac{N_s}{N_p} \times I_s$, and I_p is exactly opposite in phase to I_s.

Vector $E'_s a = R_s I_s$, and is exactly opposite in phase to I_s (parallel to I_s), because it represents the voltage reaction due to the resistance in the secondary.

Vector $aE_s = X_s I_s$, and is 90° behind I_s (perpendicular to I_s), because it represents the voltage induced by the leakage flux around the secondary.

$-E'_p b' = R_p I_p$, and is exactly in phase with I_p (parallel to I_p), because it represents the e.m.f. to overcome resistance reaction in the primary.

$b'E_p = X_p I_p$, and is 90° ahead of I_p (perpendicular to I_p), because it represents the e.m.f. to overcome reaction due to leakage inductance of the primary.

Now if we consider R_s and X_s to be zero, and their identical effects upon E_s to be produced by a resistance R' and a reactance X' which are added to the primary, the voltage drop in the secondary becomes zero, and the induced e.m.f. in the secondary becomes identical with the terminal e.m.f. E_s. But if E_s is the induced e.m.f. in the secondary, the induced e.m.f. in the primary on this assumption now becomes E''_s, which is equal to $\left(E_s \times \frac{N_p}{N_s}\right)$ in value. The vector shown as $-E''_s$ in Fig. 101 is really the component of impressed e.m.f. consumed in overcoming this assumed induced counter e.m.f. in the primary. Now, just as the difference between $-E'_p$ and E_p is produced by $R_p I_p$ and $X_p I_p$, so the difference between $-E''_s$ and $-E'_p$ is produced by $R'I_p$ and $X'I_p$. Thus, if E_p volts are impressed upon the terminals of the primary, $-E'_p$ volts remain after the reactions $R_p I_p$ due to primary resistance and $X_p I_p$ due to primary reactance have subtracted themselves (vectorially, of course). Then $-E''_s$ volts remain after the reactions due to the additional resistance R' and reactance X', representing the voltage drops

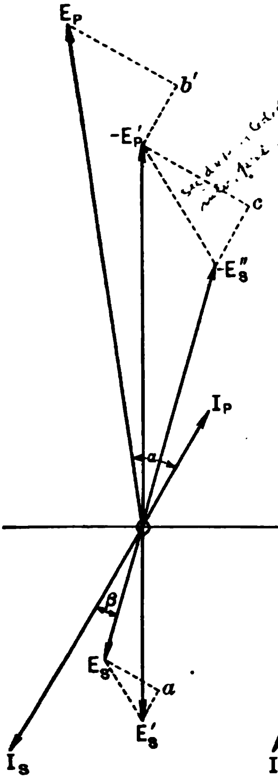

FIG. 101. The voltage drops in the secondary represented by the triangle $E'_S a E_S$ can be represented as though they took place in the primary, by the triangle $(-E''_S)\ c\ (-E'_P)$.

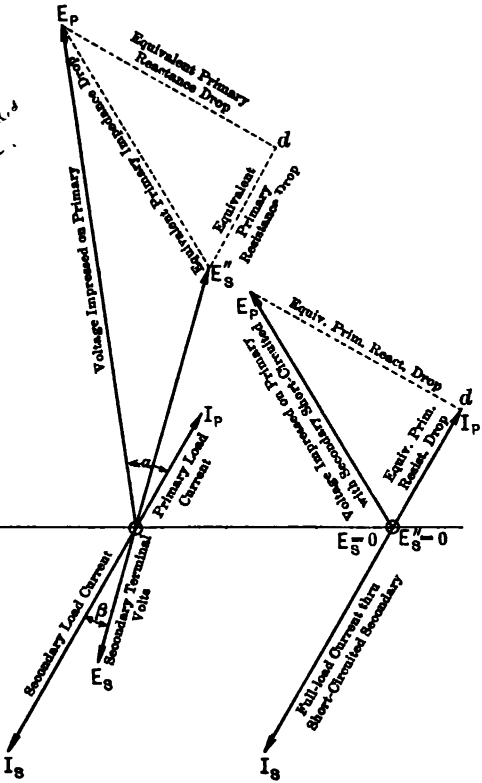
E_P
Equivalent Primary Reactance Drop
Equivalent Primary Impedance Drop
d
Voltage Impressed on Primary
Equivalent Primary Resistance Drop
E''_S
I_P
α
Primary Load Current
Secondary Load Current
Secondary Terminal Volts
β
E_S
I_S
E_P
Equiv. Prim. React. Drop
Voltage Impressed on Primary with Secondary Short-Circuited
d
Equiv. Prim. Resist. Drop
I_P
$E_S=0$
$E''_S=0$
Full-load Current thru Short-Circuited Secondary
I_S

FIG. 102. The equivalent primary voltage drops can be represented by the triangle $E''_S dE_P$, which is made up of the sides of the triangles $-E'_P b' E_P$ and $-E''_S c\ (-E'_P)$ of Fig. 101.

FIG. 103. When the secondary is short-circuited, the voltage E_P is required to overcome the equivalent primary impedance with full-load current flowing in the secondary. OD = volts to overcome equivalent primary resistance. dE_P = volts to overcome equivalent primary reactance.

in the secondary, have subtracted themselves. This remaining e.m.f. $-E_s''$ would therefore represent the induced counter e.m.f. in the primary, or rather that part of the impressed primary e.m.f. which is consumed in overcoming it. The induced e.m.f. in secondary would be equal to $\left(E_s'' \times \frac{N_s}{N_p}\right)$ and this is equal to E_s, as we assumed in drawing the diagram.

Now notice that the triangle $E_p'OE_s''$ is exactly similar to triangle $E_s'OE_s$, as $E_p' = \frac{N_p}{N_s} \times E_s'$, $E_s'' = \frac{N_p}{N_s} \times E_s$, and the angle between OE_p' and OE_s'' is exactly equal to the angle between OE_s' and OE_s.* Therefore it follows that the sides $E_p'E_s''$ and $E_s'E_s$ bear the same ratio to each other as any other similar sides of the two triangles; or,

$$(E_p'E_s'') = \frac{N_p}{N_s} \times (E_s'E_s).$$

Now let $E_p'E_s''$ be resolved into two components, one $(E_s''c)$ parallel to I_SI_P and the other $(E_p'c)$ perpendicular to I_SI_P. It will be seen, and may be proved, that the triangles $(E_p'cE_s'')$ and $(E_s'aE_s)$ are exactly similar, from which it follows that any similar sides bear the same ratio to each other. That is,

$$(E_s''c) = \frac{N_p}{N_s} \times (E_s'a) = \frac{N_p}{N_s} \times (R_sI_s)$$

and

$$(cE_p') = \frac{N_p}{N_s} \times (aE_s) = \frac{N_p}{N_s} \times (X_sI_s).$$

But since $(E_s''c)$ is parallel to I_p, and cE_p' is perpendicular to I_p, and together they represent the total drop due to $R'I_p$ (the drop across the **extra** primary resistance) and $X'I_p$ (the drop across the **extra** primary reactance), it follows that:

$$R'I_p = E_s''c = \frac{N_p}{N_s} \times R_sI_s$$

and

$$X'I_p = cE_p' = \frac{N_p}{N_s} \times X_sI_s.$$

---

* Two triangles are similar when two sides of one triangle are proportional respectively to two sides of the other and the included angles are equal. If two triangles are similar, their corresponding sides are proportional.

To simplify still further, we make use of the relation $I_s = \frac{N_p}{N_s} I_p$. Thus, substituting in the above equation, we get:

$$R' I_p = \frac{N_p}{N_s} \times R_s \times \frac{N_p}{N_s} I_p, \quad \text{or} \quad R' = \left(\frac{N_p}{N_s}\right)^2 R_s,$$

and

$$X' I_p = \frac{N_p}{N_s} \times X_s \times \frac{N_p}{N_s} I_p, \quad \text{or} \quad X' = \left(\frac{N_p}{N_s}\right)^2 X_s.$$

We are now prepared to state that:

Equivalent primary resistance $= R_p + R' = R_p + \left(\frac{N_p}{N_s}\right)^2 R_s.$

Equivalent primary reactance $= X_p + X' = X_p + \left(\frac{N_p}{N_s}\right)^2 X_s.$

Example 4. A transformer, rated 15 kv-a., with windings arranged to step down from 2400 to 240 volts, has a primary resistance of 2.335 ohms and a secondary resistance of 0.02745 ohm. What is the **equivalent primary resistance?** What would be the **equivalent secondary resistance?**

$$\begin{aligned} \text{Equivalent primary resistance} &= 2.335 + \left(\frac{2400}{240}\right)^2 \times 0.02745 \\ &= 2.335 + 2.745 \\ &= 5.080 \text{ ohms.} \end{aligned}$$

$$\begin{aligned} \text{Equivalent secondary resistance} &= \left(\frac{240}{2400}\right)^2 \times 2.335 + 0.02745 \\ &= 0.02335 + 0.02745 \\ &= 0.05080 \text{ ohm.} \end{aligned}$$

The **total equivalent impedance drop,** reduced to terms of primary, is represented by the vector E_pE_s'' in Fig. 102, which is seen to be exactly the same as Fig. 101, with some of the construction lines omitted. The total impedance drop evidently is the vector sum of the total resistance drop $[E_s''d, = (R_p + R')I_p]$, due to equivalent primary resistance and primary current, and the total reactance drop $[dE_p, = (X_p + X')I_p]$, due to equivalent primary reactance and primary current. Of course the total resistance drop and the total reactance drop are in quadrature with each other, because the former is in phase with I_p while the latter leads I_p by 90°. The total impedance drop in the primary is usually expressed as a percentage of the rated primary voltage (E_p). So also are the total resistance drop and the total reactance drop (reduced to the primary as shown in Example 4) expressed as percentage of the rated primary volts E_p. By this means it is much easier to compare transformers of different sizes with one another.

Example 5. (*a*) What would be the per cent resistance drop of the transformer in Example 4?

(*b*) If this transformer has an equivalent primary impedance of 2.0 per cent, what is the total equivalent reactance drop in per cent and in primary volts, and what is the equivalent primary reactance in ohms?

As the rated capacity is 15,000 volt-amperes and the primary voltage is 2400, the full-load current (neglecting the exciting current and the losses) is $\frac{15,000}{2400} = 6.25$ amperes in the primary or high-tension coil. As the equivalent primary resistance is 5.08 ohms, the primary equivalent resistance drop is $5.08 \times 6.25 = 31.75$ volts. This is represented by $E_s''d$ in Fig. 102. If the equivalent primary impedance is 2.0 per cent, the total impedance volts (represented by E_pE_s'' in Fig. 102) is 2.0 per cent of 2400 volts, equals 48 volts. Therefore the equivalent primary reactance drop (dE_p, in Fig. 102) must be equal to $\sqrt{(48)^2 - (31.75)^2} = 36$ volts, which is $\frac{36}{2400}$ or 1.5 per cent of the rated primary voltage. The equivalent reactance, in ohms, is equal to reactance volts divided by primary current, or $\frac{36}{6.25} = 5.76$ ohms. Or, we may say that the equivalent primary resistance drop of 31.75 volts is 1.322 per cent of the rated voltage $\left(\frac{31.75}{2400} = 0.01322\right)$, and that therefore the reactance drop must be equal to

$$\sqrt{(2.0\%)^2 - (1.322\%)^2}, \text{ or } 1.5 \text{ per cent.}$$

46. Short-circuit Test for the Impedance of a Transformer. Now let us short circuit the secondary of a transformer and apply enough voltage to the primary to send full-load current I_s through the short-circuited secondary.

The terminal voltage of the secondary is reduced to zero ($OE_s = 0$) and all voltage impressed upon the primary (OE_p) is consumed in overcoming the impedance of the transformer. This is shown in Fig. 103, where the primary impressed voltage OE_p is equal to the total equivalent primary impedance drop, consisting of resistance drop Od and reactance drop dE_p. Fig. 103 should be compared with Fig. 102 and 101, as they all represent conditions in the same transformer with same resistances and reactances and the same current flowing in each coil. The ratio of this transformer is 2 : 1. Obviously, if we divide the voltage impressed upon the primary of the short-circuited transformer by the current which it produces in the primary, we shall get the equivalent primary impedance, in ohms. If E_p is just sufficient to cause rated full-load current to flow, then $\frac{E_p}{\text{rated primary voltage}} \times$ 100 per cent is the per cent impedance of the transformer.

Example 5a. The low-tension side of a transformer, rated 5 kv-a., 2400/240 volts, 60 cycles, is short-circuited through an ammeter. The voltage across the primary terminals has to be made 72 volts in order to produce rated-load current through the ammeter. The resistances of the primary and secondary coils are measured and found to be 11.53 and 0.1153 ohms respectively. Calculate the equivalent primary reactance in per cent, volts, and ohms, and the copper loss at full load in each coil.

Full-load current in the low-tension coil means full-load current also in the high-tension coil. This is equal to $\frac{5000 \text{ volt-amperes}}{2400 \text{ volts}}$, or 2.08 amperes. The equivalent primary impedance is evidently equal to $\frac{72}{2400} = 0.03$, or 3 per cent. In ohms, this equivalent primary impedance is $\frac{72}{2.08}$, or 34.6 ohms. The equivalent primary resistance is $11.53 + \left(\frac{2400}{240}\right)^2 \times 0.1153 = 23.06$ ohms, or $23.06 \times 2.08 = 48$ volts, or $\frac{48}{2400} = 0.02 = 2$ per cent.

Therefore we should have:

Equivalent primary reactance, **in per cent,**

$$= \sqrt{(3\%)^2 - (2\%)^2} = 2.24 \text{ per cent.}$$

Equivalent primary reactance, **in volts,**

$$= \sqrt{(72)^2 - (48)^2} = 53.7 \text{ volts.}$$

Check: 2.24 per cent of 2400 volts = 53.7 volts.

Equivalent primary reactance, **in ohms,**

$$= \sqrt{(34.6)^2 - (23.06)^2} = 25.8 \text{ ohms.}$$

Check: $\dfrac{53.7 \text{ volts}}{2.08 \text{ amp.}} = 25.8$ ohms.

Copper loss at full load in high-tension coil,

$$= 11.53 \times 2.08^2 = 49.8 \text{ watts.}$$

Copper loss at full load in low-tension coil,

$$= 0.1153 \times 20.8^2 = 49.8 \text{ watts.}$$

Total copper loss = 49.8 + 49.8 = 99.6 watts.

Check: 2 per cent of 5000 watts = 100 watts.

At this point we may notice the great convenience of working with **percentage values** of impedance, resistance, and reactance of the transformer. If these be expressed in ohms, it becomes necessary for us to state whether they refer to the high-tension or the low-tension coil; but if they be expressed in percentage, the same value refers to either high-tension or low-tension coil.

Example 6. A 15 kv-a. 2400/120 volt transformer has a resistance of 2.963 ohms in the high-tension winding and 0.00685 ohm in the low-tension winding. Calculate the equivalent resistance in terms of both primary and secondary, in ohms and in per cent.

Rated-load current in high-tension coil (neglecting exciting current and losses) is $\dfrac{15{,}000}{2400} = 6.25$ amperes, and in low-tension coil it is $\dfrac{15{,}000}{120} = \left(6.25 \times \dfrac{2400}{120}\right) = 125$ amperes.

Equivalent resistance, in terms of high-tension coil,

$$= 2.963 + 0.00685 \times \left(\frac{2400}{120}\right)^2$$

$$= 5.703 \text{ ohms.}$$

Equivalent resistance, in terms of low-tension coil,

$$= 0.00685 + 2.963 \times \left(\frac{120}{2400}\right)^2$$

$$= 0.01426 \text{ ohm.}$$

Total resistance drop, in terms of high-tension coil,

$$= 5.703 \times 6.25$$
$$= 35.6 \text{ volts.}$$

This is $\left(\frac{35.6}{2400} \times 100 \text{ per cent}\right)$ or 1.485 per cent of the high-tension voltage.

Total resistance drop in terms of low-tension coil $= 0.01426 \times 125$
$= 1.782$ volts.

This is $\left(\frac{1.782}{120} \times 100 \text{ per cent}\right)$ or 1.485 per cent of the low-tension voltage.

Prob. 37–3. A transformer, rated 350 kv-a., 60 cycles, 11,000/2300 volts, has an impedance of 1.67 per cent and total I^2R loss of 1792 watts with full-load current flowing. Calculate the equivalent impedance in ohms, as referred to (*a*) high-tension coil, (*b*) low-tension coil.

Prob. 38–3. Calculate the total equivalent resistance in ohms and in per cent for the transformer specified in Prob. 37.

Prob. 39–3. Calculate the equivalent reactance of the transformer specified in Prob. 37. (*a*) In ohms referred to high-tension coil. (*b*) In ohms referred to low-tension coil. (*c*) In per cent.

Prob. 40–3. Calculate what current would flow if the transformer of Prob. 37 were to be short-circuited while connected to a generating plant and transmission line of capacity sufficient to hold the voltage up to 80 per cent of normal. (*a*) In per cent of rated current of the transformer. (*b*) In amperes on low-tension side.

Prob. 41–3. Calculate the rate of heating the copper in the transformer of Prob. 40 (*a*) in watts, (*b*) in per cent of the rate of heating at rated full-load.

Prob. 42–3. A transformer, rated 350 kv-a., 60 cycles, 34,650/430 volts, is equipped with internal magnetic shunts to increase its leakage reactance for protection against injury on short-circuits. The impedance is thereby increased to 18.2 per cent. The total copper (I^2R) loss with full-load current flowing in both coils is 1885 watts. Make same calculations as required in Prob. 37.

Prob. 43–3. Make same calculations for transformer of Prob. 42 as required in Prob. 38.

Prob. 44–3. Make same calculations for transformer of Prob. 42 as required in Prob. 39.

Prob. 45–3. Make same calculations for transformer of Prob. 42 as required in Prob. 40.

Prob. 46–3. Make same calculations for transformer of Prob. 42 as required in Prob. 41.

47. Calculation of Regulation for Constant-voltage Transformer. Having now a method for measuring the total equivalent impedance and reactance of a transformer, upon which the voltage regulation or the constancy of voltage depends, we may proceed to calculate the percentage regulation. The following statements and formulæ are copied from the Standardization Rules of the A.I.E.E., as printed in Proceedings for August, 1914:

"To compute the regulation for a constant-potential transformer, it is necessary to obtain the equivalent resistance R and impedance drop E_z. The equivalent resistance R of primary and secondary combined is found by multiplying the secondary resistance by the square of the ratio of turns and adding it to the primary resistance. The impedance voltage E_z is found by short-circuiting the secondary winding and measuring the volts necessary to send rated-load current through the primary."

The reactance drop is then

$$IX = \sqrt{E_z^2 - \left(\frac{P}{I}\right)^2},$$

where P is the "impedance watts" as measured in the short-circuit test.

Now, let

E = rated primary voltage.
IR = resistance drop in volts.
IX = reactance drop in volts.
I = rated full-load current in primary coil.
R = equivalent resistance reduced to primary.
$q_r = 100\frac{IR}{E}$ = per cent resistance drop.
$q_x = 100\frac{IX}{E}$ = per cent reactance drop.
m = power-factor of load = $\cos\theta$.
n = reactive factor of load = $\sqrt{1-m^2} = \sin\theta$.

Then the regulation is given by the following equation:

$$\text{Per cent regulation} = mq_r + nq_x + \frac{(mq_x - nq_r)^2}{200}.$$

For a load of unity power-factor, $m = 1.0$ and $n = 0.0$, and this reduces to:

$$\text{Per cent regulation} = q_r + \frac{q_x^2}{200}.$$

Example 7. Test of a 1000 kv-a. 110,000/22,000 volt 60-cycle transformer gave the following data:

Impedance watts = 7330.
Impedance volts = 5.00 per cent (of rated voltage).

Calculate the per cent voltage regulation of this transformer, (*a*) on non-inductive load, (*b*) on lagging load of 80 per cent power-factor. Resistance of high-tension winding is 43.34 ohms, and of low-tension winding 1.7337 ohms.

According to the Standardization Rules the rated primary voltage of a constant-potential transformer is equal to the rated secondary voltage multiplied by the "turn ratio." Therefore the ratio of high-tension turns to low-tension turns in this transformer is 110,000/22,000, or 5/1, and at zero load (only) this is also the ratio of terminal voltages.

The equivalent resistance referred to high-tension coil is

$$43.34 + \left(\frac{110{,}000}{22{,}000}\right)^2 \times 1.7337, \quad \text{or} \quad 86.68 \text{ ohms.}$$

The high-tension current at rated full-load (neglecting exciting current) is

$$\frac{1{,}000{,}000 \text{ volt-amperes}}{110{,}000 \text{ volts}}, \text{ or } 9.091 \text{ amperes.}$$

The total equivalent resistance drop at full load is

$$9.091 \times 86.68 = 788 \text{ volts, or } \frac{788}{110{,}000} = 0.00716, \text{ or } 0.716\%.$$

The impedance volts (at full-load current) are

$$5\% \text{ of } 110{,}000 \text{ volts, or } 5500 \text{ volts.}$$

Therefore

$$IX = \text{reactance volts} = \sqrt{(5500)^2 - \left(\frac{7330}{9.091}\right)^2} = 5441 \text{ volts.}$$

Then

$$q_x = \text{reactance drop in per cent} = \frac{5441}{110{,}000} \times 100\% = 4.945\%.$$

This value may be checked by the relation

$$\begin{aligned}\%\text{ reactance drop} &= \sqrt{(\%\text{ impedance drop})^2 - (\%\text{ resistance drop})^2}\\ &= \sqrt{(5.00)^2 - (0.716)^2} = 4.948\%.\end{aligned}$$

Substituting these values for their corresponding symbols in the equations for regulation, we have (remembering that $n = 0.6$ when $m = 0.8$)

Per cent regulation for 80 per cent power factor

$$\begin{aligned}&= (0.8 \times 0.716) + (0.6 \times 4.945) + \frac{(0.8 \times 4.945 - 0.6 \times 0.716)^2}{200}\\ &= 0.5728 + 2.967 + 0.0622\\ &\doteq 3.602 \text{ per cent.}\end{aligned}$$

Notice here that no serious error would result from neglecting the last term of the equation. In such case the regulation would appear to be 3.54 per cent.

Further, we have

$$\begin{aligned}\text{Per cent regulation for unity power-factor} &= 0.716 + \frac{(4.945)^2}{200}\\ &= 0.716 + 0.122\\ &= 0.838 \text{ per cent.}\end{aligned}$$

Here the second term is not negligible, because, although it is small, the first term is also small.

Notice that at or near unity power-factor, the regulation is nearly equal to the resistance drop in per cent, while at lower power-factors the regulation becomes more nearly equal to the reactance drop in per cent. This is in accord with our previous conclusions on basis of Fig. 98, 99, 100.

Let us now check this A.I.E.E. formula for regulation, against the vector diagram. Fig. 104 is practically the same as Fig. 102, a few unnecessary vectors having been omitted. In Fig. 104, all the vectors, except I, represent voltages, as follows:

OS = secondary terminal voltage reduced to terms of primary $\doteq 22{,}000\ T$.

T = ratio of turns $\frac{N_p}{N_s} = \frac{110{,}000}{22{,}000} = 5.0$.

$Sc = IR = 9.091 \times 86.68 = 788$ volts.

$Sa = IR \cos \alpha = 788 \times 0.80 = 630$ volts.

$cE_p = IX = 5441$ volts.

$$
\begin{aligned}
ab &= cd = IX \sin \alpha = 5441 \times 0.60 = 3265 \text{ volts.*} \\
Ob &= OS + Sa + ab \\
&= 22{,}000\,T + 630 + 3265 = (22{,}000\,T + 3895) = 113{,}895 \text{ volts.} \\
ac &= IR \sin \alpha = 788 \times 0.60 = 473 \text{ volts.} \\
dE_p &= IX \cos \alpha = 5441. \times 0.80 = 4353 \text{ volts.} \\
bE_p &= dE_p - db \\
&= 4353 - 473 = 3880 \text{ volts.} \\
OE_p &= \text{e.m.f. required to be impressed on primary at rated full load.} \\
&= \sqrt{(22{,}000\,T + 3895)^2 + 3880^2} \\
&= 113{,}961.
\end{aligned}
$$

At zero load, IR and IX both reduce practically to zero, and since E_p is maintained constant and the induced counter e.m.f. becomes equal to it, we have:

$$OS_o = \frac{113{,}961}{5.00} = 22{,}792 \text{ volts.}$$

This is the terminal e.m.f. of secondary at zero load. Therefore,

$$\text{Regulation} = \frac{22{,}792 - 22{,}000}{22{,}000} = 0.0360 = 3.60 \text{ per cent.}$$

This checks very closely with the value 3.602 per cent obtained by formula. In fact, the formula is only an algebraic expression of the numerical relations between the vectors of Fig. 104, with certain terms omitted which are relatively so small as to be negligible.

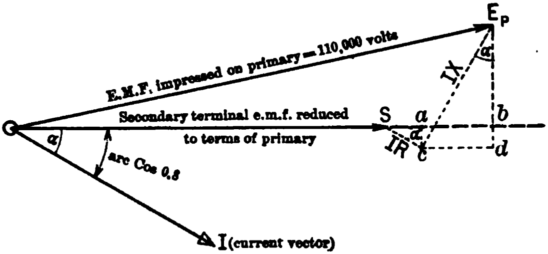
E_P
E.M.F. impressed on primary = 110,000 volts
IX
α
Secondary terminal e.m.f. reduced to terms of primary
O
S
a
b
α
IR
c
d
α
arc Cos 0.8
I (current vector)

FIG. 104. The regulation of a transformer equals $\frac{OE_P - OS}{OS}$. $OS = OE_P \ominus IR \ominus IX$.

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* Two angles are equal (or supplementary) when their corresponding sides are perpendicular. Note that dE_p is perpendicular to Sa and cE_p is perpendicular to Sc. Thus the angle between dE_p and cE_p must be equal to the angle between Sa and Sc.

Prob. 47–3. Prove that the per cent IR drop in a transformer is equal to the per cent of I^2R loss at rated full-load non-inductive.

Prob. 48–3. Using the data of Example 7, and assuming that $IR = 788$ and $IX = 5441$ in terms of primary, calculate what ratio of turns (T) must be used to give 22,000 volts at terminals of secondary, full-load 80 per cent power-factor, lagging, when 110,000 volts are impressed upon the primary.

Prob. 49–3. If the line voltage be maintained at 110,000 volts, as in Prob. 48, calculate:

(*a*) Secondary terminal voltage at zero load.

(*b*) Voltage regulation in per cent. Compare this value of regulation with that obtained by the A.I.E.E. formula.

Prob. 50–3. Assuming the value of the ratio of the number of turns (T) as calculated in Prob. 48–3, recalculate the values of

(*a*) Equivalent IR.

(*b*) Equivalent IX.

Compare these values with those of Example 7.

Prob. 51–3. Using the values of Prob. 50, repeat the calculation based on Fig. 104. That is, $OS = T \times 22{,}000$, and IR and IX as in Prob. 50. Calculate therefrom:

(*a*) The primary terminal e.m.f. (OE_p) that must be impressed.

(*b*) The per cent voltage regulation if this e.m.f. is maintained constant while load is removed. Compare this regulation with that obtained by the A.I.E.E. formula.

SUMMARY OF CHAPTER III

TRANSFORMERS change high voltage to low voltage and vice versa. They consist of stationary coils linked together by a stationary core, and are the simplest, most rugged, most efficient, and least expensive in first cost and maintenance of any electrical machine.

THE CAPACITY of a transformer is the load that it will carry without developing an injuriously high temperature at any spot in the apparatus.

TRANSFORMERS ARE COOLED by one of the following means:

(1) The case is filled with oil which carries the heat from the coils to the case which dissipates it into the surrounding air. The cooling surface may be increased by means of auxiliary pipes.

(2) By circulating water through pipes installed in the oil-filled cases.

(3) By forced circulation of the oil.

(4) By forcing air through the coils by means of a blower.

THE RATIO OF A TRANSFORMER is the ratio of the number of turns in the high-voltage coils to the number of turns in the low-voltage coils.

THE INDUCED E.M.F. IN THE COILS HAS A PHASE DIFFERENCE of 180° to the impressed e.m.f., if the impressed e.m.f. has a sine wave-form. The induced e.m.f. lags 90° behind the magnetizing current which the impressed e.m.f. forces through the primary coil. The impressed e.m.f. therefore leads the magnetizing current by 90°.

WHEN NO CURRENT IS BEING TAKEN FROM THE SECONDARY coil, the counter e.m.f. which is induced in the primary prevents the impressed e.m.f. from sending more than the exciting current through the primary coils.

WHEN A CURRENT IS TAKEN FROM THE SECONDARY of a transformer, this current sets up a counter magnetomotive force in the core, which decreases the flux and the value of counter e.m.f. induced in the primary. The impressed e.m.f. can thus send enough more current through the primary to balance this counter m.m.f. due to load current in the secondary coil. The ratio of the current in the primary to the current in the secondary at full load is approximately the inverse ratio of the number of turns in the primary and secondary coils.

THE EXCITING CURRENT (I_E) OF A TRANSFORMER is the vector sum of the magnetizing current (I_M) and the core-loss current (I_H) which is taken to supply the hysteresis and eddy-current losses in the core. These two currents are in quadrature, since I_M is reactive and I_H supplies real power to the transformer.

Therefore

$$I_E = \sqrt{I_M^2 + I_H^2}.$$

The power-factor of power taken by an unloaded transformer is usually found as follows:

$$\text{Power-factor} = \frac{I_H}{I_E} = \frac{I_H}{\sqrt{I_M^2 + I_H^2}}$$
$$= 10\% \text{ to } 50\% \text{ (usually)}.$$

THE EXCITING CURRENT has not a sine wave-form when the maximum value of the flux density lies beyond the " knee " of the magnetization curve for the core. This is in spite of the fact that the impressed e.m.f. usually has a sine wave-form.

THE MAXIMUM VALUE OF FLUX DENSITY B_m should not be carried much beyond the " knee " of the magnetization curve as all the losses are increased and greater heating results. The value of B_m can be found from the following equation:

$$B_m = \frac{10^8 E_p'}{4.44 AfN_p} = \frac{10^8 E_s'}{4.44 AfN_s}.$$

This equation shows how B_m is affected by any change in the voltage, area of core, frequency, and number of turns in the coils.

THE LOSSES IN A TRANSFORMER consist of core losses and copper losses.

Core losses are nearly constant for all loads and are composed of two separate losses.

(First). **HYSTERESIS LOSS**, the equation for which is

$$P_H = WK_H f B_m^{1.6}.$$

The hysteresis loss for transformers using annealed silicon steel varies from 0.54 to 0.82 watt per pound of steel, for a frequency of 60 cycles and a maximum flux density of 64,500 lines per square inch.

(Second). **EDDY-CURRENT LOSS**, the equation for which is

$$P_E = WK_E f^2 B_m^2 t^2.$$

The eddy-current loss for ordinary transformer silicon steel having thickness of 0.014 inch is 0.12 to 0.18 watt per pound of core, at 60 cycles, and 64,500 lines per square inch for B_m.

THE COPPER LOSS in a transformer consists of the I^2R in the primary and in the secondary winding and is usually about equally divided between the two.

The cheaper the transformer, the greater the losses and the lower the efficiency. If the yearly cost of supplying the power for the greater losses in any particular case is less than the yearly interest on the extra cost of a more efficient transformer, the cheaper type should be purchased, and vice versa.

STATION TRANSFORMERS of from 100 to 10,000 kv-a. step up the pressure generated, usually at about 6600 or 11,000 volts, to a much higher pressure (as high as 150,000 volts) for economical transmission. Transformers of the same type and size are often used to step down the line pressure to about 2300 volts for distribution throughout a town or community.

DISTRIBUTING TRANSFORMERS, usually below 50 kilovolt-amperes in size, are used to step down the 2300 volts of the distributing lines to the pressure of the consumers' apparatus, usually 110, 220, or 440 volts.

THE EFFECT OF OPERATING TRANSFORMERS AT OVERVOLTAGE is to increase greatly the core and copper losses. By a slight increase in voltage the capacity of the transformer may be raised, but the losses increase so much faster than the rate of increase of the voltage that it rarely pays to operate at more than a small percentage overvoltage.

OPERATION AT UNDER VOLTAGE means increased annual charges per kilovolt-ampere transformed.

LEAKAGE REACTANCE is the reactance which is in greatest evidence when a transformer is loaded and is due to magnetic flux lines which do not link both primary and secondary coils. Leakage reactance in the primary circuit requires part of the voltage impressed on the primaries to be used to overcome it. Thus the voltage induced in the primary coils is less than the impressed voltage. Leakage reactance in the secondary circuit requires part of the voltage induced in the secondary coils to overcome it. Thus the terminal voltage across the secondary coils is less than the induced voltage of the secondary coils. The voltage drop necessary to overcome this leakage reactance is always 90° ahead of the current which sets up the leakage lines. In a practical transformer the *IR* drops in phase with the currents, combined with the reactance drops leading the currents by 90°, cause the ratio of the primary and secondary voltages to differ slightly from the ratio of the number of turns in the coils.

The practical vector diagram of conditions in a loaded transformer shows all these relations.

THE VOLTAGE REGULATION DEPENDS UPON THE POWER-FACTOR of the load. The drop is due mainly to the resistance of the coils when the load has approximately unity power-factor, and to the leakage reactance when the power-factor is low and lagging. A low leading power-factor tends to raise the terminal voltage of the secondary above the induced e.m.f.

THE TRANSFORMER WITH THE BETTER REGULATION costs more and is more liable to injury from short-circuits and improper operation.

TO FIND THE EQUIVALENT PRIMARY RESISTANCE AND REACTANCE of a transformer, consider the resistance and reactance of the secondary side to be zero, and enough extra resistance and reactance added to the primary side to cause the secondary terminal voltage to have the same value as in the actual transformer, at any given load. The sum of the resistance of the primary plus this extra resistance is called the Equivalent Primary Resistance and the sum of the primary reactance plus this extra reactance is called the Equivalent Primary Reactance.

$$\text{The Equivalent Primary Resistance} = R_P + \left(\frac{N_P}{N_S}\right)^2 R_S.$$

$$\text{The Equivalent Primary Reactance} = X_P + \left(\frac{N_P}{N_S}\right)^2 X_S.$$

Similarly

$$\text{The Equivalent Secondary Resistance} = R_S + \left(\frac{N_S}{N_P}\right)^2 R_P.$$

$$\text{The Equivalent Secondary Reactance} = X_S + \left(\frac{N_S}{N_P}\right)^2 X_P.$$

THE IMPEDANCE OF A TRANSFORMER is the Equivalent Primary Impedance, and may be expressed in ohm

or in volts to overcome impedance drop at rated load or, better still, in percentage of impressed primary voltage necessary to overcome impedance drop at rated load. It is found by measuring the primary voltage necessary to send full-load current through the short-circuited secondary.

THE REGULATION OF A CONSTANT VOLTAGE TRANSFORMER may be computed from the impedance watts, impedance volts, and the resistance of both windings, by the vector diagram or the A.I.E.E. approximate formula.

PROBLEMS ON CHAPTER III

Prob. 52–3. (*a*) At what per cent of rated full-load (non-inductive) will the 10 kv-a. "Type S" transformer of Table A, page 149, attain its maximum efficiency?

(*b*) What will this efficiency be?

(*c*) By what percentage will the total rate of heat development in the transformer then be greater or less than at rated load?

Prob. 53–3. Solve Prob. 52 with relation to the 10 kv-a. "Type SA" transformer in Table A. Compare the results of Prob. 52 and 53.

Prob. 54–3. On the basis of 3 hours at full-load and 21 hours at zero-load each day, what will be the all-day efficiency of the 5 kv-a. "Type S" transformer in Table A? Power-factor of load 100 per cent.

Prob. 55–3. Solve Prob. 54 on the basis of a load having 80 per cent power-factor.

Prob. 56–3. A Wright demand ammeter in service several months on the primary of a 15 kv-a. transformer supplying about ten residences registered about 25 per cent of the total connected load. Assuming this transformer to have the characteristics of a "Type S" transformer in Table A, calculate the all-day efficiency on the supposition that the transformer rating is 50% of the total connected load, and that the peak load persists 4 hours per day, zero load during the other 20 hours. Power-factor of load is 100 per cent.

Prob. 57–3. Transformers for lighting service are usually selected so that at peak load they are overloaded about 25 per cent. What would be the all-day efficiency of a 2-kv-a. "Type S" transformer (Table A), supplying a load of 90 per cent average power-factor for 5 hours per day, and no load the other 19 hours?

Prob. 58–3. A group of buildings demands 25 kv-a. at 100 per cent power-factor for 3 hours, 15 kv-a. at 70 per cent power-factor for 8 hours, and zero load for 13 hours each day. If transformers having the characteristics of "Type SA" in Table A are considered, would the all-day efficiency be higher by using a 20 kv-a. size or by using a 25 kv-a. transformer?

Prob. 59–3. What should be the watts taken from a line at rated voltage and frequency by a 7½-kv-a. "Type S" transformer (Table A) with its secondary circuit open? By what percentage is this greater than the real value of core loss?

Prob. 60–3. Calculate the magnetizing and core-less components of exciting current for a ½-kv-a. and for a 50-kv-a. "Type S" transformer (Table A).

Prob. 61–3. A station transformer, rated 550 kv-a., 60 cycles, 10,500 volts high-tension, when operated at rated full-load non-inductive, yields the following data:

Efficiency = 98.31 per cent.
Constant (core) loss = 0.88 per cent of input.
Variable (copper) loss = 0.81 per cent of input.

A distributing transformer manufactured by the same company, and rated 37½ kv-a., 60 cycles, 2200 volts high-tension, yields the following data when operated at rated full-load non-inductive.

Efficiency = 98.35 %.
Constant (iron) loss = 197 watts.
Variable (copper) loss = 433 watts.

The usual operating conditions of distributing transformers will be met if the all-day efficiency is based on 5 hours at full-load (non-inductive) and 19 hours at zero load. At zero load the input is equal to the constant loss only; the output is zero and variable loss is negligibly small.

Calculate the all-day efficiency of each of these transformers on this basis. Notice that the full-load efficiency of both transformers is practically the same.

Prob. 62–3. (*a*) Repeat the calculations of Prob. 61 on basis of 15 hours at full-load and 9 hours at zero load, each day. Compare results with those of Prob. 61.

(*b*) Which operating condition is more likely to be met in the station transformer? In the distributing transformer? Which gives best all-day efficiency under each condition?

Prob. 63–3. Check Table I, page 159, against the formulas for core losses given in Art. 38, on the assumption that:

(*a*) The core loss is entirely due to hysteresis.

(*b*) The core loss consists of hysteresis watts and eddy-current watts normally in the proportion 4 : 1.

(Note that if core loss were entirely due to eddy-currents, increasing the voltage from 100 per cent to 110 per cent at constant frequency would increase B_m in the ratio $\left(\frac{1.10}{1.00}\right)$ and the eddy-current watts in the ratio $\frac{1.1^2}{1.0^2}$, or 1.21. Compare this with the value 1.23 given in the table.

To solve with regard to hysteresis loss requires a knowledge of logarithms.)

Prob. 64–3. Calculate the magnetic force exerted upon the winding on short-circuit as a percentage of the magnetic force exerted at rated full-load,

(*a*) For the transformer specified in Prob. 37.

(*b*) For the transformer specified in Prob. 42.

Prob. 65–3. (*a*) How many volts must be applied to the high-tension winding of the transformer of Prob. 37 with low-tension short-circuited, to produce full-load current through both windings?

(*b*) How many volts must be applied to the low-tension winding with the high-tension winding short-circuited?

Prob. 66–3. A short-circuit or impedance test is made on a transformer rated 300 kv-a. 34,600/2300 volts, 60 cycles. It is found that 1045 volts must be impressed on the high-tension coil to produce full-load current in the transformer and the power supplied is 2270 watts. Calculate:

(*a*) Impedance in per cent.

(*b*) Equivalent resistance in ohms (referred to high-tension) and in per cent.

(*c*) Short-circuit current at rated voltage, in per cent of rated full-load current.

(*d*) Magnetic forces exerted in per cent of those existing at rated load.

Prob. 67–3. How many henries of inductance having negligible resistance must be inserted in series with the high-tension coil of the transformer of Prob. 37 in order to limit the current to 5 times rated full-load when a short-circuit occurs on the secondary?

Prob. 68–3. Calculate the regulation of the transformer of Prob. 67, with its current-limiting reactance, at unity power-factor and at zero power-factor. Compare with that of the transformer alone.

Prob. 69–3. Repeat calculation of Prob. 67 for a reactance connected close to the transformer on the secondary side.

Prob. 70–3. Repeat Prob. 68 on the assumption of Prob. 69.

Prob. 71–3. By the method used in Example 7, calculate what ratio of turns would be required to deliver 22,000 volts at secondary terminals with 110,000 volts impressed at primary terminals, with full-load, non-inductive.

Prob. 72–3. Using the value of T calculated in Prob. 71, and the values of IR and of IX calculated in Example 7, redraw Fig. 104 and calculate what value of primary volts (OE_p) must be impressed to give 22,000 volts at secondary terminals with full load, 80 per cent power-factor.

Prob. 73–3. While the primary line voltage is maintained constant at the value calculated in Prob. 72 and the load is reduced to zero, calculate:

(*a*) The zero-load terminal voltage of secondary.

(*b*) The per cent voltage regulation. Compare this regulation with that obtained by the A.I.E.E. formula in Example 7.

Prob. 74–3. A transformer, rated 1500 kv-a., 33,000/2300 volts, 60 cycles, shows the following results on test:

Impedance volts = 4.41 per cent.

Impedance watts = 8200.

Total watts I^2R loss in primary and secondary as calculated from measured resistances and full-load currents, 6570 watts. Calculate:

(*a*) Per cent I^2R loss at full-load, non-inductive.

(*b*) Per cent IR drop at full-load.

(*c*) Per cent IX drop at full-load.

(*d*) Per cent voltage regulation for non-inductive load, by A.I.E.E. formula. Notice that there is a considerable difference between impedance watts and I^2R as calculated from measured resistances. Try to explain possible reasons for this.

Prob. 75–3. Compute the limiting values of the constant K_H in the equation for hysteresis loss, for ordinary steels having $P_H =$ 1.0 to 2.0 watts per pound, while $B_m = 64{,}500$ lines per square inch and the flux varies harmonically at 60 cycles per second.

Prob. 76–3. Under the same conditions as in Prob. 75, compute the limiting values of K_H for "silicon steels" containing from 3 to 4 per cent silicon. See data on page 145.

Prob. 77–3. Compute the limiting values of the constant K_E in the equation for eddy-current loss for annealed sheet steels containing no silicon. Conditions as in Prob. 75. Data on page 145.

Prob. 78–3. Compute the limiting values of K_E for "silicon steels" containing from 3 to 4 per cent silicon. Conditions as in Prob. 75. Data on page 145.

CHAPTER IV

TRANSFORMERS

OPERATION AND POLYPHASE CONNECTIONS

As the territory to be served becomes extended and new loads are added to the low-tension distributing system fed from the secondaries of transformers, we may install the necessary additional transformer kilovolt-ampere capacity according to either of two policies:

(*a*) Furnish a separate and independent transformer for each load or convenient group of loads.

(*b*) Connect the transformer secondaries in parallel, through a network of secondary or low-tension mains, to the loads or services which are also in parallel.

The first policy tends to a large number of relatively small transformers, while in the second system a smaller number of relatively large transformers will suffice to carry the same loads. The latter has the advantage that the cost per kv-a. of transformer capacity is less, and that both the core loss and the copper loss per kv-a. are less, since the efficiency and the economy of material are better in large transformers than in small ones.

The practice of "banking" transformers, or connecting their secondaries in parallel, has also the advantage that the total rated kv-a. of the transformers required may be considerably less than would be necessary if isolated transformers were used. Thus, relatively **small** transformers connected to a number of small individual loads usually require a transformer capacity of approximately 80 per cent of the connected load. Relatively **large** transformers connected to a number of small consumers usually require a transformer capacity of from 30 per cent to 50 per cent of the connected load. This reduction of transformer capacity necessary per kilowatt of connected load is due to the "diversity factor" of the individual loads connected. In general, the more separate services we connect together, the lower will be the ratio of the peak load of the combination to the total connected load or to the sum of the peaks of the individual loads, on account of the decreasing probability of the peaks occurring simultaneously.

As offsetting this saving in first cost and improvement of all-day efficiency in transformers, due to increase in the number and diversity of interconnected loads and to the use of fewer and larger transformers thereby made possible, we must consider the cost of the extensive low-tension distributing network of mains required, and the value of the energy losses in it. If the interconnections between transformers is carried too far, or if the average size of district served per transformer becomes too great, the increase in the annual fixed charges on the low-tension mains and the value of energy lost in them may be greater than the saving of transformer costs and losses effected by such interconnections.

Then, too, if the operating characteristics of banked transformers are not properly suited to one another, cross-currents analogous to the synchronizing currents between parallel alternators will flow between them. When this occurs the total kv-a. of transformer rating must be larger for the same load than would be necessary if the system were designed with regard to the fundamental principles involved.

A practical disadvantage in putting too many consumers on one bank of transformers is that the service to all of them may be interrupted in case of accident to one or more of the transformers. In such event the entire load, being connected to the remaining transformers, overloads them and causes their fuses to blow one after another. Where transformer secondaries are not paralleled, the failure of one could not interrupt the service from others. The maximum practicable size of distributing transformer may be limited by the strength of pole or cross-arm on which it is supported or the size of manhole in which it is placed, or by the maximum rate at which the heat losses can be dissipated from the manhole.

48. Proper Conditions for Paralleling Transformers. Stated briefly, the characteristics and adjustments necessary to make transformers operate properly in parallel are as follows:

(1) The ratio of primary to secondary voltage should be the same for all transformers in parallel.

(2) Secondary terminals of similar polarity should be connected together or to the same low-tension main.

(3) The percentage of impedance should be approximately the same for all transformers in the same bank.

(4) The ratio of resistance to reactance, or of resistance or reactance to impedance, should be the same for all of the transformers.

Consider the effects of violating each one of these requirements separately, while the others are satisfied. For simplicity take only two transformers:

(1) **Voltage Ratios Unequal.** If the voltage ratios are not equal, the transformer having the higher secondary induced voltage will force current through the other against its induced e.m.f. in such amount as will consume the difference of induced voltage in overcoming the total impedance of the local circuit formed by the two secondaries. This current will flow even when no external load is connected, and will result in increased losses and reduced capacity to carry useful load.

(2) **Wrong Polarity.** This may be considered as a greatly magnified case of (1). The resultant voltage in the local circuit between the secondaries is equal to the arithmetical **sum** rather than the **difference** of the secondary induced voltages. As the total impedance of this circuit is kept relatively small by careful design in order to avoid bad voltage regulation, this resultant voltage causes a current to flow which is enormous relative to the rated current, and

which must burn out the transformers if it does not open the fuses or other devices provided to protect the transformers against overload.

(3) Unequal Percentages of Impedance. The currents carried by the transformers, expressed as percentages of their respective rated full-load currents, will be inversely proportional to their percentages of impedance. As the terminal voltages are practically the same for all transformers in parallel, the kv-a. load in individual transformers, **expressed as percentage of their respective rated kv-a.**, will also be inversely proportional to their percentages of impedance. Thus, if transformer *A* has 4 per cent impedance and *B* has 2 per cent impedance, then when the total load is such that *A* carries 75 per cent of its rated kv-a. or rated current, transformer *B* must at the same time be carrying $\left(\frac{4 \text{ per cent}}{2 \text{ per cent}} \times 75 \text{ per cent}\right)$ or 150 per cent of its own rated kv-a. or rated current: in other words, *B* is operating at 50 per cent overload when *A* is at $\frac{3}{4}$ load, and *A* is operating at half load when *B* is at full load, regardless of what the actual kv-a. value of these rated loads may be.

(4) Unequal Ratios of Reactance to Resistance. If the ratio of (equivalent) resistance to (equivalent) reactance, or the ratios of resistance or reactance to impedance, are not equal for all the transformers whose secondaries are interconnected in parallel, the result will be that the total kv-a. of transformer capacity required will be greater than the total kv-a. of load served from the network. This is because the currents and the kv-a. in the different transformers will not be in phase with one another or with the total load, and the power-factors of the individual transformers will not be equal to one another or to the resultant power-factor of the load. If the transformers are very poorly selected, the resulting loss of load capacity and increase of energy losses in transformers and in connecting mains may become a very serious matter.

49. Polarity of Transformers. The Standardization Rules of the American Institute of Electrical Engineers specify a definite system for marking the relative polarity of all terminals of transformers, in order to minimize the chances of making improper connections between various transformers or between the various coils of any one transformer. The two ends of each high-tension coil are to be marked *A* and *B*, respectively, and the two ends of each low-tension coil are to be marked *X* and *Y*, respectively, in such manner that the alternating e.m.f. in the direction from *A* towards *B* reaches its cyclic maximum value at the same instant the e.m.f. from *X* towards *Y* reaches its cyclic maximum value. If the transformer has two high-tension coils, the terminals of the second one are to be marked A_1 and B_1, these marks having the same relative significance as *A* and *B*. Similarly, a transformer may have two low-tension coils, *XY* and X_1Y_1.

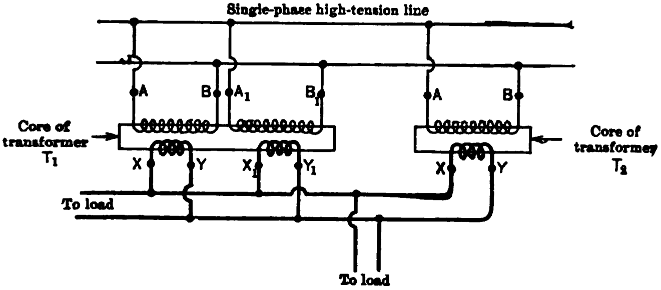


FIG. 105. Transformer T_1, having two coils in both the high-tension and the low-tension side, is connected in parallel with transformer T_2 which has but one high-tension coil and one low-tension coil. Note that the ends marked *A* are all joined to the same high-tension line wire and the ends marked *X* are all joined to the same low-tension line wire.

To connect transformers thus marked in parallel, it is only necessary to connect similarly marked terminals together to the same main. Thus, in a distributing transformer having two high-tension coils and two low-tension coils, parallel connections would be made as in T_1, Fig. 105. In this figure, T_1 is shown connected in parallel with another transformer, T_2, which has only one high-tension and one low-tension coil.

50. Determination of Polarity in Transformers. Suppose we are given a transformer like T_1, Fig. 105, whose terminals have not been marked. The data given on the rating plate are as follows:

Kv-a. = 5.0, or Watts = 5000.
Volts = 2200/220 – 110.
Frequency = 60 cycles per second.

There are four low-tension leads of heavy wire protruding from the transformer case, and, although there are only two high-tension leads of light wire protruding, there are four coil-terminals inside which must be connected in a certain manner to the two high-tension leads by means of brass or copper links on a porcelain terminal block within the transformer-case.

According to the rating plate, the e.m.f. impressed upon the high-tension primary circuit at 60 cycles should not exceed 2200 volts effective, and at this value each of the secondary coils will have 110 volts induced in it, giving 220 volts between low-tension terminals if the secondary coils are connected in series, or 110 volts if they are connected in parallel.

Label either terminal of the high-tension winding A and the other B_1 (Fig. 106). Connect these primary terminals to low-tension mains (say, 220 volts alternating) or to the low-tension secondary of another transformer. Then, by connecting various paired combinations of the four secondary terminals to a suitable voltmeter or testing lamp, or even to pieces of small fuse-wire, determine which are the two terminals of each low-tension coil S_1 and S_2. Connect an end of one of these coils (S_2) to the B_1 end of the primary, as in Fig. 106. If then the e.m.f. from A to the remaining end of S_2 is greater than the e.m.f. from A to B_1, we know that it must be the X end of

S_2 which is connected to B_1, and the other end is Y, as marked in Fig. 106. The e.m.f.'s in all coils are in time-phase with one another, being in quadrature with the core flux common to all coils. The voltmeter shows in Fig. 106 that S_2 is in additive series with the primary AB_1, indicating that the joined terminals are of opposite polarity at all instants.

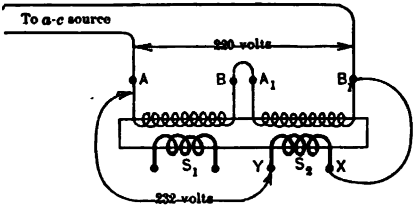


Fig. 106. The test for determining the polarity of a transformer. Terminals A and B_1 are marked arbitrarily. The higher voltmeter reading between the terminals A and Y indicates that coil S_2 is joined in additive series to the high-tension coils. Thus the positive direction through S_2 must be from X to Y.

The polarity of S_1 may be determined in similar manner. As a check, we may connect the X_1 of S_1 to the X of S_2. There should be zero resultant voltage, and a piece of small fuse wire connected between Y and Y_1 should not blow. Or, if we connect the Y of S_2 to the X_1 of S_1, we should find that the e.m.f. across the series is equal to the arithmetical sum of the e.m.f.'s across S_1 and S_2.

Prob. 1-4. Each of the two low-tension coils XY and X_1Y_1 in a 5-kv-a. 60-cycle distributing transformer has $\frac{1}{10}$ as many turns as each of the high-tension coils AB and A_1B_1. The maximum permissible core flux generates 110 volts in each low-tension coil. If we connect B to A_1, and Y to X_1, what is the greatest line pressure (effective volts) which may be applied between A and B_1, and what voltage would be obtained across XY_1?

Prob. 2-4. In the transformer of Prob. 1, if we connected B to B_1 and then connected A and A_1 to the same high-tension mains, without altering the low-tension connections, what would be the result?

Prob. 3-4. If we connect A and A_1 to one of the same high-tension line wires, and B and B_1 to the other, and leave the low-tension coils connected together as in Prob. 1, what will be the voltage between low-tension terminals of the transformer? How will the values of total flux, B_{max}, and magnetizing current compare with the corresponding values for Prob. 1?

Prob. 4–4. Draw sketch of connections for the transformer of Prob. 1, with high-tension coils in series and low-tension coils delivering power to a three-wire 110–220-volt system of secondary mains. Label clearly all coil terminals with proper letters.

Prob. 5–4. In the transformer of Prob. 1, connect X to Y_1, then connect Y and X_1 to 110-volt mains. If we leave the high-tension coils connected as in Prob. 1, what will be the pressure between secondary mains AB_1?

Prob. 6–4. With the high-tension coils connected together as in Prob. 1 and to the same line, the secondary coils are to be paralleled. Show the connections and calculate voltage between secondary mains.

51. Effect of Unequal Voltage Ratios in Parallel Transformers. The two transformers shown connected in parallel in Fig. 107 have unequal ratios of turns, X_2Y_2 having the lower voltage induced in secondary coils. Fig. 108 is a simplified diagram of secondary connections only, and Fig. 109 illustrates vectorially the relations of phase and value between the e.m.f.'s and currents in the closed secondary network formed by the coils X_1Y_1 and X_2Y_2 with their connecting mains m and n. The equivalent resistance and reactance reduced to basis of low-tension side are r_1 and x_1, respectively, for transformer No. 1, and r_2, x_2, respectively, for transformer No. 2. The secondary mains m, n are assumed to have negligibly small impedance, for simplicity.

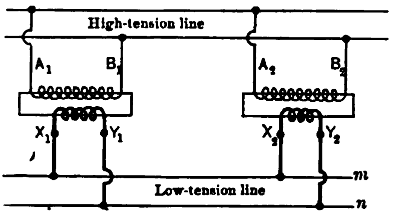


FIG. 107. The two transformers having unequal voltage ratios are connected in parallel.

In Fig. 109, vector E_1 represents the e.m.f. from X_1 toward Y_1, and E_2 represents the smaller e.m.f. from X_2 toward Y_2. The vector E_2' (or E_2 reversed) represents the e.m.f. from Y_2

to X_2, in the same direction as E_1 in coil X_1Y_1, through the series circuit formed by the two secondary coils with the mains m,n, or in the direction indicated by the dotted arrow in Fig. 108. Adding E'_2 to E_1 vectorially as in Fig. 109, we obtain the resultant e.m.f. E_R acting in the circuit of secondary coils, in the direction of the larger e.m.f. E_1 from X_1 to Y_1, and from Y_2 to X_2. This resultant e.m.f. E_R produces a current I through both coils, whose value is given by the equation:

$$I = \frac{E_R}{\text{Total equivalent impedance}} = \frac{E_1 \ominus E_2}{z_1 \oplus z_2} = \frac{E_1 \ominus E_2}{\sqrt{(r_1 + r_2)^2 + (x_1 + x_2)^2}}.$$

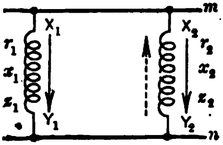


FIG. 108. A simple diagram to represent the properties of the two transformers connected in parallel as in Fig. 107. The dotted arrow represents the positive direction of the e.m.f. of transformer X_2Y_2 with respect to the circuit formed by the coils X_1Y_1, X_2Y_2, and mains m and n.

The lengths of vectors E_1 and E_2 are obtained by dividing the voltage E of the high-tension line by the respective ratios of high-tension to low-tension turns

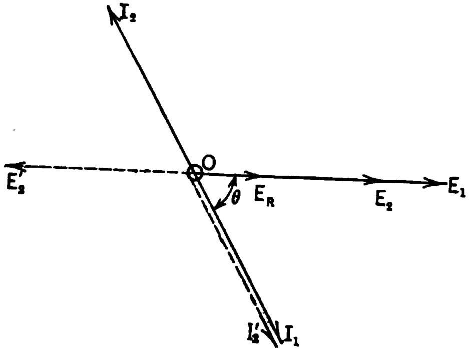


FIG. 109. The vector diagram for the current and e.m.f.'s in Fig. 107 and 108. The full lines represent phase relations with respect to positive directions in both coils toward the same main. The dotted lines refer to positive directions which are the same within the circuit formed by the coils and the mains m and n.

for the two transformers. If we assume negligible impedance drop in the high-tension line between transformers, E is identical in value and in phase for both of them; therefore E_1 and E_2 are in phase with each other, and their vector difference has the same value as their arithmetical difference.

The current I lags behind E_R by an angle θ such that

$$\tan\theta = \frac{x_1 + x_2}{r_1 + r_2}.$$

The equivalent low-tension resistances and reactances for the two transformers are r_1 and x_1, r_2 and x_2, respectively. As E_R is in phase with E_1 (the greater e.m.f.), the current I_1 in coil X_1Y_1 will lag $\theta°$ behind E_1. This same current, represented by the dotted vector I'_2 in Fig. 109, flows through X_2Y_2 in the direction indicated by the dotted arrow in Fig. 108, which is the direction of the resultant e.m.f. E_R in the series circuit of secondaries. Thus, if the vector I'_2 represents the current in transformer No. 2 with respect to a positive direction from Y_2 toward X_2, it follows that the vector I_2 (directly opposite to I'_2) represents the same current but with respect to a positive direction from X_2 to Y_2, or with respect to the induced e.m.f. E_2 in the secondary of transformer No. 2.

Notice from Fig. 109 that the current produced by inequality of voltage ratios in the transformers lags behind the higher secondary e.m.f. by θ degrees and leads the lower secondary e.m.f. by $(180 - \theta)$ degrees. As θ must be less than 90 degrees, it follows that the secondary of transformer No. 1 (having the higher secondary induced e.m.f., corresponding to the lower ratio of high-tension turns to low-tension turns) gives out power ($\cos\theta$ being of positive sign), and that the other secondary X_2Y_2 takes in power. Thus:

$$P_1 = E_1 I_1 \cos\theta = \left(\frac{E}{a_1}\right) \times \left(\frac{\dfrac{E}{a_1} \ominus \dfrac{E}{a_2}}{\sqrt{(r_1 + r_2)^2 + (x_1 + x_2)^2}}\right) \times \left(\frac{r_1 + r_2}{\sqrt{(r_1 + r_2)^2 + (x_1 + x_2)^2}}\right)$$

$$P_2 = E_2 I_2 \cos(180^\circ - \theta) = -\left(\frac{E}{a_2}\right) \times \left(\frac{\dfrac{E}{a_1} \ominus \dfrac{E}{a_2}}{\sqrt{(r_1 + r_2)^2 + (x_1 + x_2)^2}}\right) \times \left(\frac{r_1 + r_2}{\sqrt{(r_1 + r_2)^2 + (x_1 + x_2)^2}}\right)$$

in which a_1 and a_2 represent the ratios of high-tension turns to low-tension turns in transformers No. 1 and No. 2, respectively. P_1 and P_2 when positive represent power given out by secondary induced e.m.f., the negative sign signifying power received thereby.

The total amount of power consumed in heating the copper in the transformers or in overcoming their resistance would be as follows:

In transformer No. 1,

$$I_1^2 r_1 = \frac{r_1\left(\dfrac{E}{a_1} \ominus \dfrac{E}{a_2}\right)^2}{(r_1 + r_2)^2 + (x_1 + x_2)^2} = \frac{E^2 r_1 (a_2 - a_1)^2}{a_1^2 a_2^2 [(r_1 + r_2)^2 + (x_1 + x_2)^2]}.$$

In transformer No. 2,

$$I_2^2 r_2 = \frac{r_2\left(\dfrac{E}{a_1} \ominus \dfrac{E}{a_2}\right)^2}{(r_1 + r_2)^2 + (x_1 + x_2)^2} = \frac{E^2 r_2 (a_2 - a_1)^2}{a_1^2 a_2^2 [(r_1 + r_2)^2 + (x_1 + x_2)^2]}.$$

Thus, transformer No. 1, having the lower transformer ratio a_1 or higher secondary induced voltage, takes P_1 watts from the high-tension line, consumes $I_1^2 r_1$ watts in heating its conductors, and gives $(P_1 - I_1^2 r_1)$ watts out from its terminals and into the low-tension terminals of transformer No. 2. The latter consumes $I_2^2 r_2$ watts in heating its own conductors, and the remainder $(P_1 - I_1^2 r_1 - I_2^2 r_2)$ or $-P_2$ watts is returned to the high-tension line from the coil A_2B_2, after having circulated through the two transformers. We should find, as a check on the accuracy of our work, that $(P_1 + P_2)$, the algebraic sum of power taken from the high-tension line by the coils A_1B_1 and A_2B_2, is exactly equal to $(I_1^2 r_1 + I_2^2 r_2)$, the total power consumed in overcoming the copper losses of the two transformers. The iron losses are not considered, as they are supplied by the exciting current which flows in each high-tension coil in addition to the currents which we have been considering.

A further useful application of our vectors, as in Fig. 110, gives us the secondary terminal voltage E_t between the mains

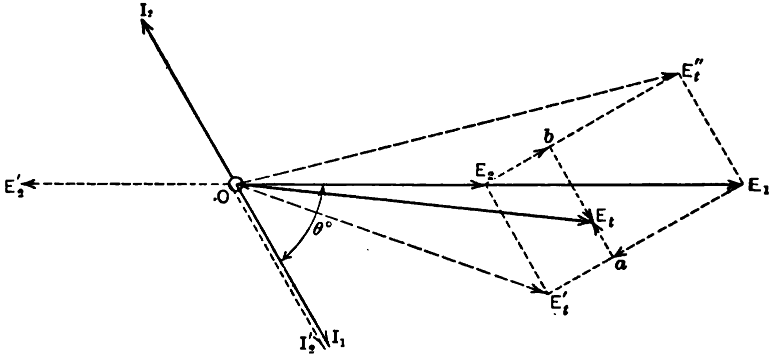


FIG. 110. Terminal voltage of the unloaded transformers having unequal voltage ratios. The vector OE_t represents the terminal voltage of both transformers, and must equal $E_2 \oplus x_2I_2 \oplus r_2I_2$ and $E_1 \oplus x_1I_1 \oplus r_1I_1$. The vector OE_t' would represent the terminal voltage if No. 1 possessed the combined reactance of the two transformers and no resistance, while E_t'' is the terminal voltage if No. 1 possessed all the resistance of the two transformers, and no reactance.

m, *n* of Fig. 107 and 108. Here the vectors E_1, E_2, I_1, I_2, E_2' and I_2' have the same significance as in Fig. 109, although the difference between E_1 and E_2 has been magnified in order to make the diagram clearer. Starting at the end of vector E_1 we lay off E_1a of length proportional to x_1I_1 volts, lagging behind the vector I_1 by 90 degrees, and representing the counter e.m.f. due to total equivalent leakage reactance in transformer No. 1. Then from a we lay out vector aE_t of length proportional to the total equivalent resistance reaction, in volts, of transformer No. 1, and opposite in phase to I_1. This locates the end of the vector OE_t, which represents terminal e.m.f. of transformer No. 1. Next, starting from E_2 lay out E_2b of length proportional to x_2I_2 and lagging 90 degrees behind I_2, representing the reacting e.m.f. in transformer No. 2 due to its leakage reactance. Then from b lay out bE_t of length proportional to r_2I_2 and opposite in phase to I_2, representing the reacting e.m.f. in transformer No. 2 due to its equivalent resistance. As the terminal e.m.f. in the transformer is the resultant or vector sum of the total induced e.m.f. and the reacting e.m.f.'s due to current, it follows that OE_t is the terminal e.m.f. of transformer No. 2. As the impedance of the secondary mains m, n between X_1Y_1 and X_2Y_2 has been assumed negligible, the current I_1 or I_2 will automatically adjust itself in value and phase so that the terminal e.m.f. of X_1Y_1 is equal to that of X_2Y_2, as indicated by coincidence of vectors OE_t in Fig. 110.

If we consider the total resistance $(r_1 + r_2)$ and the total reactance $(x_1 + x_2)$ in the local circuit of secondaries to be fixed while the distribution of either quantity between the transformers is varied, we find that the terminal e.m.f. between secondary mains m, n may vary in value and phase all the way between OE_t' and OE_t'' in Fig. 110. The former (OE_t') represents the e.m.f. between the secondary mains when transformer No. 1 has all of the reactance and no resistance — a hypothetical limiting condition; while OE_t'' is the corresponding e.m.f. when transformer No. 1 has all of the resistance and no reactance. The point E_t may, in fact, come anywhere within the rectangle $E_1E_t''E_2E_t'$, whose diagonal is the resultant e.m.f. $(E_1 - E_2)$, one side parallel to I and equal to $(r_1 + r_2)I$, and other side perpendicular to I and equal to $(x_1 + x_2)I$. The value of I and its phase angle (θ°) with respect to E_1 or E_2 or $(E_1 - E_2)$ are fixed by the values of $(r_1 + r_2)$ and of $(x_1 + x_2)$, as previously demonstrated.

It is instructive to consider the effect of unequal voltage ratios in parallel transformers upon the distribution of load between them when an external circuit is completed between secondary mains. In Fig. 111, the vectors E_1, E_2, I_1 and I_2

are exactly the same as in Fig. 110. Now suppose that there is connected between the secondary mains an external circuit of such resistance and reactance that, if the voltage ratios of the transformers were equal, it would draw I'_L amperes at power-factor cos α_1, from the secondary of transformer No. 1, and I''_L amperes at power-factor cos α_2 from the secondary

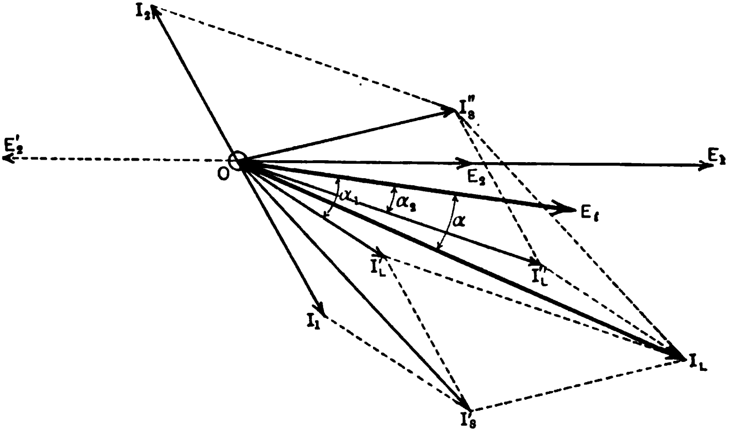


FIG. 111. Current in the transformers having unequal voltage ratios, when loaded. The vectors E_1, E_2, I_1 and I_2 are the same as in Fig. 110. No. 1 is now delivering a load current I'_L and No. 2 a load current I''_L. These combine to produce a total load current of I_L. The total current I'_S in No. 1 equals I'_L (the load current) vectorially added to I_1 the local current. Total current I''_S in No. 2 equals I''_L added vectorially to the local current I_2. $I'_S \oplus I''_S = I_L$ just as $I'_L \oplus I''_L = I_L$.

of transformer No. 2. The total load current I_L (resultant of vectors I'_L and I''_L) is taken from the secondary mains at pressure E_t volts and power-factor cos α.

If now we add (vectorially, or by parallelogram constructions as shown) the local current I_1 in transformer No. 1 to the load current I'_L in the same transformer, we get I'_S, the total secondary current in this transformer. If then we add vectorially the local current in transformer No. 2, represented by vector I_2, to the load current I''_L in the same transformer, the resultant is the total current I''_S in the secondary of this transformer. The effect of the circulating current I_1 or I_2 due to unequal voltage ratios is, therefore, to alter the total current in transformer No. 1 from I'_L to I'_S, and to alter the total current in transformer No. 2 from I''_L to I''_S. As a check, we may combine I'_S with I''_S, and we should find the resultant to be exactly coincident with I_L, as the external circuit and total current remain unchanged.

Note: All problems in this group are to be worked on the assumption that there is no load connected to the secondary mains.

Prob. 7–4. One transformer having a ratio of 10:1 with equivalent resistance 1 per cent and impedance 2 per cent is paralleled with another having ratio 9.6:1 with resistance 1.5 per cent and impedance 5.0 per cent. Each transformer is rated 25 kv-a. 60 cycles, and for 6600 volts on high-tension side. Calculate:

(*a*) Secondary open-circuit voltage of each transformer.

(*b*) Equivalent low-tension resistance and reactance of each transformer, in ohms.

(*c*) Current flowing in secondaries when connected through mains of negligible impedance, expressed in amperes and in per cent of rated current of each transformer.

Prob. 8–4. Calculate I^2R loss in each transformer of Prob. 7, in kilowatts and in per cent of its I^2R loss at rated full load.

Prob. 9–4. Calculate kilowatts taken by each transformer of Prob. 7 from the high-tension line, neglecting core losses. There is no load connected to the secondary mains. Find whether the algebraic sum of these two amounts is equal to the sum of I^2R losses in Prob. 8. If not, explain why.

Prob. 10–4. Calculate the voltage between terminals of secondaries in Prob. 7, and also the power-factor of the current which is being pumped from the secondary of the 9.6:1 transformer into the secondary of the 10:1 transformer.

Prob. 11–4. Prove algebraically from the equations in the preceding article that, in general,

$$P_1 + P_2 = I_1^2r_1 + I_2^2r_2.$$

Prob. 12–4. Perform the calculations of Prob. 7 and 8 on the assumption that the 10:1 transformer has 1.5 per cent resistance and 5.0 per cent impedance while the 9.6 : 1 transformer has 1 per cent resistance and 2 per cent impedance.

Prob. 13–4. Perform the calculations of Prob. 9 and 10 on the assumption stated in Prob. 12.

52. Effect of Unequal Percentages of Impedance in Parallel Transformers. For simplicity, let us continue to assume that the impedances are negligible in those portions of the high-tension and of the low-tension mains connecting the transformers. It follows, then, that the terminal voltage is the same for all high-tension coils, and also that all low-tension coils have the same terminal voltage. This means that the total internal drop due to equivalent impedance must be the same for all of the parallel transformers — that is, equal in value and coincident in phase. In other words, the vectors OS, OE_P and SE_P of Fig. 104 (Chap. III) are identically the same in value and in phase relations for all transformers in parallel.

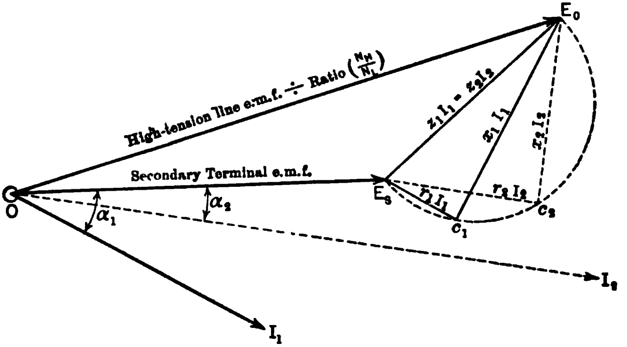


FIG. 112. Although the impedance z_1 of No. 1 is not equal to the impedance z_2 of No. 2, the currents in the two transformers in parallel will always adjust themselves so that the impedance drop $z_1 I_1$ of No. 1 is equal to and in phase with the impedance drop $z_2 I_2$ of No. 2. The terminal voltage of each then is $OE_S = E_0 \ominus z_1 I_1 = E_0 \ominus z_2 I_2$.

Thus, in Fig. 112, OE_0, common to all the transformers in parallel, is equal to the high-tension line e.m.f. divided by the ratio of high-tension turns to low-tension turns, or it is the e.m.f. that would be induced in secondary if all of the equivalent resistance and reactance were concentrated in the secondary, there being then no reactions in the primary. OE_S is the terminal e.m.f. of secondary, identical for all of the transformers in parallel. Transformer No. 1 is delivering to the load I_1 amperes from its secondary, lagging, with power-factor of $\cos \alpha_1$. $E_Sc_1 = r_1I_1$ is the e.m.f. consumed in overcoming the reaction due to equivalent resistance, while $c_1E_0 = x_1I_1$ is the e.m.f. consumed in overcoming the reaction due to equivalent reactance in transformer No. 1. Of course, I_1 is the current in the **secondary** coil of transformer No. 1, and r_1 and x_1 are the equivalent resistance and reactance, respectively, of this transformer reduced to terms of secondary. The vector sum $(OE_S \oplus r_1I_1 \oplus x_1I_1)$ is equal to OE_0, the total induced voltage in secondary; and z_1I_1 is the total equivalent impedance drop, in secondary volts.

Now consider the other transformer No. 2, which is operating in parallel with No. 1. Having identical vectors OE_S and OE_0, as in Fig. 112, it must also have identical impedance drop E_SE_0. Given the values of equivalent resistance r_2 and of equivalent reactance x_2 for this transformer, the current I_2 in its secondary coils must automatically adjust itself to such value and such phase relations that the vector sum of r_2I_2 parallel to I_2, and x_2I_2 perpendicular to I_2, will be equal to the vector $z_2I_2 = z_1I_1 = E_0 \ominus E_S = E_SE_0$. In fact, I_2 will keep changing and we shall not have equilibrium, until this relation is attained. Transformer No. 2 will then be delivering to the load I_2 secondary amperes

$$\left(I_2 = \frac{E_Sc_2 \text{ volts}}{r_2 \text{ ohms}} = \frac{c_2E_0 \text{ volts}}{x_2 \text{ ohms}}\right)$$

at power-factor $\cos \alpha_2$, while transformer No. 1 is delivering I_1 secondary amperes $\left(= \frac{E_Sc_1 \text{ volts}}{r_1 \text{ ohms}} = \frac{c_1E_0 \text{ volts}}{x_1 \text{ ohms}}\right)$ at power-factor $\cos \alpha_1$.

Notice from Fig. 112 that the secondary currents (expressed in amperes) in the transformers must be in the inverse ratio of the impedances (expressed in ohms). That is, since z_1I_1 volts $= z_2I_2$ volts, it must follow that

$$\frac{I_2 \text{ amperes}}{I_1 \text{ amperes}} = \frac{z_1 \text{ ohms}}{z_2 \text{ ohms}}.$$

From this relation it is easy to show that, if transformer No. 1 has 4 per cent impedance and transformer No. 2 has 2 per cent impedance, then transformer No. 1 cannot be operated at more than half of its rated load without overloading transformer No. 2, and No. 2 will carry 100 per cent overload when No. 1 is just loaded to its full rated kilovolt-amperes. When No. 1 carries its rated full-load kv-a. or current, the impedance drop in it is 4 per cent of line voltage; when No. 2 carries its rated kv-a. or current, the impedance drop in it is only 2 per cent of the same line voltage. But the impedance drop in volts must be the same for both transformers, hence No. 1 will carry only half as large a fraction of its rating as No. 2 will carry simultaneously, regardless of what the rating may be or of what actual currents or what kilovolt-amperes the transformers carry.

Example 1. Transformers A, B, C, having characteristics as tabulated below, are operated in parallel. Determine:

(*a*) Which transformer will reach its full-load first, as the total load is increased?

(*b*) What will be the load on each of the other transformers, in kv-a. and in percentage of its respective rated capacity, when the transformer of part (*a*) is operating at its full rated capacity?

Transformer number.	High-tension volts.	Low-tension volts.	Frequency.	Impedance volts in per cent.	Impedance watts.	Total I^2R calculated from resistances.	Rated kv-a.	$\left(\frac{\text{Rated kv-a.}}{\text{Impedance \%}}\right) = K$.
A	6600	2300	60	2.20	2700	2080	500	227.2
B	6600	2300	60	2.45	3095	2860	500	204.0
C	6600	2300	60	4.14	5570	4250	750	181.0

(*c*) When transformer C having the highest percentage of impedance reaches its full-load, what will be the load on B and A, respectively, in kv-a. and in per cent of their rated capacities?

(*a*) We have already explained that the transformer having lowest percentage of impedance (in this case, A) will be first to reach its rated full load as the total load is increased. We have also seen that the transformer having highest percentage of impedance (in this case, C) will be the last to attain its rated full load.

(*b*) It should be evident from the preceding explanations that the actual loads, in amperes or in kv-a., on each of several transformers in parallel will be inversely proportional to the percentages of impedance and directly proportional to the rated values of kv-a. or of currents. Thus, in the above example, C will carry $\left(\frac{2.20}{4.14}\right)$ times its own rated kv-a. when A is carrying just its rated kv-a.; but inasmuch as the rated kv-a. of C is $\left(\frac{750}{500}\right)$ times that of A, it follows that the actual kv-a. which C carries will be $\left(\frac{2.20}{4.14} \times \frac{750}{500}\right)$ times as large as that which A carries, whatever may be the numerical values of these loads. Thus

$$\frac{\text{Kv-a. in } C}{\text{Kv-a. in } A} = \frac{\%\text{ impedance of } A}{\%\text{ impedance of } C} \times \frac{\text{Rated kv-a. of } C}{\text{Rated kv-a. of } A},$$

whence

$$\frac{\text{Kv-a. in } C}{\text{Kv-a. in } A} = \frac{\dfrac{\text{Rated kv-a. of } C}{\%\text{ impedance of } C}}{\dfrac{\text{Rated kv-a. of } A}{\%\text{ impedance of } A}} = \frac{K_C}{K_A},$$

where K is a factor obtained for each transformer by dividing its rated kv-a. by its percentage of impedance.

Having calculated the values of K for the transformers in this example as shown in the last column of the table, we may state the following proportionality: kv-a. of A : kv-a. of B : kv-a. of C :: 227.2 : 204.0 : 181.0, or that $I_A : I_B : I_C = 227.2 : 204.0 : 181.0$.

Thus, at any and all loads, C will deliver $\left(\frac{181.0}{227.2}\right)$ times as many amperes as A, and B will deliver $\left(\frac{204.0}{227.2}\right)$ times as many amperes as A. Therefore, when A is delivering its rated full load of 500 kv-a. or $\left(\frac{500{,}000}{2300} = 217\right)$ amperes low-tension, C will be delivering

$\left(\frac{181}{227.2} \times 217\right)$ or 173 amperes, and *B* will be delivering $\left(\frac{204}{227.2} \times 217\right)$ or 195 amperes. But 173 amperes is only $\left(\frac{173}{750{,}000 \div 2300}\right)$ or 53.1 per cent of the rated capacity of *C*, and 195 amperes is only $\left(\frac{195}{217}\right)$ or 89.8 per cent of the rated capacity of *B*. $\left(\text{Check: } \frac{2.20}{4.14} = 53.1 \text{ per cent for } C, \text{ and } \frac{2.20}{2.45} = 89.8 \text{ per cent for } B.\right)$

(*c*) When *C* reaches its rated full load, or 326 amperes low-tension, *B* will be delivering $\left(\frac{204}{181} \times 326 = 367.4\right)$ amperes or $\left(\frac{4.14}{2.45} = 169\right)$ per cent of its rated capacity; while *A* will be delivering $\left(\frac{227.2}{181} \times 326 = 409\right)$ amperes or $\left(\frac{4.14}{2.20} = 188\right)$ per cent of its rated capacity.

Prob. 14–4. When transformer *B* of Example 1 is carrying its rated full load, what kv-a. are being delivered by *A* and by *C*?

Prob. 15–4. How many henrys of inductance having negligible resistance must be inserted between the high-tension line and the primary of transformer *A*, Example 1, in order to make it reach its full load when *C* reaches its full load?

Prob. 16–4. How many henrys of inductance having negligible resistance must be inserted between the low-tension line and the secondary of transformer *B*, Example 1, in order to make it reach its full load when *C* reaches its full load?

Prob. 17–4. Determine the rating of the reactances of Prob. 15 and of Prob. 16, (*a*) in kv-a., (*b*) in per cent.

53. Effect of Unequal Reactance Factors in Parallel Transformers. If Fig. 112 were drawn to represent the relations between any number of transformers operating in parallel, it would be found that all the points c (as c_1, c_2, each being the junction point of the rI and xI vectors for the same transformer) lie on the circumference of a circle having E_SE_0 or zI as diameter, because the angles $\underline{/E_Sc_1E_0}$, $\underline{/E_Sc_2E_0}$, $\underline{/E_ScE_0}$ must all be right angles. This rests upon a well-known proposition in geometry. The relation is useful in determining the actual load which each individual transformer will assume when a given total load is drawn from the secondary mains.

Thus, suppose that transformers A and C only, of Example 1, are operated in parallel, the total load having 72 per cent power-factor and such value in kilovolt-amperes that transformer A carries its rated full load. We are required to determine the total kilovolt-amperes and kilowatts delivered from secondary mains, as well as the kilowatts, kilovolt-amperes and power-factor for each of the individual transformers, and the voltage between the secondary mains when the high-tension line impresses full rated voltage of 6600 volts at 60 cycles on both transformers.

Figure 113 illustrates a method of solution. Start with any convenient line, such as E_SE_0, representing the voltage drop in each transformer due to equivalent impedance on low-tension basis. Divide E_SE_0 into 100 equal parts, and draw a semicircle upon it as diameter. Now calculate the ratio r/z for each transformer, as follows:

Transformer A: The measured "impedance watts" equal the true "copper loss"; hence,

$$\frac{\text{2700 watts copper loss with full-load current}}{\text{500,000 watts input (approx.) with full-load current}} = 0.0054.$$

That is, 0.54 per cent of the impressed voltage is consumed in overcoming resistance, or the equivalent resistance is 0.54 per cent. The impedance of this transformer is 2.20 per cent, therefore

$$\frac{r_1I_1}{z_1I_1} = \frac{0.54}{2.20} = 0.245, \text{ or } r_1I_1 = 24.5 \text{ per cent of } zI.$$

Similarly, for Transformer C:

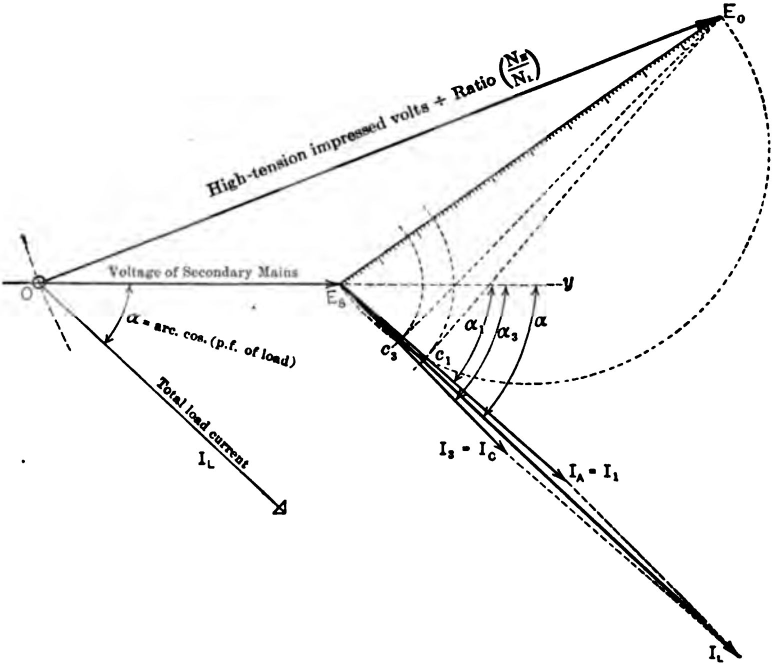
$$\frac{\text{5570 watts copper loss}}{\text{750,000 watts input (approx.)}} = 0.00743.$$

That is, the total equivalent resistance drop is 0.743 per cent of the applied voltage. Then

$$\frac{r_3I_3}{z_3I_3} = \frac{0.743}{4.14} = 0.179, \text{ or } r_3I_3 = 17.9 \text{ per cent of } zI.$$

Now, with E_S as center and radius 24.5 per cent of E_SE_0, swing an arc intersecting the semicircle at c_1, and through E_Sc_1 draw a vector

I_1 or I_A representing to any suitable scale the number of amperes in transformer A, namely $\frac{500,000}{2300}$ or 217 in this example. Similarly, strike another arc with radius 17.9 per cent of E_SE_0 intersecting the semicircle at c_3, and through E_Sc_3 draw vector I_3 or I_C representing to the same scale the amperes in transformer C, namely 173. Add the currents I_1 and I_3 vectorially, and obtain the resultant current I_L which is being delivered from the secondary mains. Next find the angle α whose cosine is the power-factor given for the total load.



E₀
High-tension impressed volts ÷ Ratio (Nₛ/N_L)
Voltage of Secondary Mains
O
Eₛ
y
α = arc. cos. (p.f. of load)
Total load current
I_L
c₃
c₁
α₁
α₃
α
I₃ = I_C
I_A = I₁
I_L

FIG. 113. Diagram for finding currents I_A, I_C, angles α_1 and α_3 and terminal voltage OE_S of parallel transformers A and C. Line E_SE_0 is first laid off to represent impedance drop of each transformer. The total current I_L is then found and from this and the angle α the direction of the line OE_Sy is determined. The length of the induced-voltage vector OE_0, to same scale as E_SE_0, determines the length of OE_S, the terminal-voltage vector.

If the total-load current I_L is lagging, as usual, lay out a line OE_{Sy} of indefinite length through E_S on each side of E_S and leading the vector E_SI_L by α degrees. Then from E_0, with radius representing the total induced voltage (high-tension line e.m.f. divided by ratio of transformation) to the same scale that E_SE_0 represents impedance volts in transformer A with I_1 amperes flowing, strike an arc intersecting OE_{Sy} at O. By applying the scale to OE_S we measure the terminal voltage between secondary mains.

Transformer A will under these conditions be delivering power at power-factor equal to cos α_1, and transformer C will deliver power at power-factor cos α_3, where α_1 and α_3 are the angles separating OE_S or E_{Sy} from I_1 and I_3, respectively. The relations between real power outputs of individual transformers and of secondary mains should furnish a check on the accuracy of the work. Thus:

$$\begin{aligned} P_A &= E_SI_A \cos \alpha_1, \\ P_C &= E_SI_C \cos \alpha_3, \\ P_L &= \text{total power taken from secondary mains} \\ &= P_A + P_C = E_SI_L \cos \alpha. \end{aligned}$$

From the construction given it may be seen that, if the power taken from the secondary mains be reduced to $\frac{1}{2}$ P_L watts, but still at the same power-factor cos α, the effect is to reduce the length of I_L, I_A and I_C each to $\frac{1}{2}$ its former value, but not to alter their phase relations. The vector E_SE_0 now represents half as many volts impedance drop as before, therefore OE_0 representing the same impressed voltage will be twice as long as before, relative to the length of E_SE_0. Of course OE_S will now be measured by the new voltage scale adopted for E_SE_0. We can also see from Fig. 113 that for low power-factors with I_L lagging, the impedance drop is more nearly parallel to OE_0 and therefore the secondary terminal voltage is lower and the voltage regulation is higher or poorer.

It is apparent that if the ratios x/r or r/z are nearly the same for all the transformers which are connected in parallel, the individual currents, as I_1 and I_3, will be nearly in phase with one another, and the angles α_1, α_3, α will be nearly equal. From this it results that the total or resultant current I_L delivered from the secondary mains will be then very nearly equal to the arithmetical sum of the individual currents, or that the total kilovolt-amperes of transformer capacity required will be very little greater than the total or resultant kilovolt-amperes of load taken from the secondary mains.

A further result of less consequence is that all such transformers operate at nearly the same power-factor as the mains. Conversely, if the reactance factors x/z are much different for the various transformers in parallel, the total kv-a. rating required will be considerably larger than the resultant kv-a. taken from the mains; that is, some of the transformer kv-a. will be consumed in useless currents circulating between the transformers. Evidently, therefore, the money investment in banked transformers intended to carry a given load may be uselessly increased by failure to select transformers having similar characteristics.

Example 2. Let us proceed to find numerical values for the quantities referred to above, with relation to transformers A and C of Example 1.

$$\underline{/E_0E_SI_1} = \text{arc cos}^* \left(\frac{r_1I_1}{z_1I_1}\right) = \text{arc cos } 0.245 = 75° \, 49'.$$

$$\underline{/E_0E_SI_3} = \text{arc cos} \left(\frac{r_3I_3}{z_3I_3}\right) = \text{arc cos } 0.179 = 79° \, 40'.$$

$$\underline{/I_1E_SI_3} = 79° \, 40' - 75° \, 49' = 3° \, 51'. \qquad \cos 3° \, 51' = 0.9977.$$

$I_3 = 173$ when $I_1 = 217$ (full load of transformer A). See page 217.

$$I_L^2 = (173)^2 + (217)^2 + 2 \times 173 \times 217 \times \cos 3° \, 51' = 151{,}940.$$

$$I_L = \sqrt{151{,}940} = 389.8, \text{ say } 390 \text{ amperes.}$$

Thus I_L is almost exactly equal to the arithmetical sum of I_A and I_C notwithstanding the fact that the transformers differ in percentage resistance and impedance.

$$\frac{\sin \underline{/I_1E_SI_L}}{\sin \underline{/E_SI_1I_L}} = \frac{I_3}{I_L} = \frac{173}{389.8} = 0.444. \quad \text{(See First Course, page 511.)}$$

$$\sin \underline{/I_1E_SI_L} = 0.444 \sin (180° - 3° \, 51') = 0.444 \times 0.06714 = 0.02983.$$

$$\underline{/I_1E_SI_L} = 1° \, 43'.$$

If the power-factor of I_L is 0.72, then $\alpha = \text{arc cos } 0.72 = 43° \, 57'$, $\alpha_1 = 43° \, 57' - 1° \, 43' = 42° \, 14'$, and $\alpha_3 = 42° \, 14' + 3° \, 51' = 46° \, 5'$.

---

* The expression "arc cos" means "the angle the cosine of which is."

Power-factor of load on A is cos 42° 14′ = 0.7404.
Power-factor of load on C is cos 46° 5′ = 0.6936.

$$\underline{\angle yE_SE_0} = \underline{\angle E_0E_SI_1} - \underline{\angle \alpha_1} = 75^\circ\,49' - 42^\circ\,14' = 33^\circ\,35'.$$

$$\overline{OE_0}^2 = \overline{OE_S}^2 + \overline{E_SE_0}^2 + 2 \times OE_S \times E_SE_0 \times \cos 33^\circ\,35'.$$

(See First Course, page 509.)

But OE_0 = secondary voltage at zero load; let us assume it to be 2300 volts.

Also, $E_SE_0 = z_1I_1$ = 2.20 per cent of 2300 volts when A is carrying full load
= 50.6 volts.

Then $$\overline{2300}^2 = \overline{OE_S}^2 + \overline{50.6}^2 + 2 \times 50.6 \times 0.833 \times OE_S$$

or $$\overline{OE_S}^2 + 84.3 \times OE_S - 5{,}287{,}440 = 0.$$

Whence

$$OE_S = \frac{-84.3 \pm \sqrt{(84.3)^2 + (4 \times 1 \times 5{,}287{,}440)}}{2 \times 1} = \frac{-84.3 \pm 4600}{2}.$$

OE_S = 2257.8 volts.

Kv-a. output of A = 217 × 2257.8 ÷ 1000 = 490.0.
Kv-a. output of C = 173 × 2257.8 ÷ 1000 = 390.6.
Kw. output of A = 490 × 0.7404 = 363.0 = P_A.
Kw. output of C = 390.6 × 0.6936 = 271.0 = P_C.

$$\text{Check}\begin{cases} P_A + P_C = 363.0 + 271.0 = 634 \text{ kw.} \\ P_L = 389.8 \times 2257.8 \times 0.72 = 634 \text{ kw.} \end{cases}$$

If we assume that the secondary terminal voltage is to be 2300 at full load, with the same pressure of 6600 impressed upon the primary, we shall require a different ratio of turns in the transformer. Now, OE_S = 2300 and OE_0 becomes the unknown e.m.f. to be determined from the trigonometrical relations. Thus:

$$\overline{OE_0}^2 = \overline{2300}^2 + \overline{50.6}^2 + 2 \times 2300 \times 50.6 \times 0.833,$$

whence OE_0 = 2345 volts. Then:

$$\frac{N_H}{N_L} = \frac{6600}{OE_0} = \frac{6600}{2345} = 2.814.$$

Therefore, at zero load the secondary terminal voltage becomes

$$\frac{6600}{2.814} = 2345 \text{ volts,}$$

as zI then reduces practically to zero.

In this case the voltage regulation of the bank of transformers is

$$\frac{2345 - 2300}{2300} = 1.95 \text{ per cent.}$$

In the other case, the voltage regulation of the bank was

$$\frac{2300 - 2257.8}{2257.8} = 1.87 \text{ per cent.}$$

It should be evident from this example that such problems cannot be solved graphically with necessary accuracy. The vector diagram serves only as a basis for the trigonometric calculations as used above. The treatment is easily extended to include any given number of transformers in parallel.

Prob. 18–4. By study of Fig. 113 state in clear English what relation of impedances and of equivalent resistances and reactances of the two transformers *A* and *C* would cause the maximum difference to exist between the total kv-a. necessary rating of the transformers, and the kv-a. supplied to the secondary mains. The total transformer capacity required is then what percentage of the kv-a. load on mains?

Prob. 19–4. In Electric Journal, May, 1909, an author states, with regard to two transformers in parallel, that if the reactance of neither transformer is more than 4 times its resistance nor less than 1 times its resistance, then the current or kv-a. capacity required per transformer does not exceed by more than 4 per cent that which would be necessary if this ratio (x/r) were the same in both transformers. Prove either that this statement is correct or that it is incorrect. Is it true if both transformers have the same per cent impedance?

Prob. 20–4. If the total load on the two transformers of Example 2 is non-inductive and of such value that *C* delivers its rated current, calculate:

(*a*) Power-factor of current delivered by *A*;
(*b*) Power-factor of load on *C*;
(*c*) Volts between secondary mains.

Prob. 21–4. What will be the kv-a. delivered by each of the three transformers of Example 1 when the total kv-a. load on the secondary mains is equal to the arithmetical sum of the several capacities — namely, 1750 kv-a.?

Prob. 22–4. Considered simply as an equivalent group of parallel impedances, what is the total equivalent impedance, in ohms, of the three transformers of Example 1?

Prob. 23–4. If no transformer can be permitted to carry a greater overload than 25 per cent, calculate the greatest load in kilowatts at 80 per cent power-factor that can be taken from the secondary mains of Example 1.

54. Autotransformers. Consider a wire wound into a continuous coil of N_H turns, on an iron core, as between A and B in Fig. 114. We impress an alternating e.m.f. of E_H volts across AB; this causes an exciting current to flow in AB which produces an alternating flux in the core. If we assume the magnetic system to be perfect, so that there is no leakage of flux out of the core, the same flux will link with every one of the N_H turns, and the e.m.f. thereby induced in every individual turn will be of the same value and in the same direction. If the copper circuit is well designed the resistance drop will be negligible in comparison with the applied e.m.f. or the induced counter e.m.f.; in the ideal case we assume that the counter e.m.f. is equal to the applied e.m.f. Therefore, across each individual turn we shall have (E_H/N_H) volts.

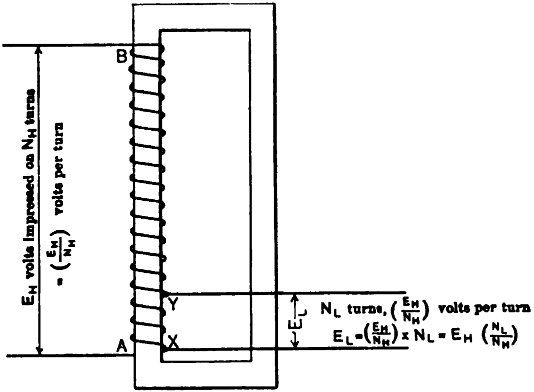

B
E_H volts impressed on N_H turns
$= \left(\frac{E_H}{N_H}\right)$ volts per turn
Y
X
A
E_L
N_L turns, $\left(\frac{E_H}{N_H}\right)$ volts per turn
$E_L = \left(\frac{E_H}{N_H}\right) \times N_L = E_H \left(\frac{N_L}{N_H}\right)$

FIG. 114. Diagram of a step-down autotransformer. The low-tension coil is a part of the high-tension coil.

Now suppose we connect a load circuit to any two taps XY on this coil, between which there are N_L turns. As each turn has (E_H/N_H) volts across it and all these e.m.f.'s are in phase with one another, the total e.m.f. across XY is $N_L\left(\frac{E_H}{N_H}\right)$ or $E_H\left(\frac{N_L}{N_H}\right)$. That is, in general, $\frac{E_L}{E_H} = \frac{N_L}{N_H}$. The relative polarity of the high-tension and low-tension sides is indicated according to the A.I.E.E. rule; the external e.m.f. from X toward Y reaches its maximum value at the same instant that the external e.m.f. from A toward B reaches its maximum value.

Figure 115 illustrates a step-up autotransformer, as distinguished from Fig. 114, which is a step-down autotransformer. In Fig. 115, we impress E_L volts upon N_L turns, which causes an e.m.f. of (E_L/N_L) volts to be induced in each turn of the entire coil, which has altogether N_H turns, N_H being larger than N_L. The total e.m.f. between A and B, which includes N_H turns, is, therefore, equal to $\left(\frac{E_L}{N_L} \times N_H\right)$ or $\left(\frac{N_H}{N_L} \times E_L\right)$ volts. Some minds may inquire how the induced e.m.f. between Y and B (Fig. 115) adds itself to the impressed e.m.f. between X and Y, to produce the total e.m.f.

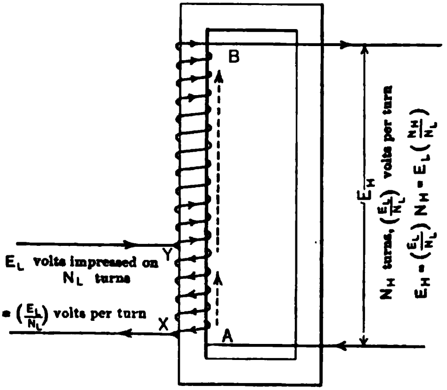

B
A
Y
X
A
E_L volts impressed on N_L turns
$= \left(\frac{E_L}{N_L}\right)$ volts per turn
E_H
N_H turns, $\left(\frac{E_L}{N_L}\right)$ volts per turn
$E_H = \left(\frac{E_L}{N_L}\right) N_H = E_L\left(\frac{N_H}{N_L}\right)$

FIG. 115. A step-up autotransformer. The voltage between any two taps is the sum of the voltage across all the turns between the taps.

$(N_H/N_L)\ E_L$ between A and B. Let us assume that at some chosen instant of time the direction of the impressed e.m.f. is toward the coil at Y and away from it at X, as indicated by the arrowheads on the primary mains. Then the counter e.m.f. induced in coil XY at this instant has a value of E_L volts and direction as shown by the dotted arrow pointed vertically upward from A. The induced e.m.f. between Y and B has a similar direction (from Y toward B as shown also by dotted arrow), and a value of $(N_H - N_L)\left(\frac{E_L}{N_L}\right)$ or $\left(\frac{N_H - N_L}{N_L}\right) E_L$ volts. It is easily seen that the induced e.m.f. Y to B is in additive series with the impressed e.m.f. between Y and X, and, as it is also in phase with the latter, the total e.m.f. between A and B is equal to $E_L + \left(\frac{N_H - N_L}{N_L}\right) E_L$ or $E_L\left(\frac{N_H}{N_L}\right)$ volts. An autotransformer, therefore, differs from the regular transformer only in that the primary and secondary coils are in series, or rather, that one of these is a part of the other.

We may economize material and improve the operating characteristics of the autotransformer by calculating the relation between the currents which flow in the coils and proportioning the size of the wire thereto. For convenience, let us neglect the small exciting current that flows in the primary section which is connected to the source of power, and study only the currents due to load. Thus, in Fig. 115, let us draw 10 amperes current from the secondary AB to a load circuit, and assume constants as follows:

$N_H = 2000$. $N_L = 200$. $E_L = 220$. $E_H = 220 \times (\frac{2000}{200}) = 2200$.

Now, 10 amperes drawn from BA will flow through the turns from Y to B in the direction shown by the arrows, which is the direction of the total e.m.f. acting to produce this current. The m.m.f. acting on the magnetic circuit due to this current of 10 amperes is $I_H\ (N_H - N_L)$ or $10\ (2000 - 200)$ equals 18,000 ampere-turns. This m.m.f. tends to decrease the flux due to the exciting current, which flows in the primary coil from Y to X in the direction of the arrowheads marked on the turns. To maintain the primary counter e.m.f. equal (approximately) to the impressed e.m.f. E_L requires the same flux; therefore, a load current will flow from the primary mains in the same (marked) direction as the exciting current. Enough load current will flow from Y to X to produce a m.m.f. equal and opposite to that due to the secondary load current, or 18,000 ampere turns. The amount of load current in the primary coil XY will, therefore, be $I_L = \frac{I_H(N_H - N_L)}{N_L}$, or $\frac{18{,}000}{200} = 90$ amperes in this example. The distribution of load currents will then be as shown in Fig. 116 and we have:

Input to primary XY = 100 amp. at 220 volts
= 22,000 volt-amperes.

Output from secondary AB = 10 amp. at 2200 volts
= 22,000 volt-amperes.

For reasons explained in Art. 35, the power-factor of this input will be equal to that of the output. The actual current flowing in XY will be the vector sum of the load component (90 amperes in this case) and the exciting current which is practically unchanged at all loads.

Inasmuch as the current in XY of Fig. 115 and 116 is 90 amperes when the current in YB is 10 amperes, it would be folly to use the same size of wire in all turns of the autotransformer. To preserve a uniform current density in all parts of the copper circuit, the conductor from X to Y should have 9 times as much cross-sectional area (circular-mils) as the conductor from Y to B. Obviously, this ratio will depend upon the ratio of voltages; in fact, it will be $\frac{I_L}{I_H} = \frac{E_H - E_L}{E_L}$. Particularly when the pressure is to be changed only slightly, the autotransformer is very much more economical of material, and therefore cheaper, than the ordinary transformer. This is illustrated by Prob. 27–4.

It is instructive to check further our knowledge of these relations by considering the results of reversing the connections of that section of the winding between *Y* and *B* in Fig. 116. The new relations are indicated in Fig. 117, from which we see that we must put in 80 amperes at 220 volts, or 17,600 volt-amperes, when we take out 10 amperes at 1760 volts, or 17,600 volt-amperes. In this case the section *YB* "bucks" the section *XY*, and the voltage across *AB* is equal to the difference between the voltages *YB* and *XY*, instead of their sum as in the previous example.

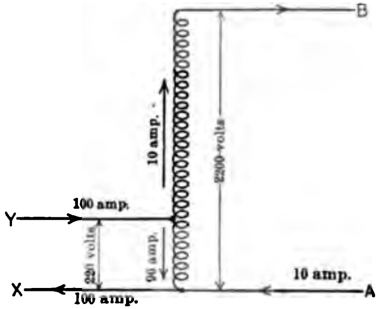


Fig. 116. Diagram showing the distribution of the load currents in an autotransformer.

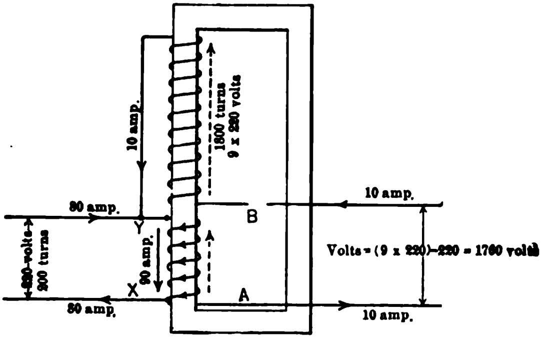


Fig. 117. The autotransformer of Fig. 116, with the winding between taps *Y* and *B* reversed.

If we were to use the same size copper in all turns of Fig. 115 or 116 we should either waste the carrying capacity of one coil or burn the insulation of the other, except when the ratio of transformation is 1:2. If all the wire were heavy enough to carry the low-tension current without injury to its insulation, the copper in *YB* would be underloaded and used to poor advantage; and if all the copper were just heavy enough to carry rated secondary current, the low-tension section would be burned out.

Note: In the following problems H_1 and H_2 are the high-tension line wires and L_1 and L_2 are the low-tension mains.

Prob. 24-4. An ordinary distributing transformer rated 10 kv-a., 2200-1100/220-110 volts, 60 cycles, has two low-tension coils X_1Y_1 and X_2Y_2 and two high-tension coils A_1B_1 and A_2B_2. Calculate the rated (effective) full-load values of the following: (*a*) Amperes per coil, high-tension, (*b*) Amperes per coil, low-tension, (*c*) Volts per coil, high-tension, (*d*) Volts per coil, low-tension.

Prob. 25-4. The transformer of Prob. 24 is connected as follows: A_1 to H_1; B_1 to A_2; B_2 to X_1 and L_1; Y_1 to X_2; Y_2 to L_2 and H_2. Calculate the values of the following quantities, when core flux has the same value as in Prob. 24:

(*a*) Volts between high-tension mains, H_1 and H_2.

(*b*) Volts between low-tension mains, L_1 and L_2.

(*c*) Largest current that can be delivered to low-tension mains without injuring any coil in the transformer.

(*d*) Current taken from high-tension line corresponding to (*c*).

(*e*) Current in each low-tension coil corresponding to (*c*).

(*f*) Current in each high-tension coil corresponding to (*c*).

(*g*) Largest kv-a. rating of the autotransformer with these connections.

Prob. 26-4. The transformer of Prob. 24 is connected as follows: A_1 to H_1; B_1 to A_2; B_2 to X_1, X_2 and L_1; Y_1 and Y_2 to L_2 and H_2. Answer the questions of Prob. 25, assuming the same core flux. Does this connection make the best use of materials?

Prob. 27-4. An autotransformer is designed economically to take 10 kv-a. from 2200-volt mains and deliver it at a pressure 10 per cent higher. If the "low-tension" coil were to be cut apart from the other coil, what would be the proper rating (volts and kv-a.) of the straight transformer so formed?

Prob. 28-4. A person standing on wet earth and touching the secondary circuit of the transformer of Prob. 24-4, would receive what voltage between contacts, under the following conditions?

A_1 is connected to H_1; B_1 to A_2; B_2 to H_2; X_1 to L_1; Y_1 to X_2; Y_2 to L_2.

(*a*) H_1 grounded; hand touching L_1.
(*b*) H_1 grounded; hand touching L_2.
(*c*) H_2 grounded; hand touching L_1.
(*d*) H_2 grounded; hand touching L_2.

Prob. 29–4. A person standing on wet earth and touching the secondary circuit of the transformer of Prob. 24–4 would receive what voltage between contacts when the transformer is used as an autotransformer at normal flux density under the following conditions?

A_1 to H_1; B_1 to A_2; B_2 to X_1 and L_1; Y_1 to X_2; Y_2 to L_2 and H_2.

(*a*) H_1 grounded; hand touching L_1.
(*b*) H_1 grounded; hand touching L_2.
(*c*) H_2 grounded; hand touching L_1.
(*d*) H_2 grounded; hand touching L_2.

Compare these results from the autotransformer arrangement with the corresponding results in Prob. 28 for the ordinary transformer arrangement, noting that the ratio of primary to secondary volts is nearly the same in both cases; and on this basis discuss relative merits of the two arrangements as regards the personal safety of the energy consumer and the fire risk.

55. Transformers for Polyphase Systems. Under this general heading we must distinguish several different problems, as follows:

(*a*) Transformation from 3 phases at a given voltage into the same number of phases at some other voltage.
(*b*) Transformation from 3 phases at a given voltage into a different number of phases at the same voltage.
(*c*) Transformation from 2 or 3 phases at a given voltage into a different number of phases at a different voltage.
(*d*) Polyphase transforming systems which cannot be operated in parallel.
(*e*) Systems in which all phases are housed in a single "polyphase transformer" as distinguished from polyphase transforming systems consisting of aggregates of separate single-phase transformers.

Before proceeding to connect any transformers in polyphase systems, it is practically necessary to determine carefully and to mark according to a fixed conventional system (such as prescribed by the A.I.E.E. Standardization Rules) the relative polarity of the high-tension and low-tension coils belonging to each of the phases. Thereafter the interconnections of both high-tension and low-tension sides are comparatively simple.

56. Transformer Arrangements for Three Phases. In the most common case, three separate, but similar, single-phase transformers are used to change three-phase at one voltage into three-phase at another voltage. Four distinct combinations are possible:

(*a*) High-tension coils *Y*-connected; low-tension coils *Y*-connected.

(*b*) High-tension coils *Y*-connected; low-tension coils Δ-connected.

(*c*) High-tension coils Δ-connected; low-tension coils *Y*-connected.

(*d*) High-tension coils Δ-connected; low-tension coils Δ-connected.

Let us assume, for example, that the highest operating voltage that will not injure the insulation of the transformer or make its core losses excessive is 12,000 volts on the high-tension coil and 6000 volts on the low-tension coil of each of the three distinct single-phase transformers which are to be grouped together in a three-phase system, as indicated above. Fig. 118 shows the phase relations among all of the coils. Thus, the e.m.f. from X_1 towards Y_1 (6000 volts) is in phase with the e.m.f. from A_1 towards B_1 (12,000 volts), because both of these coils are on the same iron core in transformer No. 1 or phase No. 1; X_2Y_2 and A_2B_2 are correspondingly in phase with each other, and lagging 120° behind X_1Y_1 and A_1B_1; X_3Y_3 and A_3B_3 are correspondingly in phase with each other, and lagging 120° behind X_2Y_2 and A_2B_2. Note that in Figs. 118a and 118b no connections have yet been made between coils.

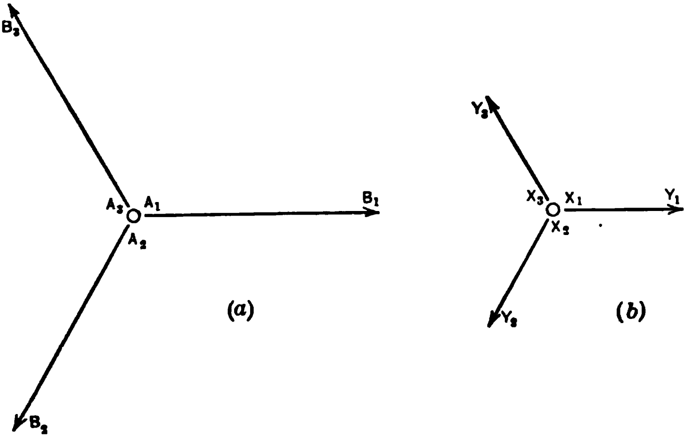


FIG. 118. Vector diagrams showing the relations between the high-tension and low-tension sides of three single-phase transformers connected independently to the various phases of a three-phase source of supply. The vector X_1Y_1 represents the low-tension e.m.f. of transformer No. 1. The e.m.f. of the high-tension side of this transformer is represented by vector A_1B_1.

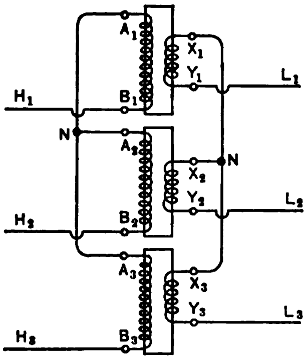


FIG. 119. Diagram of the *YY* connection. The high-tension terminals marked *A* are joined to form a neutral, and the corresponding low-tension terminals marked *X* are joined to form the neutral on the low-tension side.

The *YY* connection is shown in Fig. 119, while Fig. 120a shows the e.m.f. relations in the high-tension coils, and Fig. 120b shows the e.m.f. relations in the low-tension coils. Notice that in the vector diagrams, as well as in the connection diagrams, the letters indicate or identify the ends of coils as well as the relative directions of e.m.f.'s or the polarity of coils.

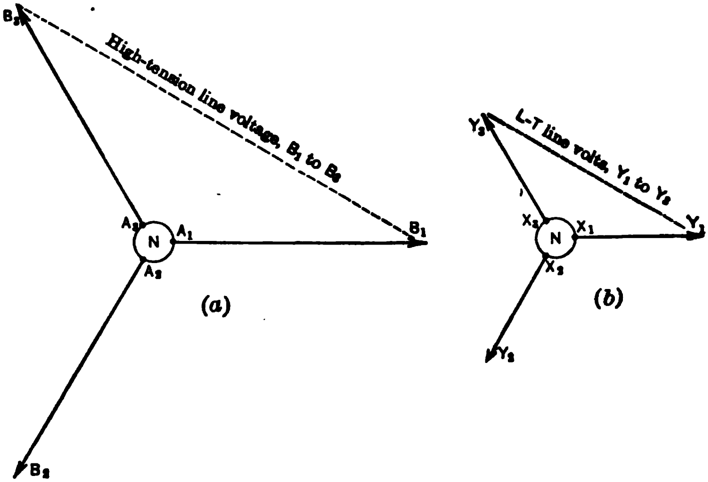


FIG. 120. The vector diagrams for the *YY* connection shown in Fig. 119.

From Fig. 120 we can see that the largest permissible operating voltages between line wires are $\sqrt{3} \times A_1B_1 = \sqrt{3} \times 12{,}000 = 20{,}800$ volts high-tension and $\sqrt{3} \times 6000 = 10{,}400$ volts low-tension. The high-tension line wires are $H_1H_2H_3$; the low-tension mains are $L_1L_2L_3$, in Fig. 119.

The *Y* (high-tension) Δ (low-tension) connections are shown in Fig. 121, while Fig. 122 shows the corresponding e.m.f. relations. Using the same transformers, the highest permissible operating voltage between line wires is $\sqrt{3} \times 12{,}000 = 20{,}800$ volts high-tension, and 6000 volts low-tension.

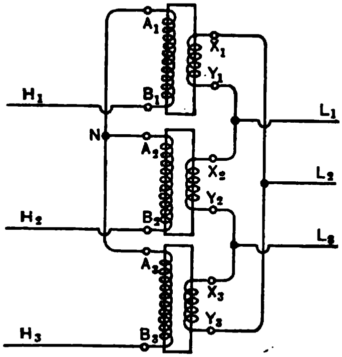


FIG. 121. The $Y\Delta$ connection. The high-tension sides are connected Y and the low-tension sides, Δ.

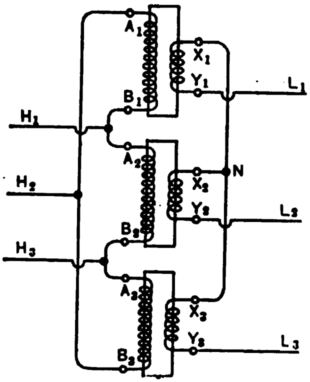


FIG. 123. The ΔY connection. The high-tension sides are connected Δ and the low-tension, Y.

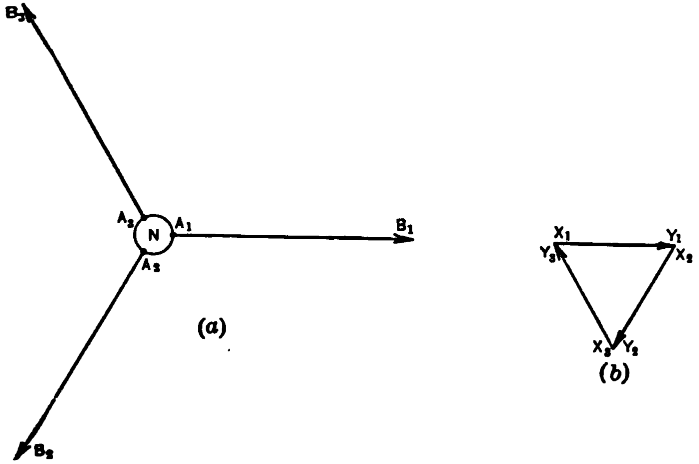


FIG. 122. The vector diagrams for the e.m.f. relations in the $Y\Delta$ connections of Fig. 121.

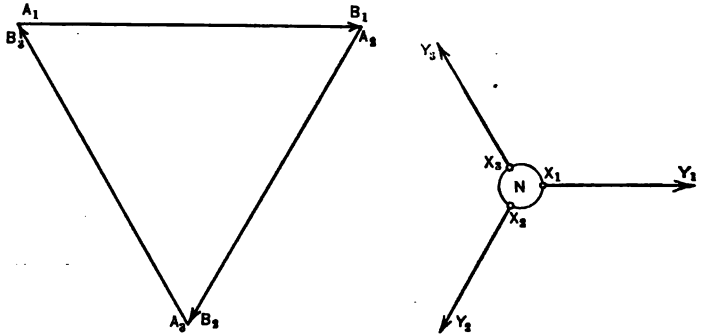


FIG. 124. The vector diagrams for the ΔY connection shown in Fig. 123.

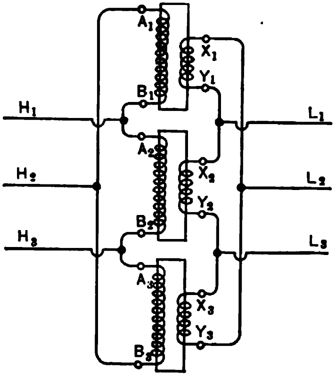


FIG. 125. The ΔΔ connection.

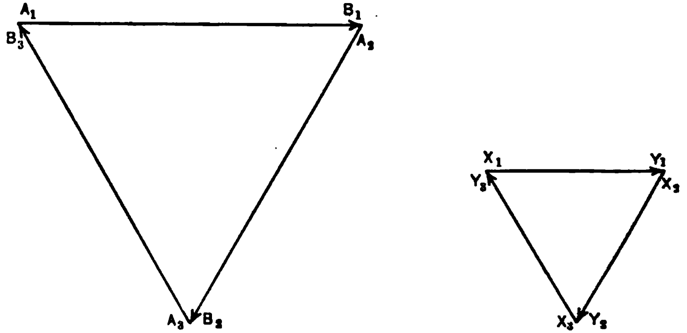


FIG. 126. The vector diagrams for the ΔΔ connection shown in Fig. 125.

The ΔY connections are shown in Fig. 123, and Fig. 124 shows the corresponding e.m.f. relations. With the same transformers, the highest permissible operating voltage between line wires is 12,000 volts high-tension and $\sqrt{3} \times 6000 = 10,400$ volts low-tension.

The $\Delta\Delta$ connections are shown in Fig. 125, and Fig. 126 shows the corresponding e.m.f. relations. With the same transformers, the highest permissible operating voltage between line wires is 12,000 volts high-tension, and 6000 volts low-tension.

Prob. 30–4. (*a*) What should be the rated high-tension and low-tension voltage, and the voltage ratio, in each of three single-phase transformers intended to take power from a three-phase line with 23,000 volts between wires, and deliver it to three-phase low-tension mains with 2300 volts between wires? Transformers are connected $Y\Delta$.

(*b*) If 500 kv-a. are to be delivered altogether, what will be the current in each line wire and each transformer coil on the high-tension side?

(*c*) On the low-tension side?

Prob. 31–4. Answer the questions of Prob. 30 on the basis of same conditions, except that transformers are connected ΔY.

Prob. 32–4. Answer the questions of Prob. 30 on the basis of the same conditions except that transformers are connected YY.

Prob. 33–4. Answer the questions of Prob. 30 on the basis of same conditions, except that transformers are connected $\Delta\Delta$.

57. Transforming Three Phases to Six Phases. The three-phase three-wire system is used, almost exclusively, to transmit large amounts of power by alternating currents. When the alternating current must be converted into direct- or continuous-current by means of synchronous converters, however, we find that a machine of given size and cost can convert a considerably greater amount of power if it is tapped for six phases and six alternating-current collecting rings than if it is tapped for three phases and three rings. In fact, it is demonstrated theoretically in Chapter IX, and proved by experience, that whereas a three-ring (three-phase) converter can safely deliver 134 per cent as much direct-current power as it could deliver if mechanically driven as an ordinary direct-current generator, the same machine tapped for six phases gives a corresponding ratio of 197 per cent. In other words, the capacity of the same converter is increased in the ratio 197/134, or 147 : 100, by tapping it for six phases instead of for three phases. These figures are based upon a power-factor of 100 per cent, which is usually easily maintained by adjustment of the field excitation of the converter.

Thus, although the transmission of power by six phases and six line wires is not practiced because of the complications and the lack of economic advantage, there exists a very real advantage in using six phases within the converter substation. Converters are most commonly employed to deliver continuous current at about 600 volts for street or interurban electric railways. The ratio of (effective) alternating voltage between adjacent or consecutive collecting rings, to the voltage between the positive and the negative terminals on the direct-current side, is $\frac{61.2}{100}$ for the three-ring converter and $\frac{35.4}{100}$ for the six-ring converter (see Art. 126). Thus, to deliver 600 volts at the direct-current terminals we should impress (0.612×600) or 367 volts between consecutive rings of a three-phase converter, and 212 volts between consecutive rings of a six-phase converter. As the three-phase transmission line usually operates at from 2300 to 23,000 volts between line wires, we must use regular alternating-current transformers to step down the pressure of the alternating current before it enters the converter. Obviously it would be most convenient, if possible, to arrange the transformer connections so as to change the number of phases at the same time that we change the pressure. In this article these special transformer connections are discussed from the viewpoint of the transformer. For a discussion of the same subject from the converter viewpoint see Art. 128.

The fundamental six-phase groupings are **star** and **mesh**, corresponding to the Y and Δ groupings for three phases. Thus, in Fig. 127, the vectors 1, 2, 3, 4, 5, 6 represent six different coils connected in star. The inherent phase relations between the e.m.f.'s in these coils, and the choice of which end of each coil is to be connected to the common neutral point, are such that e.m.f. No. 1 reaches its maximum value in the direction away from N (neutral) just 60° before No. 2 reaches its maximum value in the direction away from N; No. 2 reaches its maximum value just 60° before No. 3, No. 3 just 60° before No. 4, and so on, the e.m.f.'s being all equal in value and the phase differences between consecutive phases or coils being all of equal value and progressively leading or lagging, **on the basis of similar positive direction with respect to neutral.** From the six coils we now have seven terminals, of which one (N) is common to all and is called "neutral" because it exhibits the same potential difference with respect to each of the other six terminals. These other six terminals are connected to the six-phase line wires L_1, L_2, L_3, L_4, L_5, L_6, or to the rings of the synchronous converter, in proper sequence. The e.m.f. from L_1 to L_2 is equal in value to the e.m.f. from L_2 to L_3 and lagging 60° behind it; the e.m.f. from L_2 to L_3 is equal in value to the e.m.f. from L_3 to L_4 and lagging 60° behind it, and so forth. Also, the e.m.f. between any two consecutive line wires, as L_1L_2 or L_2L_3, is equal in value to the e.m.f. between any line wire and neutral, as NL_1 or NL_2. If vector diagrams be drawn to prove these statements, it should be remembered that (for instance) the e.m.f. acting from L_1 toward L_2 in the external circuit is represented by vector No. 1 minus vector No. 2, or by vector No. 1 plus vector No. 2 reversed; or that the e.m.f. acting from L_1 toward L_2 in the internal circuit of phases is represented by vector No. 2 minus vector No. 1, or by vector No. 2 plus vector No. 1 reversed. Thus, vector NL_6 represents not only the e.m.f. in coil No. 6 but also the e.m.f. from L_1 toward L_2 through the external circuit, and vector NL_1 represents not only the e.m.f. in coil No. 1 but also the e.m.f. from L_2 toward L_3 through the external circuit.

The typical six-phase mesh connection is described vectorially by Fig. 128. Thus, with respect to a positive direction which is the same for all coils through the closed series in which they are connected, the e.m.f. of coil No. 2 is equal to the e.m.f. of coil No. 1 and leads it by 60°; the e.m.f. of coil No. 3 is equal to that of coil No. 2 and leads it by 60°, and so on. It is evident that the e.m.f.'s between consecutive line wires, as L_1L_2 or L_2L_3, being identical with the corresponding e.m.f.'s across consecutive coils, as No. 2 or No. 3, bear also corresponding relations to each other, as regards both value and phase.

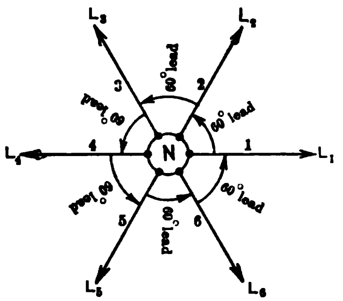


FIG. 127. Vector diagram of the e.m.f. relations in a six-phase star connection.

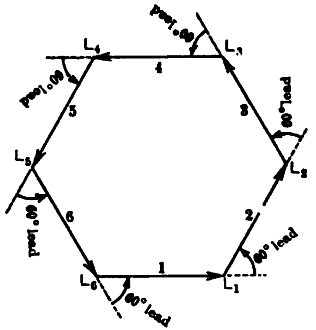


FIG. 128. Vector diagram of the e.m.f. relations in a six-phase mesh connection.

Regarding Fig. 127 and 128, we note that e.m.f.'s No. 1 and No. 4 reach their maximum values at the same instant (though in opposite directions with respect to the common neutral point or with respect to the common positive direction through the mesh). In reality, therefore, there are only three different phases in the six-phase system, and the difference between the systems is merely due to arrangement. If, as in Fig. 129, we divide each of three phases into two equal parts and distinguish by corresponding letters the terminals having similar relative polarity, we shall get a correct six-phase star by connecting X_1, X_3, X_5, Y_4, Y_6, Y_2 all to neutral. Then the six phases of the external circuit will be, in the order given, from Y_1 to X_2, from X_2 to Y_3, from Y_3 to X_4, from X_4 to Y_5, from Y_5 to X_6, and from X_6 to Y_1.

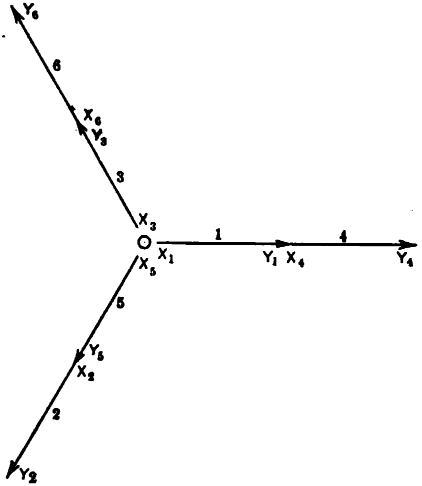


FIG. 129. Vector diagram for the low-tension coils of three transformers on a three-phase circuit, each coil being divided into two equal parts.

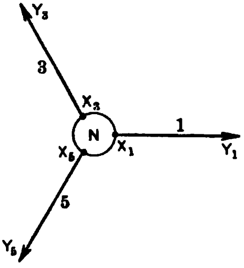


FIG. 130.

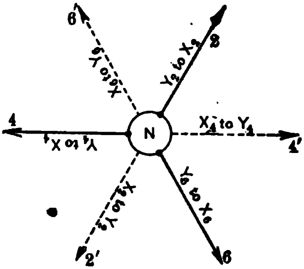


FIG. 131.

Another vector representation of the e.m.f. relations in a six-phase star, produced by the coil arrangement of Fig. 129.

Perhaps this result is more clearly explained in Fig. 130

and 131. Connecting X_1 to X_3 to X_5 to neutral, the e.m.f. relations are as represented by vectors 1, 3 and 5 in Fig. 130. Now as coil No. 4 is on the same transformer core with coil No. 1, the e.m.f. from X_4 toward Y_4 is represented by the same identical vector which represents the e.m.f. from X_1 toward Y_1; the e.m.f. X_6 to Y_6 and the e.m.f. X_3 to Y_3 are represented by the same vector, and so on. But if the e.m.f. X_4 to Y_4 is represented, as in Fig. 131, by the vector 4′ parallel to vector 1 of Fig. 130, it follows that the e.m.f. from Y_4 toward X_4 is represented, as in Fig. 131, by the vector 4 which is equal and opposite to vector 4′. That is, if e.m.f. X_4 to Y_4 reaches its maximum value at the same instant as e.m.f. X_1 to Y_1, then e.m.f. Y_4 to X_4 reaches its maximum value 180° before or after e.m.f. X_1 to Y_1. Therefore, when we connect Y_4, Y_2 and Y_6 to the same neutral as X_1, X_3 and X_5, it is as though we drew the vectors 4, 6 and 2 of Fig. 131 between the vectors 1, 3 and 5 of Fig. 130, making a diagram like Fig. 127. That is, with respect to positive directions which are in all cases chosen to be outward from neutral (X_1 to Y_1, Y_2 to X_2, X_3 to Y_3, Y_4 to X_4, X_5 to Y_5, Y_6 to X_6) these e.m.f.'s are consecutively 60° apart in the order named. If we had connected all X terminals to neutral, the e.m.f. relations would have been as described by Fig. 132, which tells us that the line e.m.f.'s Y_1 to Y_4, Y_3 to Y_6, and Y_2 to Y_5 would all be equal to zero, and we could distinguish only three instead of six line e.m.f.'s.

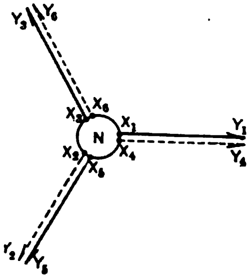


FIG. 132. If all the X terminals of the transformer coils of Fig. 129 were connected to neutral, only a three-phase arrangement would result.

Fig. 133 shows the transformer connections corresponding to Fig. 127, or to Fig. 130 and 131. There are three distinct transformers, each having two low-tension coils. The six-phase low-tension line e.m.f.'s L_1 to L_2, L_2 to L_3, L_3 to L_4, L_4 to L_5, L_5 to L_6 and L_6 to L_1 are equal to each other in value, and each lags behind the preceding one by 60°.

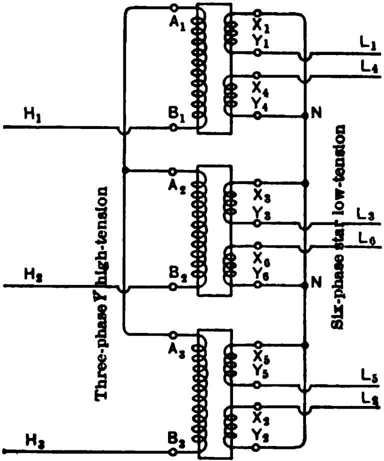


FIG. 133. Transformer connections to produce the e.m.f. effects of Fig. 127, 130 and 131. Six-phase star.

Fig. 134 shows the transformer connections corresponding to Fig. 128, namely the six-phase mesh. We first select one coil of each transformer and mark it 1, 3, 5, respectively. The other coil of each transformer we mark 4, 6, 2, respectively, in accordance with the preceding vector diagrams. Let the relative polarity of the two low-tension coils in each transformer be denoted in the conventional manner by letters X and Y, and let us consider the direction from X toward Y as the positive direction in each coil. Starting at X_1 or S, and tracing our way continuously through the series of coils, we pass consecutively through coil No. 1 positively, No. 2 negatively, No. 3 positively, No. 4 negatively, No. 5 positively, and No. 6 negatively. We have, therefore, obeyed the injunctions of Fig. 128 by passing through coils 1, 3 and 5 in direction parallel to the corresponding vectors in Fig. 129, and intermediately through coils 2, 4 and 6, in direction opposite to the corresponding vectors in Fig. 129. If we have made no errors we will find that the e.m.f. between the finishing end of the series (marked X_6 or F in Fig. 134) and the starting end (marked X_1 or S) is zero, and we may therefore with safety connect X_6 to X_1, or F to S, thus completing the closed mesh. The junction points Y_1Y_2, X_2X_3,

Y_3Y_4, X_4X_5, Y_5Y_6 and X_6X_1 are connected to the six-phase line wires L_1, L_2, L_3, L_4, L_5 and L_6, respectively.

If any appreciable voltage exists between the terminals F and S in Fig. 134, they should not be connected together, because then this voltage would cause a current to circulate in the mesh even when no load is connected externally, which

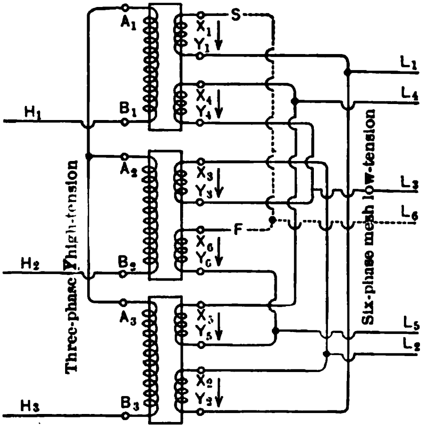


FIG. 134. Transformer connections for producing the six-phase mesh effect shown in Fig. 128.

would at least reduce seriously the load capacity of the transformers and might injure them. Such resultant voltage may be due to unequal ratios in the transformers, or to irregular wave-forms in the various coils even if the effective voltages are equal. However, if no voltage exists between the terminals F and S, it does not necessarily prove that the connections are correct or that a six-phase system may be obtained from the mesh even though it is safe to complete the mesh. Thus, Fig. 135 and 136 both show safe series connections between these same six coils (which, if joined in proper polarity and sequence, would give a correct six-phase system). It is manifestly impossible to get six phases or e.m.f.'s equal in value and 60° apart between consecutive pairs of secondary mains connected to the junction points of coils in Fig. 135 and 136, regardless of how we number these mains or junction points; yet the resultant e.m.f. between F and S is zero in both cases. In checking six-phase connections, we should find that the e.m.f. from any terminal to another, removed by one, should be equal to $\sqrt{3}$ times the e.m.f. between consecutive terminals; and the e.m.f. from any terminal to another, removed by two, should be equal to 2 times the e.m.f. between consecutive terminals.

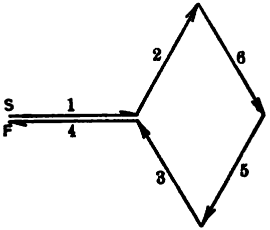


Fig. 135.

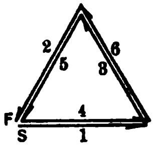


Fig. 136.

Vector diagrams to show that the transformer coils of Fig. 133 and 134 can be so connected as to produce zero voltage between S and F and still not produce a six-phase mesh connection.

Fig. 137 shows transformer secondaries connected in six-phase star to the collecting rings of a six-phase two-pole synchronous converter, these rings being tapped to the closed armature winding at L_1, L_2, L_3, L_4, L_5, L_6. This six-phase star is exactly equivalent to that shown in Fig. 133 and 127, and differs only in that each numbered vector of Fig. 127 or each coil e.m.f. in Fig. 133 has been thrown into exactly opposite phase relation with respect to neutral. If we break the (dotted) neutral connection in Fig. 137 it will make no difference in the operation of the system. We shall then have three equal e.m.f.'s (Y_2 to X_5, Y_6 to X_3, Y_4 to X_1) each lagging 120° behind the one preceding, impressed across three corresponding pairs of rings or taps (L_2L_5, L_6L_3, L_4L_1, respectively) on the converter. It should be clear, from Chap. VII of the First Course, that the e.m.f.'s induced in the converter winding from L_2 to L_5, from L_6 to L_3 and from L_4 to L_1 are equal in value and 120° apart consecutively with respect to these positive directions. The system of Fig. 137, without the neutral, is known as the "diametral connection."

It is necessary to note that the stability of the diametral six-phase (like Fig. 137 with neutral connection broken) is due entirely to the six-phase converter to which it is connected. The e.m.f.'s L_1L_2 and L_2L_3 induced in the converter winding act as links to keep the L_2 end of vector L_2L_5 in its proper position with respect to L_1 and L_3. Thus, it can be readily seen that if all connections between transformers and collecting rings were broken, there would only be three e.m.f.'s (120° apart) obtainable, namely Y_2X_5, Y_6X_3 and Y_4X_1, because these three coils are insulated from each other. Only when these e.m.f.'s are held in a certain relation with respect to each other by some additional means, like the converter itself or the neutral connection, do we have six-phase from them.

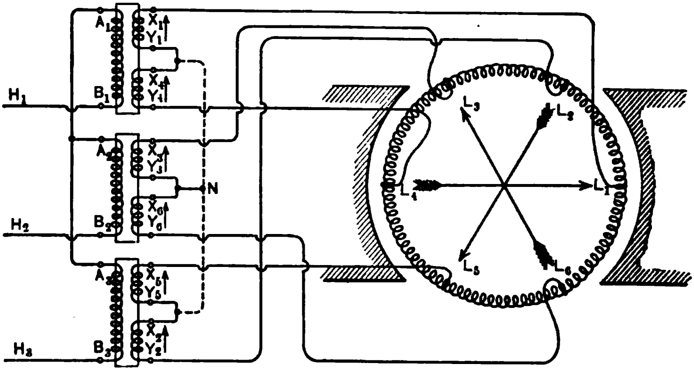
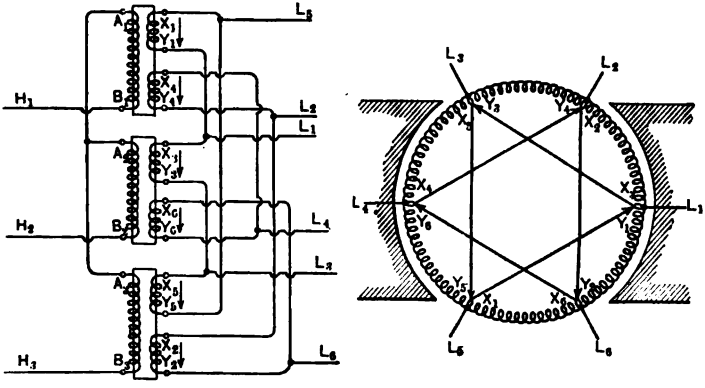


FIG. 137. The six-phase star connection used to operate a six-phase converter. When no connection is made to neutral, this arrangement is called the "diametral" or diametrical connection.

The diametral is one of the two most common six-phase connections, the other being the **double-delta** shown in Fig.

138a. Study of the vector diagram in Fig. 138b should explain the system. Notice that the six phases (L_1 to L_2, L_2 to L_3, L_3 to L_4, etc.) are obtained on account of a certain relative position of two three-phase deltas (Y_1X_3 to Y_3X_5 to Y_5X_1, and X_2Y_4 to X_4Y_6 to X_6Y_2). This relative position is maintained by reason of the e.m.f.'s induced in the converter windings, acting as links of certain length and angular relation to keep the apices of the two deltas in proper relation to each other. In Fig. 138b notice that vector X_4Y_4 is equal to vector X_1Y_1 and in phase with it as it should be on account of the symmetry of the two similar portions of armature winding subtended by these two vectors under opposite poles of the converter. So it is also with vectors X_5Y_5 and X_2Y_2, and with vectors X_6Y_6 and X_3Y_3.



(a) (b)

FIG. 138. The double-delta connection of three transformers for use with a six-ring converter.

Thus, if we pick out coils 1, 3 and 5 to form one three-phase delta (joining Y_1 to X_3, Y_3 to X_5, and Y_5 to X_1, as described in Art. 50), it follows that coils 2, 4 and 6 must form the other delta. Now we might connect Y_4 to X_6, Y_6 to X_2 and Y_2 to X_4, but this would give us a delta practically identical with that composed of transformer coils 1, 3 and 5 and we could get only three phases, not six. So we have connected coils 2, 4 and 6 in the only other way which will permit a closed delta, and it is seen that this arrangement gives us six phases if the apices of the delta are held in proper relative position.

It would be impossible to get six phases before connecting both deltas to the converter, because they are otherwise electrically insulated from each other. We may connect delta 1, 3, 5 to any three alternate rings as L_1, L_3, L_5. This done, there is only one proper way to connect the other delta to the remaining three rings, as shown in Fig. 138b. Thus, if we were to connect the X_6Y_2 terminal instead of the X_2Y_4 terminal to ring L_2, we should have the e.m.f. relations illustrated by vector triangles A and C in Fig. 139. If we

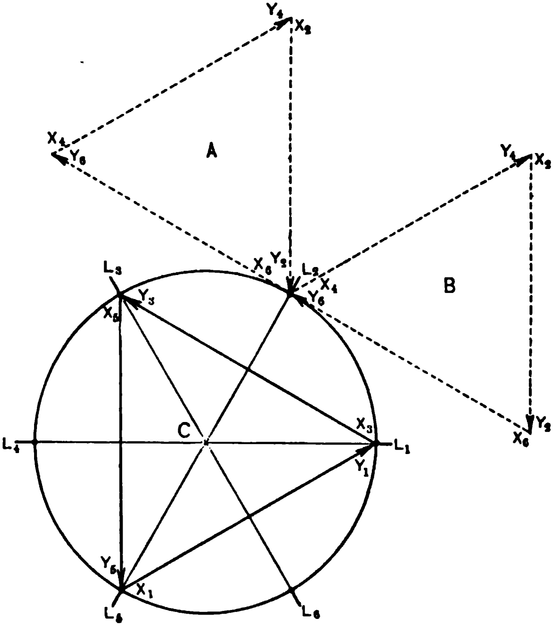


FIG. 139. Diagram showing the e.m.f.'s produced by incorrect connections in an attempt to produce a double-delta arrangement.

were to connect the X_4Y_6 terminal instead of the X_2Y_4 terminal to L_2, we should have relations as shown between triangles B and C in Fig. 139. In neither of these latter cases would there be zero e.m.f. from either of the rings L_4 or L_6 to any of the transformer points X_4Y_6, X_6Y_2 or X_2Y_4, hence a short-circuit would be formed if we attempted to complete the six-phase connections to the converter.

NOTE. In solving the following problems, assume the relations between effective values of alternating e.m.f. between collecting rings on the converter to be as follows (referring to Fig. 137 or 138):

Volts L_1L_2 : volts L_1L_3 : volts L_1L_4 : volts between d-c. terminals = 35.4 : 61.2 : 70.7 : 100. (See **Art. 126.**)

Prob. 34-4. What should be the voltage across each high-tension coil and each low-tension coil in a bank of three single-phase transformers connected in Y to a 23,000-volt high-tension line, secondaries star as in Fig. 133 and 127 to a six-ring converter intended to deliver 500 volts d-c., at zero load?

Prob. 35-4. Solve Prob. 34 with secondaries connected in mesh as illustrated by Fig. 128 and 134.

Prob. 36-4. Specify what connections to make between the secondary coils of the transformers in Fig. 133 and 134, in order to realize the vector relations shown in Fig. 135.

Prob. 37-4. Solve Prob. 36 but with reference to the vector diagram of Fig. 136.

Prob. 38-4. Draw the vector diagram corresponding to the following connections between secondaries of the transformers of Fig. 133 or 134: Y_1 to Y_2, X_2 to X_3, Y_3 to Y_6, X_6 to X_5, Y_5 to Y_4. Calculate the ratio of the voltage between Y_1 and each of the junction points in the series, to the voltage across a single coil.

58. Transforming Two-Phase to Three-Phase. There have been numerous occasions to notice that when two coils having e.m.f.'s that differ in phase are joined in series, the resultant e.m.f. between the terminals of the series differs in phase from both of the component e.m.f.'s. This fact is the basis of the T-connection for transforming two-phase to three-phase, commonly called the Scott system in recognition of its inventor.

In Fig. 140, the coil *ab* represents an autotransformer or the secondary of an ordinary transformer, with a tap *c* brought out from its mid-point. The coil *cd* represents another transformer secondary or autotransformer, with one end connected to the mid-point *c* of the other transformer, and a tap brought out at *e* so as to include between *c* and *e* a number of turns equal to 86.6 per cent of the total number of turns between *c* and *d*. If now we impress one phase of a two-phase system across *ab* and the other phase across *cd*, we shall have a correct three-phase system between the points *a*, *b* and *e*. That is, if the e.m.f. *a* to *b* is equal to the e.m.f. *c* to *d* and 90° behind it, then the e.m.f.'s *a* to *b*, *b* to *e* and *e* to *a* are equal to each other and 120° apart consecutively with respect to positive directions as stated (*a* to *b* to *c*).

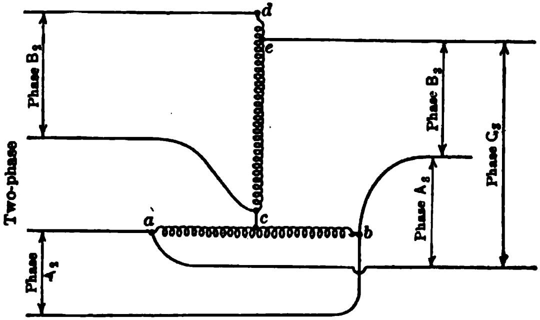


FIG. 140. The arrangement of a Scott transformer for transformation from two-phase to three-phase. Between *c* and *e* there are 86.6 per cent of the turns between *c* and *d*.

The e.m.f. relations of the Scott system are shown in Fig. 141 and 142 wherein the vectors represent the e.m.f.'s in the coils, or parts of coils, which are lettered correspondingly in Fig. 140. Let us assume that these vectors represent the e.m.f.'s which act internally through the coils between the points lettered like the ends of the vectors. Now the e.m.f. acting through the external circuit from *a* toward *b* is the same as the e.m.f. acting internally from *b* toward *a*, which is the reverse of vector *ab* in Fig. 141, as shown in Fig. 142. This gives us vector OA_3. The e.m.f. acting through the external circuit from *b* toward *e* is equal to the resultant of the e.m.f.'s acting internally from *e* to *c* and from *c* to *b*, which is the vector sum of *ce* reversed and *cb* direct from Fig. 141. This gives us the vector OB_3 in Fig. 142. Finally, the e.m.f. acting through the external circuit from *e* toward *a* is equal to the resultant of the e.m.f.'s acting internally from *a* to *c* and from *c* to *e*, which is the vector sum *ac* direct and *ce* direct from Fig. 141. This resultant is the vector OC_3 in Fig. 142.

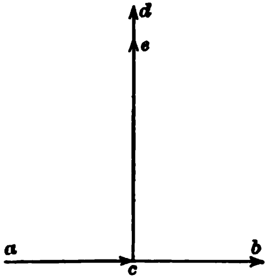


FIG. 141. Vectors of internal e.m.f.'s of the Scott connection shown in Fig. 140.

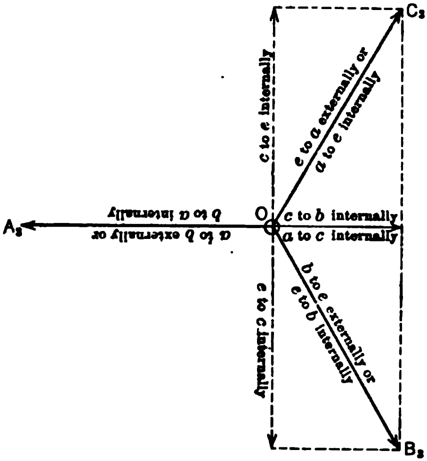


FIG. 142. Vectors showing how the internal e.m.f.'s of Fig. 140 and 141 combine to produce the external three-phase e.m.f.'s (*b* to *e*), (*e* to *a*) and (*a* to *b*).

Thus the e.m.f.'s *a* to *b*, *b* to *e* and *e* to *a* in the external circuit are represented respectively by the vectors OA_3, OB_3 and OC_3 in Fig. 142. It is easily proved by simple trigonometry that these three e.m.f.'s will be equal to each other and 120° apart (that is, line wires connected to terminals *a*, *b* and *c* will compose a three-phase system) if the following conditions be fulfilled:

(*a*) E.m.f. *a* to *c*, or *c* to *b*, equal to one-half of e.m.f. *a* to *b*.
(*b*) E.m.f. *c* to *e* equal to $\sqrt{3}/2$ or 0.866 times the e.m.f. *c* to *d*.
(*c*) E.m.f. *a* to *b* 90° out of phase with e.m.f. *c* to *d*, and equal to it.

It is not necessary that there be any particular relation between the number of turns in the coils *ac* or *ab* and the number of turns in the coils *ce* or *cd* — only that the e.m.f. relations within each of the two phases be as stated above.

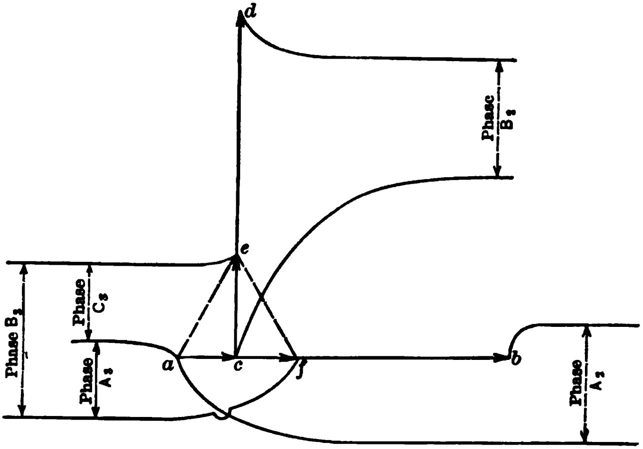


FIG. 143. A method of changing from two-phase to three-phase at different voltage by autotransformers, using only three taps — *e*, *c* and *f*.

It will be seen readily that a scheme similar to this could be applied to change any given polyphase system into some different polyphase system, although it could not change a single-phase into a polyphase system. It has received its principal application, however, in connecting two-phase systems to three-phase systems. It is impossible to use autotransformers for the Scott connection with a three-wire two-phase system, because the interconnection of the two phases short-circuits one of the coils *ac* or *cb*. We may, however, use ordinary transformers with Scott connection between their secondaries.

When connections are as in Fig. 140, 141 and 142, we shall have the same voltage between line wires of the three-phase system that we have between line wires of the two-phase system. However, we may easily transform from two-phase at one voltage to three-phase at some different voltage, as shown in the following example.

Example 3. The coil *ab* in Fig. 140 has 3000 turns, and the coil *cd* has 2400 turns. The e.m.f.'s *ab* and *cd* are each equal to 6600 volts and are 90° out of phase with each other. How should taps *c* and *e* be located with respect to *ab* and *cd* respectively, in order to obtain three-phase with 2300 volts between any two line wires? Where should the three-phase line wires be connected to the coils?

There are two solutions, as illustrated by Fig. 143 and 144, in which the following relations hold:

Figure No.	Between points.	Number of turns.	Voltage.
143	*a* to *b*	3000	6600
	c to *d*	2400	6600
	a to *f*	$\frac{2300}{6600}\times 3000 = 1045.2$	2300
	a to *c*	$\frac{1}{2}\times 1045.2 = 522.6$	1150
	c to *e*	$\frac{\sqrt{3}}{2}\times\frac{2300}{6600}\times 2400 = 725.5$	$\frac{\sqrt{3}}{2}\times 2300 = 1994$
	f to *e*		2300
	e to *a*		2300
144	*a* to *b*	3000	6600
	c to *d*	2400	6600
	g to *f*	$\frac{2300}{6600}\times 3000 = 1045.2$	2300
	g to *c* or *c* to *f*	$\frac{1}{2}\times 1045.2 = 522.6$	1150
	c to *e*	$\frac{\sqrt{3}}{2}\times\frac{2300}{6600}\times 2400 = 725.5$	$\frac{\sqrt{3}}{2}\times 2300 = 1994$
	f to *e*		2300
	e to *g*		2300

Practically the only difference between these two schemes is that Fig. 144 requires four taps to be made in the autotransformers or transformer secondaries, whereas Fig. 143 requires only three taps to be made.

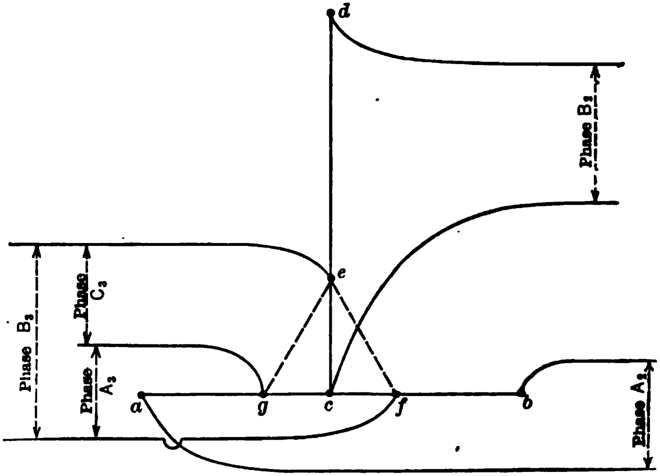


FIG. 144. This method requires four taps, *g*, *c*, *f* and *e*, to accomplish the same result as is shown in Fig. 143.

59. Current Relations in Tee-connected Transformers. So far we have illustrated the application of the Scott-connection to autotransformers, only. In Fig. 145a, however, are shown two ordinary transformers taking power from a two-phase line and delivering three-phase power. For simplicity, the secondary coils and connections alone are shown in Fig. 145b.

Fig. 146 represents the e.m.f. and current relations for the particular case of balanced load at unity power-factor delivered from the three-phase terminals. We start out by drawing the three vectors I_3', I_3'', and I_3''', representing the currents flowing within the coils from *b* toward *a*, from *e* toward *b* and from *a* toward *e*, respectively, due to the e.m.f.'s OA_3, OB_3 and OC_3, respectively (of Fig. 142), acting upon circuits of equal resistance and zero reactance. Fig. 145b represents the positive directions of the internal currents in the transformers when power is being delivered to the three-phase circuit. Noting from Fig. 145b that the internal current from *c* toward *b* (in direction of arrows in Fig. 141) is equal to the vector difference between the currents (*e* to *b*) and (*b* to *a*) or equal to the vector sum of currents (*e* to *b*) and (*a* to *b*), we reverse I_3' in Fig. 146 and then add it to I_3'', thus obtaining I_b, which is the current flowing internally in the coil connected to terminal *b*. By similar reasoning we find I_a by subtracting (vectorially) I_3' from I_3''', and we get I_e by subtracting I_3'' (vectorially) from I_3'''. If our reasoning and our work have been done correctly, we should find that the vector sum of I_b and I_e, both of which have positive directions away from the junction point *c*, should be exactly equal to the vector I_a, which has positive direction toward *c*; or, the vector sum of I_a reversed, and I_b, and I_e, as in Fig. 146, should be equal to zero.

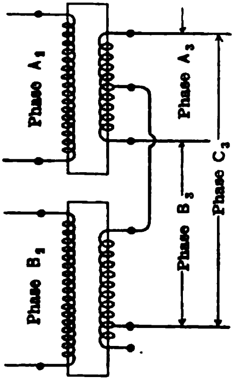


FIG. 145a. Two transformers with primaries connected to two-phase line, and secondaries tapped and connected according to Scott system delivering three-phases (A_3, B_3, C_3). This scheme may be used with a three-wire two-phase line, which is impossible with auto-transformers.

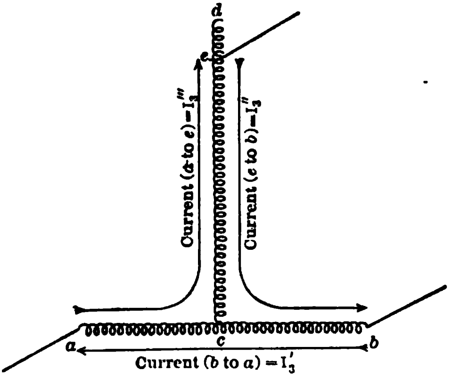

d
e
Current (a to e) = I_3'''
Current (e to b) = I_3''
a
c
b
Current (b to a) = I_3'

FIG. 145b. Positive directions of the currents in the transformer coils of Fig. 145a as power is delivered to the three-phase line.

Current from *c* to *b* = $I_3'' \ominus I_3'$
Current from *a* to *c* = $I_3''' \ominus I_3'$
Current from *c* to *e* = $I_3''' \ominus I_3''$

It is to be noted that Fig. 146 applies only to the three-phase side of two-coil transformers. To apply it to auto-transformers, we must draw in the vector representing the two-phase component of current in each coil, and combine it with the component of three-phase current in the same coil.

Example 4. What transformer capacity, in kv-a., would be required to transform 2300 volts two-phase to 2300 volts three-phase, delivering a balanced non-inductive load of 300 kv-a. from the three-phase terminals?

Kv-a. delivered to each of three external circuits = $\frac{300}{3}$ = 100.
In Fig. 142, $OA_3 = OB_3 = OC_3$ = 2300 volts.

In Fig. 146, $I_3' = I_3'' = I_3''' = \dfrac{100{,}000 \text{ volt-amperes}}{2300 \text{ volts}} = 43.5$ amperes.

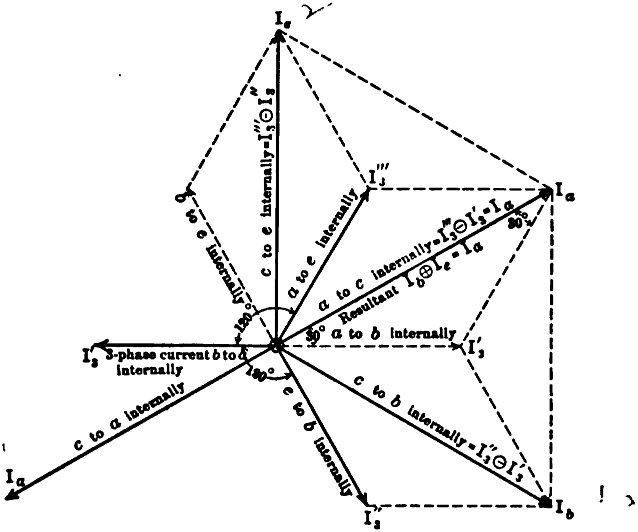

I_e
I_3'''
I_a
c to e internally $= I_3''' \ominus I_3$
b to e internally
a to e internally
a to c internally $= I_3''' \ominus I_3' = I_a$
Resultant $I_b \oplus I_e = I_a$
30°
120°
30°
a to b internally
I_3'
I_3
3-phase current b to c internally
180°
c to a internally
e to b internally
c to b internally $= I_3'' \ominus I_3'$
I_a
I_3''
I_b

FIG. 146. Vector diagram of internal currents in the coils of Scott-connected secondaries of two-coil transformers like Fig. 145a.

From the relations developed in Fig. 146 we see that under the above conditions,

$$I_a = I_b = I_e = \sqrt{3}\, I_3 = 1.732 \times 43.5 = 75.3 \text{ amperes.}$$

Each of the two transformers must therefore have a capacity of 75.3 amperes at 2300 volts, or 173.2 kv-a., and the total transformer capacity required is 2 × 173.2 equals 346.4 kv-a., in order to deliver 300 kv-a. at the three-phase terminals.

It is to be noted that this problem applies only to two-coil transformers like Fig. 145a; it does not apply to autotransformers connected according to the Scott system, as in Fig. 140, 143, 144.

If the power-factor of the three-phase circuit becomes less than unity, say 0.87, while the load remains balanced, it does not affect the transformer capacity required for a given kv-a. output. The effect is merely that the vector relations of Fig. 146 remain exactly the same while the whole diagram of currents shifts around so that the currents I_3', I_3'' and I_3''' make angles of 30° (= arc cos 0.87) with the respectively corresponding e.m.f. vectors OA_3, OB_3 and OC_3 of Fig. 142.

It is to be noticed that in the transformer *ab* (Fig. 141), the part *ac* carries a current **leading** the e.m.f. in this part by 30° (see Fig. 146), while the part *cb* carries a current **lagging** behind the e.m.f. in this part by 30°. This not only has the effect to require total transformer capacity in excess of the actual load delivered, as shown above, but also affects the voltage regulation of the transformers, and the balance of voltages in the phases. By comparing Fig. 146 with Fig. 142 we see that the phase relation of current to e.m.f. within the coil *ce* is exactly the same as the phase relation of I_3'' to OB_3, or of I_3''' to OC_3; that is, when the external load is non-inductive the current in *ce* is in phase with the e.m.f. in *ce*.

Prob. 39-4. If the tap *c* in Example 3, Fig. 143, is misplaced so that the number of turns from *a* to *c* is 572.6 instead of 522.6, all other connections being correct as calculated, what will be the values and phase relations of the e.m.f.'s between three-phase terminals?

Prob. 40-4. If the tap *e* in Example 3 is misplaced so that the number of turns from *c* to *e* is 775.5 instead of 725.5, all other connections being correct as calculated, what will be the values and phase relations of the e.m.f.'s between three-phase terminals?

Prob. 41-4. We desire to transform from two-phase at 4400 volts to three-phase at 2200 volts by means of T-connected autotransformers, as in Fig. 143. The design of the iron cores is such that at 60 cycles the maximum permissible flux density gives two volts per turn in the windings. Mark the required number of turns on all parts of coils in a connection diagram similar to Fig. 143.

Prob. 42-4. We desire to transform from two-phase at 4400 volts to three-phase at 6600 volts by means of T-connected autotransformers, as in Fig. 144. The design of the iron cores is such that at 60 cycles the maximum permissible flux density gives 5 volts per turn in the windings. Mark the required number of turns on all parts of coils in a connection-diagram similar to Fig. 144.

Prob. 43-4. Calculate the carrying capacity in amperes required for each part of the windings in Prob. 41, to deliver 200 kv-a. at 2200 volts three-phase.

Prob. 44-4. Calculate the carrying capacity in amperes required for each part of the windings in Prob. 42, to deliver 200 kv-a. at 6600 volts three-phase.

60. Transformers in Open-Delta Connection. Current Relations. If we have three transformers connected in delta to three-phase lines as in Fig. 125, it is valuable to know that one of these transformers, say No. 3, can be removed entirely from the system, giving a so-called "open delta," without interrupting the three-phase service on the low-tension mains L_1, L_2, L_3. That is, the e.m.f.'s L_1 to L_2, L_2 to L_3 and L_3 to L_1 remain approximately equal to each other as when the delta was complete, and approximately 120° apart as to phase in the sequence indicated. However, the current in each of the two remaining transformers is compelled to increase to a value $\sqrt{3}$ times as great as the current carried by each of three transformers in closed delta carrying the same load.

That is, if a balanced load of 300 kv-a. were being drawn from the secondary mains L_1, L_2, L_3, each of three 100-kv-a. transformers being fully loaded in closed delta (Fig. 125), the removal of one of the transformers would cause each of the remaining two to carry 173 kv-a. without altering the total load appreciably. If the transformers were each of 100-kv-a. rated capacity it would be necessary to reduce the total load from 300-kv-a. to ($\frac{100}{173}$) times 300 kv-a., or 173 kv-a., if we wished to avoid overloading the transformers.

If we wish to use two transformers in open delta to supply this balanced 300-kv-a. three-phase load as a regular operating condition, this may be done, provided each of the two transformers has a rated capacity of 173 kv-a. This would require a total capacity of 2 × 173, or 346 kv-a., in transformers to carry a 300-kv-a. load, for which 300 kv-a. of transformer capacity would be sufficient if the closed delta (three transformers) had been used. That is, we should need two equal transformers in open delta each of $\left(\frac{K}{3} \times \sqrt{3}\right)$ kv-a. capacity $\left(\text{total capacity} = \frac{2\sqrt{3}}{3}K\right)$, instead of three equal transformers in closed delta each of ⅓ K kv-a. capacity, to carry the same total balanced load of K kv-a. three-phase at any power-factor.

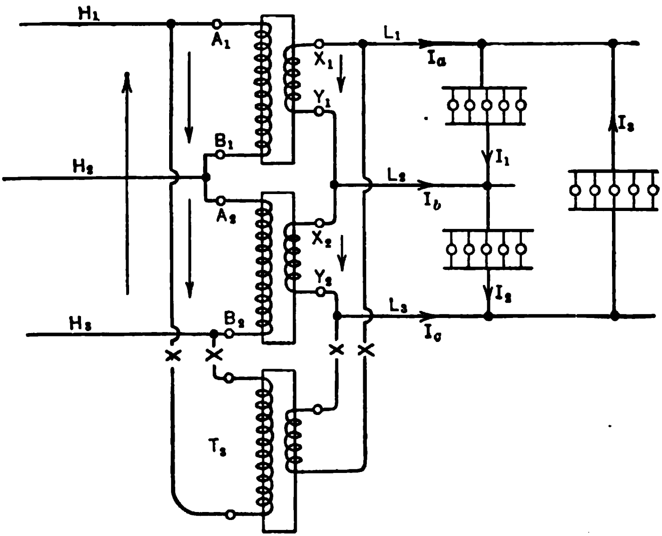


FIG. 147. The open-delta connection. Note the transformer T_3 is disconnected and that the other two are carrying the three-phase load. The line currents are:

$$I_a = I_1 \ominus I_3$$
$$I_b = I_2 \ominus I_1$$
$$I_c = I_3 \ominus I_2$$

In Fig. 147, transformer T_3 is shown disconnected from the delta of Fig. 125, leaving the open-delta arrangement of transformers No. 1 and No. 2. Fig. 148 shows the e.m.f. X_1Y_1 from main L_1 toward main L_2 in the external circuit, added vectorially to the e.m.f. X_2Y_2 from main L_2 toward main L_3, giving the (dotted) vector X_1Y_2 as representing the e.m.f. acting in the external circuit from L_1 toward L_3. But the convention according to which the three phases are uniformly 120° apart is that we proceed in orderly fashion from main to main, thus: L_1 to L_2, L_2 to L_3, L_3 to L_1, as indicated by the arrows representing positive directions for currents I_1, I_2 and I_3 in Fig. 147. The third phase will, therefore, be from L_3 toward L_1, and the e.m.f. acting on it through the external circuit is as represented by the (full line) vector Y_2X_1, in Fig. 148, which is equal and opposite to the e.m.f. acting externally from L_1 toward L_3 as represented by the vector X_1Y_2.

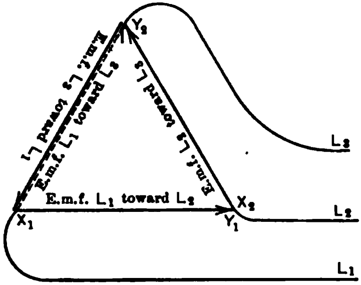


FIG. 148. Vector diagram of the e.m.f. relations in Fig. 147. The vector Y_2X_1 represents the e.m.f. across the open-delta.

In Fig. 149, we have assumed currents of I_1, I_2, I_3 amperes, equal in value and of the same power-factor, to be flowing in the three phases of the external circuit with positive directions chosen as indicated by arrows in Fig. 147. That is, the angle between the current I_1 in Fig. 149 and the e.m.f. X_1Y_1 or L_1L_2 in Fig. 148 is the angle whose cosine is the power-factor of the external circuit between mains L_1 and L_2, and so on. If now we draw arrows in the same (either) direction on mains L_1,L_2,L_3 of Fig. 147, to represent what we shall consider to be positive directions of current therein, we are enabled to write the relations of I_a, I_b, I_c in the mains to the load currents I_1, I_2, I_3. This is merely Kirchhoff's law relating to currents meeting at a point, applied to vector summations, instead of to algebraic summations as in the case of direct currents.

Thus, with positive directions as marked in Fig. 147, I_a is equal to the vector difference between I_1 and I_3, or equal to the vector sum of I_3 reversed and I_1, since the (vector) sum of all currents with positive directions toward a junction point must be equal to the (vector) sum of all currents with positive directions away from the same point. Applying this relation to each of three junction points of a main with two loads, we find the values of the main currents I_a, I_b, I_c as in Fig. 149. Although this figure relates only to a **balanced** load of **unity power-factor**, the same manipulations of vectors would bring correct results for any state of unbalance as to current or power-factor among the three loads.

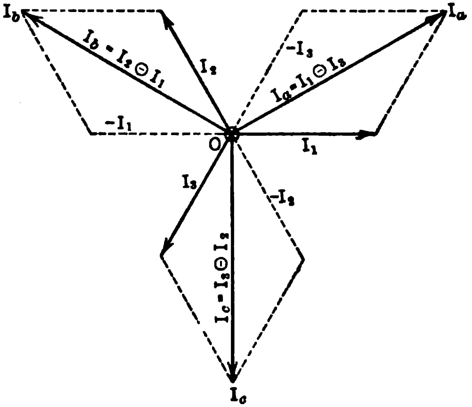


FIG. 149. Vector diagram of the current relations in Fig. 147, assuming a balanced load.

For **balanced load of any power-factor**, it is apparent from Fig. 149 that:

$$I_a = I_b = I_c = \text{current per line wire}$$
$$= \sqrt{3} \times \text{current per phase of closed delta.}$$

Then, since voltage across each transformer equals voltage across each load circuit, and current in each transformer of open-delta equals line current,

Kv-a. per transformer in open delta = $\sqrt{3} \times$ kv-a. per phase of three phases.

Prob. 45–4. What should be the kv-a. capacity of each transformer in an open delta to carry a balanced three-phase load whose total value is 120 kilowatts at 80 per cent power-factor?

Prob. 46–4. Two transformers each of 10 kv-a. rated capacity are connected in open delta. What maximum total load in kilowatts at 90 per cent power-factor, balanced among three phases, can be delivered allowing 25 per cent overload on transformers during the limited period of peak load? Neglect voltage drop in transformers.

Prob. 47–4. Each of the transformers of Prob. 46 has equivalent resistance of 0.5 per cent, impedance of 4 per cent, and ratio of turns 10 : 1. Low-tension voltage is 230 between each pair of mains. Calculate voltage across each pair of high-tension line wires.

Prob. 48–4. The loads on the system of Fig. 147 using transformers as specified in Prob. 46 and 47 are as follows: I_1 is 2 kv-a. at 90 per cent power-factor, I_2 is 6 kv-a. at 90 per cent power-factor. How many kv-a. at 90 per cent power-factor may be taken at I_3? In this problem neglect the voltage drops within the transformers.

Prob. 49–4. Using the data of Prob. 47 under the conditions of Prob. 48, calculate the voltages H_1H_2, H_2H_3 and H_3H_1 respectively, with 230 volts across each load. Assume current in each transformer same as in Prob. 48.

Prob. 50–4. If three transformers, like those specified in Prob. 46 and 47, were connected in closed delta to carry the load specified in Prob. 48, how many kilovolt-amperes could be taken at I_3? Power-factor equals 90 per cent in each phase of the load.

61. Parallel Connection of Three-phase Banks. When we desire to operate in parallel, polyphase transformers or groups of transformers, certain conditions must be fulfilled that are concisely presented in the following paragraph which we quote from the "American Handbook for Electrical Engineers":

"If there are several banks of transformers in the same system connected in parallel on one side, then to connect the other sides in parallel the connections must be such that the voltage between any two lines on this side will have the same phase in all the banks. From this relation result the following rules:

(*a*) With *YY* on one bank, the other bank must be *YY* or ΔΔ.

(*b*) With *Y*Δ on one bank, the other bank must be *Y*Δ or Δ*Y*.

(*c*) With Δ*Y* on one bank, the other bank must be Δ*Y* or *Y*Δ.

(*d*) With ΔΔ on one bank, the other bank must be ΔΔ or *YY*.

Even when these relations are satisfied a short-circuit will result unless the three phases of each bank are connected in the proper sequence."

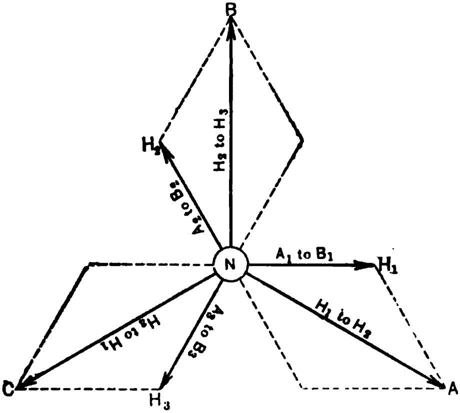


FIG. 150. The vector diagram of the high-tension side of a bank of *Y*-connected transformers. The *A* ends have been connected to neutral. The vectors H_1, H_2 and H_3 represent the voltages across the high-tension coils of the transformer. The vectors *A*, *B* and *C* represent the voltages between the high-tension line wires.

The reasons for the above statements should appear if the vector diagrams of Fig. 150 to 159 are studied carefully. The relative polarity of the high-tension and the low-tension coils in each phase of each bank of transformers is assumed to have been determined and marked by the conventional letters *AB*, *XY* (see Art. 49). Fig. 150, 151 and 152 refer to a *YY* bank; Fig. 153 and 154 refer to a *Y*Δ bank; Fig. 155, 156 and 157 refer to a Δ*Y* bank; Fig. 158 and 159 refer to a ΔΔ bank.

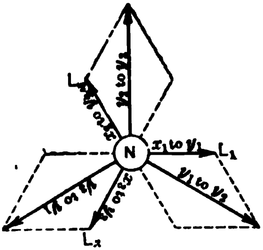


Fig. 151a. The vector diagram for the low-tension star-connected coils of the transformer of Fig. 150. The *X* terminals are connected to neutral and the *Y* terminals to the low-tension line wires.

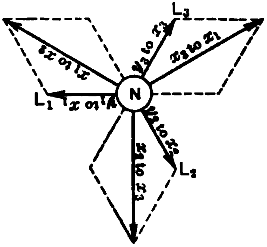


Fig. 151b. The low-tension coils of a *Y*-connected transformer bank, in which the *Y* ends of the coils are connected to neutral, and the *X* ends to the low-tension mains.

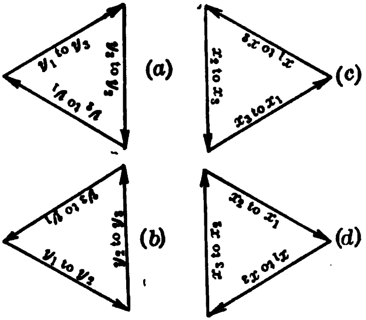


Fig. 152. The Δ-voltages between low-tension mains L_1, L_2, L_3, from the bank of transformers, the high-tension *Y*-connected coils of which are shown in Fig. 150.

From Fig. 150 we read that the high-tension coils of the *YY* bank have been connected in *Y*, the *A* terminals to neutral and the *B* terminals to the line wires H_1, H_2, H_3. The vectors *A*, B, *C* represent the high-tension e.m.f.'s H_1 toward H_2, H_2 toward H_3, and H_3 toward H_1, respectively. In Fig. 151a, the low-tension coils are connected in star, with *X* terminals to neutral and *Y* terminals to the low-tension mains L_1, L_2, L_3. The vectors Y_1 to Y_2, Y_2 to Y_3 and Y_3 to Y_1 represent the corresponding e.m.f.'s between mains L_1, L_2 and L_3. Fig. 152a represents the e.m.f.'s acting from Y_1 to Y_3, from Y_3 to Y_2 and from Y_2 to Y_1 (or L_1 to L_3, L_3 to L_2, L_2 to L_1) in three loads connected in delta to the low-tension mains. Fig. 152b shows the e.m.f.'s acting on the same loads but in the directions from L_3 to L_1, from L_1 to L_2, from L_2 to L_3. Fig. 151b represents the same low-tension coils connected in star, but with the Y terminals to neutral and X terminals to the low-tension mains L_1, L_2, L_3. After the manner indicated above, Fig. 152c represents the e.m.f.'s acting from X_1 to X_2, from X_2 to X_3, and from X_3 to X_1 in the delta loads connected to the low-tension mains, while Fig. 152d represents the e.m.f.'s acting from X_1 to X_3, from X_3 to X_2 and from X_2 to X_1 in the same delta-connected loads. Notice that the X_1Y_1 e.m.f. in each transformer is in phase with the A_1B_1 e.m.f., and so forth.

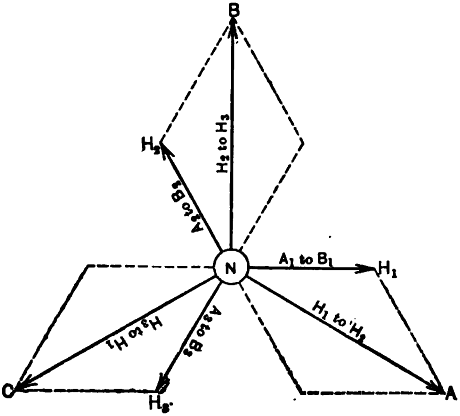


FIG. 153. The vector diagram of the high-tension e.m.f.'s in the Y-connected transformer bank. This is merely Fig. 150 repeated for convenience in comparing the relations of the low-tension e.m.f.'s.

Similarly, Fig. 154 represents phase relations between the e.m.f.'s acting upon three loads connected in delta to low-tension mains when the high-tension coils are connected in

Y as in Fig. 153 and the low-tension coils are connected in Δ. Fig. 154a and 154b refer to the same connections between coils, but to different direction of progress around the delta (Fig. 154a referring to the direction from Y_1X_2 to Y_2X_3 to Y_3X_1, while Fig. 154b refers to the direction from Y_1X_2 to Y_3X_1 to Y_2X_3). Fig. 154c and 154d refer correspondingly to the two different directions of progress around the delta but with the alternative scheme of interconnections between coils (that is, having junction points X_1Y_2, X_2Y_3, X_3Y_1, which gives us zero resultant within the delta just as well as the connection of Fig. 154a and 154b).

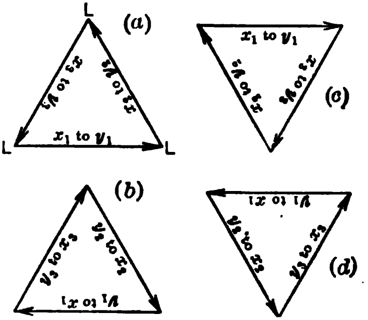


FIG. 154. The possible e.m.f.'s between low-tension mains L_1, L_2, L_3 from a $Y\Delta$-connected transformer bank, of which the high-tension Y-connected coils are seen in Fig. 153.

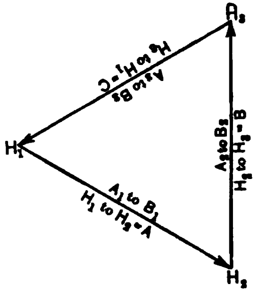
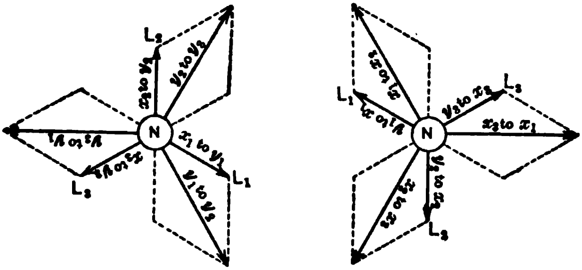


FIG. 155. The e.m.f. relations in the high-tension coils when the transformer bank is connected in delta on the high-tension side. Notice that corresponding line voltages A, B, C in Fig. 153 and 155 are in phase.

Fig. 157 illustrates in corresponding fashion the delta e.m.f.'s between low-tension mains when the low-tension coils are connected in star (as in Fig. 156a and 156b) and the high-tension coils in delta (as in Fig. 155). Fig. 157a and 157b refer to the two directions of progress around delta loads connected to secondary mains, when similar coil-terminals (X_1, X_2, X_3) are connected to neutral in both cases as in Fig. 156a. Fig. 157c and 157d refer to the corresponding cases when Y_1, Y_2 and Y_3 are connected to neutral as in Fig. 156b.

Again, Fig. 159 illustrates the delta e.m.f.'s between low-tension mains when the low-tension coils are connected in



(a) (b)

Fig. 156. The vector diagrams of the e.m.f.'s in the Y-connected low-tension coils corresponding to Fig. 155. Notice that the e.m.f. X_1Y_1 in Fig. 156 is parallel to the e.m.f. A_1B_1 in Fig. 155, and so forth.

delta, while the high-tension coils are also in delta as in Fig. 158. Fig. 159a and 159b refer to the two directions of progress around the same delta formed by the junction points Y_1X_2, Y_2X_3, Y_3X_1 between low-tension coils, while Fig. 159c and 159d refer to the corresponding cases for the other delta connection of low-tension coils which gives junction points Y_1X_3, Y_3X_2, Y_2X_1.

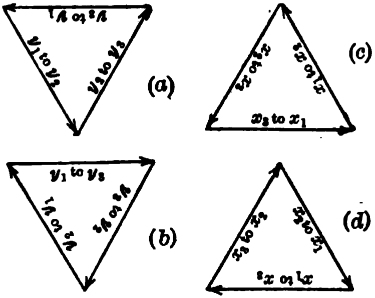


Fig. 157. The vector diagrams for the e.m.f.'s between low-tension mains L_1,L_2,L_3, corresponding to Fig. 156.

Notice that in all connections of high-tension coils, represented by Fig. 150, 153, 155 and 158, the e.m.f.'s from H_1 toward H_2 (vectors A) are parallel to or in phase with one another, the e.m.f.'s H_2 toward H_3 (vectors B) are in phase with one another, and the e.m.f.'s H_3 toward H_1 (vectors C)

are in phase with one another. This is necessarily so if corresponding high-tension coils are connected in parallel between identical line wires. The voltage rating of the high-tension coils and the low-tension coils of each transformer must of course be such that the delta voltages between low-tension mains as represented by the sides of the triangles in Fig. 152, 154, 157 and 159 are all equal to one another when the delta voltages A, B, C between high-tension line wires are equal to each other as represented in Fig. 150, 153, 155 and 158.

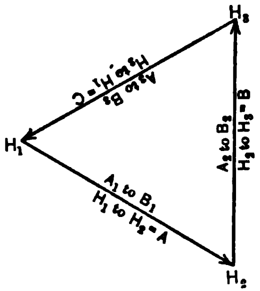


FIG. 158. The e.m.f. relations in the high-tension coils when the transformer bank is connected in delta on the high-tension side. Fig. 155, repeated.

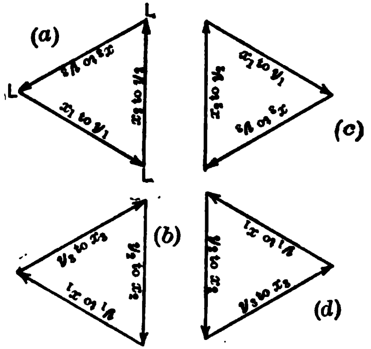


FIG. 159. The e.m.f. relations in the low-tension delta-connected coils of the transformer of Fig. 155. Compare with Fig. 152 for same low-tension relations with the high-tension coils connected in star.

Now obviously the low-tension mains of Fig. 152, 154, 157 or 159 may be connected in parallel whenever it is possible to superpose the corresponding triangles one over another **without turning them around.** Thus, there appear to be four possible parallel combinations of the ΔΔ connection (Fig. 159) with the YY connections (Fig. 152); but none are possible between either ΔΔ or YY and either ΔY or YΔ, on account of the fixed difference of angular relation between the triangles. There also appear to be four possible parallel combinations of the ΔY connection (Fig. 157) with the YΔ

TABLE IV

PROPER CONNECTIONS FOR PARALLELING THREE-PHASE BANKS OF TRANSFORMERS, YY WITH $\Delta\Delta$

Combination.	Vector diagrams.	Connections of the YY bank.				Connections of $\Delta\Delta$ bank.	
		High-tension.		Low-tension.		High-tension.	Low-tension.
		To neutral.	To line wires.	To neutral.	To mains.		
YY with $\Delta\Delta$	Fig. 152a with Fig. 159b or Fig. 152b with Fig. 159a	$A_1A_2A_3$	B_1 to H_1 B_2 to H_2 B_3 to H_3	$X_1X_2X_3$	Y_1 to L_1 Y_2 to L_2 Y_3 to L_3	B_3A_1 to H_1 B_1A_2 to H_2 B_2A_3 to H_3	X_1Y_3 to L_1 X_2Y_1 to L_2 X_3Y_2 to L_3
	Fig. 152c with Fig. 159d or Fig. 152d with Fig. 159c	$A_1A_2A_3$	B_1 to H_1 B_2 to H_2 B_3 to H_3	$Y_1Y_2Y_3$	X_1 to L_1 X_2 to L_2 X_3 to L_3	B_3A_1 to H_1 B_1A_2 to H_2 B_2A_3 to H_3	X_3Y_1 to L_1 X_1Y_2 to L_2 X_2Y_3 to L_3

TABLE V

PROPER CONNECTIONS FOR PARALLELING THREE-PHASE BANKS OF TRANSFORMERS, $Y\Delta$ WITH ΔY

Combination.	Vector diagrams.	Connections of $Y\Delta$ bank.			Connections of ΔY bank.		
		High-tension.		Low-tension.	High-tension.	Low-tension.	
		To neutral.	To line wires.			To neutral.	To mains.
$Y\Delta$	Fig. 154a with Fig. 157c or Fig. 154b with Fig. 157d	$A_1A_2A_3$	B_1 to H_1 B_2 to H_2 B_3 to H_3	X_2Y_1 to L_1 X_3Y_2 to L_2 X_1Y_3 to L_3	B_3A_1 to H_1 B_1A_2 to H_2 B_2A_3 to H_3	$Y_1Y_2Y_3$	X_1 to L_1 X_2 to L_2 X_3 to L_3
with ΔY	Fig. 154c with Fig. 157b or Fig. 154d with Fig. 157a	$A_1A_2A_3$	B_1 to H_1 B_2 to H_2 B_3 to H_3	X_1Y_2 to L_1 X_2Y_3 to L_2 X_3Y_1 to L_3	B_3A_1 to H_1 B_1A_2 to H_2 B_2A_3 to H_3	$X_1X_2X_3$	Y_1 to L_1 Y_2 to L_2 Y_3 to L_3

connection (Fig. 154); but none are possible between either ΔY or $Y\Delta$ and either $\Delta\Delta$ or YY, for the reason already mentioned. Examining the diagrams more closely, we find that of the four combinations between $\Delta\Delta$ and YY, only two are really different; and of the four combinations between ΔY and $Y\Delta$, two are identical with the other two. This leaves four really different combinations in all, which are specified completely in Tables IV and V following. It is understood, of course, that these tables show only the parallel connections that are possible between banks of different grouping, and that other connections are possible between banks of similar grouping, as between YY and YY, between $\Delta\Delta$ and $\Delta\Delta$, between ΔY and ΔY, or between $Y\Delta$ and $Y\Delta$, provided only that the voltage ratio, polarity and sequence of phases of the two banks are alike.

Note. In the following problems there are no electrical connections between neutral points.

Prob. 51-4. Parallel connections between two YY banks of transformers which have the same ratio and characteristics have progressed thus far: In first bank, A_1', A_2', A_3' to neutral; B_1' to H_1; B_2' to H_2; B_3' to H_3; X_1', X_2', X_3' to neutral; Y_1' to L_1; Y_2' to L_2; Y_3' to L_3. In second bank, A_1'', A_2'', A_3'' to neutral; B_1'' to H_1; B_2'' to H_2; B_3'' to H_3; X_1'', X_2'', X_3'' to neutral; Y_1'' to L_2. Calculate as percentages of low-tension line voltage the following e.m.f.'s: (*a*) Y_2'' to L_1; (*b*) Y_2'' to L_3; (*c*) Y_3'' to L_1; (*d*) Y_3'' to L_3.

Prob. 52-4. Incomplete parallel connections between the YY banks of Prob. 51 are as follows: On high-tension side, both banks connected as in Prob. 51; X_1', X_2', X_3' to neutral; Y_1' to L_1; Y_2' to L_2; Y_3' to L_3; Y_1'', Y_2'' and Y_3'' to neutral; and X_2'' to L_2. Calculate the following e.m.f.'s as percentages of the e.m.f. between low-tension mains: (*a*) X_3'' to L_1; (*b*) X_3'' to L_3; (*c*) X_1'' to L_1; (*d*) X_1'' to L_3.

Prob. 53-4. Two YY banks of transformers are connected as in Prob. 52 except that X_3'' connects to L_2. Is it permissible to connect X_2'' to L_3? If so, calculate the e.m.f. X_1'' to L_1 as percentage of the e.m.f. between low-tension mains.

Prob. 54-4. A $\Delta\Delta$ bank is connected to high-tension and low-tension mains as follows: B_1A_2 to H_2; B_2A_3 to H_3; B_3A_1 to H_1; $X_1'Y_3'$

to L_1; $X_2'Y_1'$ to L_2; $X_3'Y_2'$ to L_3. A ΔY bank having the same voltage between its low-tension terminals is partly connected to the same mains as follows: high-tension same as $\Delta\Delta$ bank; X_1'', X_2'', X_3'' to neutral; Y_3'' to L_1. Calculate the following e.m.f.'s as percentages of the e.m.f. low-tension mains: (*a*) Y_1'' to L_2; (*b*) Y_1'' to L_3; (*c*) Y_2'' to L_2; (*d*) Y_2'' to L_3.

Prob. 55–4. A $\Delta\Delta$ bank is connected as in Prob. 54, and a ΔY bank has the same connections on the high-tension side. On the low-tension side of this ΔY bank, we have the following connections: Y_1'', Y_2'', Y_3'' to neutral; X_3'' to L_1. Calculate the following e.m.f.'s as percentages of the e.m.f. between low-tension mains: (*a*) X_1'' to L_2; (*b*) X_1'' to L_3; (*c*) X_2'' to L_2; (*d*) X_2'' to L_3.

Prob. 56–4. A $Y\Delta$ bank has the following connections on the high-tension side: B_1', B_2', B_3' to neutral; A_1' to H_1; A_2' to H_2; A_3' to H_3. A ΔY bank has the following connections on the high-tension side: $B_2''A_1''$ to H_1; $B_1''A_3''$ to H_2; $B_3''A_2''$ to H_3. Specify connections which shall place the low-tension sides of these banks properly in parallel with each other, the ratios of the transformers being suitable for parallel operation.

62. Three-phase Transformers. A saving of approximately 16 per cent in the amount of iron required per kilovolt-ampere of three-phase power transformed, and an increase in efficiency of from 0.15 to 0.40 per cent, as well as a very substantial reduction in the floor space required per kilovolt-ampere of transformer capacity, may be accomplished by combining parts of the magnetic circuits of three single-phase transformers so as to form a single structure known as a "three-phase transformer." Three types are distinguished, known as the "core type" (Fig. 167), the "shell type" (Fig. 163 and 165), and the "hexagonal type" (Fig. 160), which is really only a modification of the shell type. Any comparisons that may be drawn should of course be between a single three-phase transformer and three single-phase transformers having the same total kilovolt-ampere capacity.

The flux relations may be brought out most clearly, perhaps, by considering the hexagonal type as in Fig. 160. The flux in all parts of the core will vary harmonically because the e.m.f. applied to the primary winding, and therefore the counter e.m.f. induced in it by the flux, varies harmonically. Consequently, we may properly represent the fluxes, as to both value and phase relations, by vectors just as we represent harmonic e.m.f.'s and currents. In Fig. 160, ϕ_a, ϕ_b, ϕ_c, represent the fluxes in the three iron cores on which the winding of the individual phases are placed, while ϕ_1, ϕ_2, ϕ_3

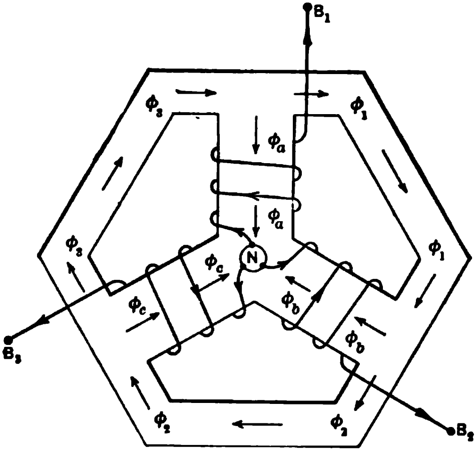


FIG. 160. Diagram showing the high-tension windings and the positive direction of the fluxes in the cores, in a three-phase transformer.

$$\begin{array}{ll}
\phi_1 = \phi_2 = \phi_3 & \phi_a = \phi_3 \ominus \phi_1 \\
\phi_1 \oplus \phi_2 \oplus \phi_3 = 0 & \phi_b = \phi_1 \ominus \phi_2 \\
\phi_a \oplus \phi_b \oplus \phi_c = 0 & \phi_c = \phi_2 \ominus \phi_3 \\
\phi_a = \phi_b = \phi_c &
\end{array}$$

represent the corresponding fluxes in the yokes connecting these cores. Only the primary or high-tension windings of the three phases are shown in Fig. 160. Marking the coil terminals in such manner that a current from A_1 toward B_1, or from A_2 toward B_2, or from A_3 toward B_3 will produce a magnetic flux in the same relative direction through the several cores (i.e., either all towards the junction of cores as in Fig. 160, or all away from the junction), we connect the A

ends of the coils together to form electrical neutral (N) and connect the B ends to line wires for a Y primary, or we connect as in Fig. 155 or 158 for a Δ primary.

Then, as the e.m.f.'s and exciting currents are equal in value and differ in phase consecutively by 120° with respect to positive direction from N toward B, it follows that the fluxes ϕ_a, ϕ_b and ϕ_c will be equal to one another and have corresponding phase differences of 120° when all are considered with respect to positive directions as marked by the arrows in the cores of Fig. 160. Choosing positive directions as in Fig. 160 for yoke fluxes ϕ_1, ϕ_2, ϕ_3, we are enabled to write the following relations.

From symmetry of the phases and of the construction:

$$\phi_1 = \phi_2 = \phi_3. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

From phase relation of e.m.f.'s and absence of coils on yoke:

$$\phi_1 \oplus \phi_2 \oplus \phi_3 = 0. \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

From relations of value and phase of e.m.f.'s and exciting currents in windings:

$$\phi_a \oplus \phi_b \oplus \phi_c = 0. \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$
$$\phi_a = \phi_b = \phi_c. \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

From consideration of flux relations at junctures of cores with yokes, assuming no magnetic leakage:

$$\phi_a = \phi_3 \ominus \phi_1. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$
$$\phi_b = \phi_1 \ominus \phi_2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$
$$\phi_c = \phi_2 \ominus \phi_3. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

According to the convention we have adopted with reference to e.m.f.'s and currents, ⊕ and ⊖ denote addition and subtraction of vectors whose arithmetical (maximum instantaneous) values are denoted by ϕ.

Fig. 161 is drawn in accord with equations (1) to (7). Starting with three equal vectors ϕ_1, ϕ_2, ϕ_3, 120° apart (whose vector sum is zero), we add ϕ_3 to ϕ_1 reversed and obtain ϕ_a in accordance with equation (5), and so on. We are thus informed that the flux in each core is equal to $\sqrt{3}$ times the flux in each yoke, or that the maximum value attained by flux in each yoke is equal to $\frac{1}{\sqrt{3}}$ or 0.577 times the maximum value of flux in each core. That is, if the magnetic circuit is of the same material throughout (as is usual, with stamped steel laminations), the cross-section area of the yoke should be about 58 per cent of the cross-section area of the core, in order to keep within the same limiting value of flux density B_m. Fig. 160 has been drawn to scale on this basis. If we make the yokes of smaller section they will overheat, and if we make them of larger section we shall waste material; obviously, we could not know the correct proportions without some such analysis as the preceding.

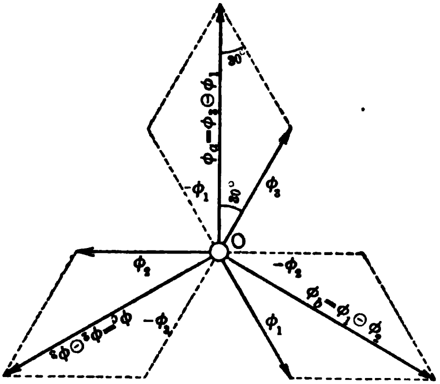

$\phi_a = \phi_3 \ominus \phi_1$
30°
$-\phi_1$
30°
ϕ_3
O
ϕ_2
$-\phi_2$
$\phi_b = \phi_1 \ominus \phi_2$
ϕ_1
$-\phi_3$
$\phi_c = \phi_2 \ominus \phi_3$

FIG. 161. Vector diagram showing the flux relations of Fig. 160. All fluxes are assumed to vary harmonically, thus the flux in the cores (ϕ_a or ϕ_b or ϕ_c) is equal to $\sqrt{3}$ times the flux in the yoke (ϕ_1 or ϕ_2 or ϕ_3).

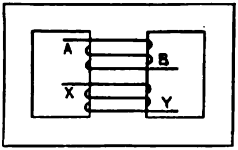


FIG. 162. Diagram of a single-phase transformer, shell-type.

Fig. 162 shows a shell-type transformer, and Fig. 163

shows three such transformers, one for each phase, piled up to form a "three-phase transformer." The end sections of the yoke of the middle transformer may be saved, as shown in Fig. 163, if we are careful to reverse the electrical connections of this phase with respect to the other two phases. (The arrow on the winding of the middle section, Fig. 163, merely shows the positive direction through the coil; note that the coil itself is reversed.) If we wind and connect all three primary coils in exactly similar manner, the saving of iron, though real, is so much reduced as to be hardly appreciable, as shown in comparison by Fig. 165. The reasons are developed in vector diagrams of Fig. 164 and 166.

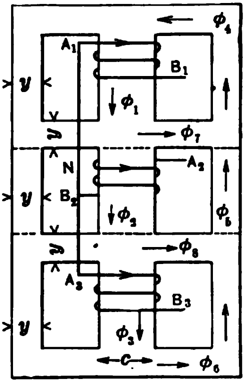


FIG. 163. Three single-phase transformers such as in Fig. 162 piled up to form a three-phase transformer shell-type. The end sections of the middle transformer can be cut out and still the flux density will be no greater than in Fig. 162.

$$\phi_4 = \phi_1 = \phi_5 \oplus \phi_7$$
$$\phi_6 = \phi_3 = \phi_5 \ominus \phi_8$$
$$\phi_7 = \phi_1 \ominus \phi_2 = \phi_4 \ominus \phi_5$$
$$\phi_8 = \phi_2 \ominus \phi_3 = \phi_5 \ominus \phi_6$$
$$\phi_5 = \phi_6 \oplus \phi_8$$

Consider first Fig. 165. With electrical connections as shown, the exciting currents reach maximum values in direction from *A* toward *B*, or from neutral toward line wires, 120 time-degrees apart consecutively. This causes the core fluxes ϕ_1, ϕ_2, ϕ_3 to reach their respective maximum values, in directions shown by the arrows, 120° apart. Therefore, in Fig. 166, we draw the vectors ϕ_1, ϕ_2, ϕ_3 (equal in value as phases are assumed to be of equal voltage) 120° apart. We now choose positive directions for the yoke fluxes ϕ_4, ϕ_5, ϕ_6, ϕ_7, ϕ_8. On the basis of directions as chosen in Fig. 165, and the explanation given with respect to Fig. 160, we may write the following equations for building the vector diagram of Fig. 166:

$$
\begin{aligned}
\phi_4 &= \phi_1 = \phi_6 \oplus \phi_7\\
\phi_6 &= \phi_3 = \phi_5 \ominus \phi_8\\
\phi_5 &= \phi_6 \oplus \phi_8 = \phi_4 \ominus \phi_7\\
\phi_7 &= \phi_1 \ominus \phi_2 = \phi_4 \ominus \phi_5\\
\phi_8 &= \phi_2 \ominus \phi_3 = \phi_5 \ominus \phi_6
\end{aligned}
$$

Using these vectorial relations and starting with ϕ_1, ϕ_2, ϕ_3, we find from Fig. 166 that ϕ_7 and ϕ_8 are each equal to $\sqrt{3}$ times the core flux. That is, to keep within the same limiting value of B_m or ϕ_m for the entire magnetic circuit, the total cross-section area of the yokes which carry ϕ_7 and ϕ_8 should be 1.732 times the area of the cores which carry ϕ_1, ϕ_2, ϕ_3.

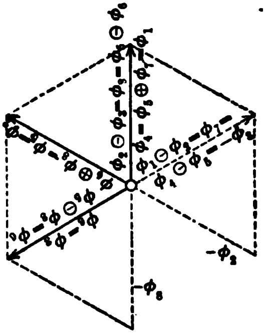


FIG. 164. Vector diagram for the flux relations in the transformer of Fig. 163.

As the yokes on the two sides and ends are in parallel, we find that, in Fig. 165, y_1 should be $\sqrt{3}/2$ or 0.87 times c, and y_2 should be 1/2 of c. Fig. 166 is drawn so as to indicate how the various vector relations stated in the equations may be checked or verified.

If now we reverse the connections of the middle phase as shown in Fig. 163, we reverse the vector ϕ_2 as seen by comparing Fig. 164 with Fig. 166. That is, ϕ_2 is now 120° out of phase with both ϕ_1 and ϕ_3 with respect to a positive direction which is opposite to their positive directions, or is only 60° out of phase with both ϕ_1 and ϕ_3 with respect to a common positive direction for all as shown by arrows in Fig. 163 and 165. The relations between chosen positive directions being now the same as in Fig. 165, the same equations hold between vector quantities, and we proceed to manipulate the vectors ϕ_1, ϕ_2, ϕ_3 of Fig. 164 exactly as we manipulated the corresponding vectors in Fig. 166. We find now, however, that the flux in every yoke is the same as in every core; that is, in

Fig. 163 we shall have the same value of B_m throughout the core if the dimension y in all parts of the (divided) yoke is equal to one-half the dimension c. Fig. 164 is drawn so as to show how all the equations may be checked. The arrangement of Fig. 163 is obviously preferable to that of Fig. 165 on account of its greater economy of iron.

The "core type" of three-phase transformers is illustrated in Fig. 167. With electrical connections of primary coils and with chosen positive directions as shown by arrows in Fig. 167, the fluxes ϕ_1, ϕ_2, ϕ_3 in the three cores are 120° apart in phase as shown in Fig. 168. The vector sum $\phi_1 \oplus \phi_2 \oplus \phi_3$ is equal to zero, which means that no flux is forced out of the iron into air, but that at every instant the flux downward (negative) in one core is exactly equal to the arithmetical sum of fluxes upward (positive) in the other two cores, or that with zero flux in one core, the downward (negative) flux in one of the other cores is exactly equal to the upward (positive) flux in the third core. This is also shown in Fig. 168, where the vector sum of ϕ_1 and ϕ_3 is exactly equal and

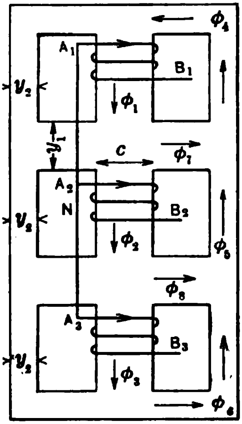


FIG. 165. A three-phase shell-type transformer with the coils all connected in a similar manner. The saving in iron over three single-phase transformers is slight. Note that in Fig. 163 the middle coil is reversed.

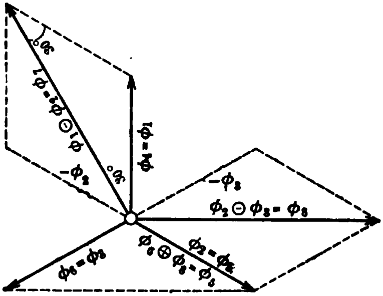


FIG. 166. Vector diagram of the flux relations in the three-phase shell-type transformer shown in Fig. 165.

opposite to ϕ_2, the vector sum of ϕ_2 and ϕ_3 is exactly equal and opposite to ϕ_1 (or ϕ_4), and the vector sum of ϕ_1 and ϕ_2 is exactly equal and opposite to ϕ_3 (or ϕ_5). We see also hereby that the fluxes ϕ_4 or ϕ_5 in the yokes are exactly equal to the fluxes ϕ_1 or ϕ_2 or ϕ_3 in the cores, which means that the cross-section area of the yoke should be exactly equal to the area of the core.

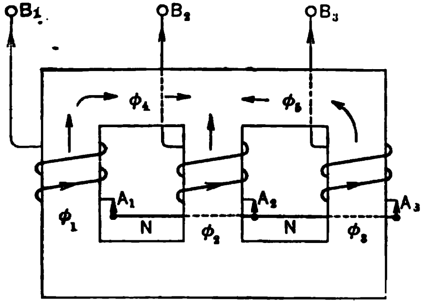


FIG. 167. The core-type three-phase transformer. The cross-section area of the yoke is the same as that of the core.

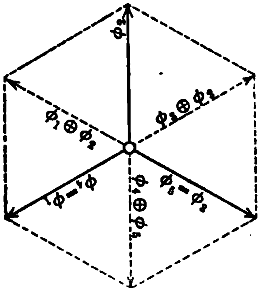


FIG. 168. The vector diagram for the flux relations in the core-type three-phase transformer of Fig. 167.

Prob. 57-4. Draw a vector diagram to represent flux relations that would result from reversing the connections of the middle coil in Fig. 167, and state in words what you interpret your diagram to mean. What will happen to the exciting current and to the value of B_m in each part of the core and yoke, for the same voltage between line wires?

Prob. 58-4. In the *Electric Journal* of May, 1913, the following statement is made: "It may be seen that by increasing the amount of copper by exactly 50 per cent and the amount of iron slightly more than 50 per cent, a three-phase transformer is obtained having 50 per cent greater capacity than the original single-phase transformer." Verify or disprove these statements by sketching a single-phase transformer for the same voltage and kv-a. capacity as two phases of the three-phase transformer of Fig. 167, and making a comparison.

Prob. 59–4. Draw a vector diagram to illustrate the result of combining two harmonically varying and equal magnetic fluxes 120° out of phase in opposite directions through the same core. What is the ratio between the maximum value of the resultant flux and the maximum value of either of the components?

Prob. 60–4. In the *Electric Journal* of May, 1913, the following statement is made: "By increasing the amount of copper of a single-phase shell-type transformer exactly 200 per cent, and of the iron approximately 150 per cent, a three-phase shell-type transformer is obtained, having three times the capacity of the single-phase transformer." By study of Fig. 163, attempt to prove or disprove these statements.

Prob. 61–4. Check the following statement: "A three-phase $\Delta\Delta$-connected shell-type transformer may be operated temporarily at 58 per cent of the combined capacity of the three windings provided only one phase has been disabled. In such a case, both the high-tension and low-tension windings of the disabled phase are disconnected from the lines and other phases, and each of these windings is short-circuited upon itself."

(*a*) Check the numerical value of 58 per cent stated above.

(*b*) Explain why the disabled windings are short-circuited and what would result if they were not.

(*c*) Why would not the current be excessive in the short-circuited coils?

63. Feeder Voltage Regulators. As explained in Art. 7, page 22, it is in general highly desirable that the consuming devices shall be served with electric power at constant voltage. But as the pressure drop on the various transmitting and distributing lines emanating from a generating station will depend upon the length and other features of the individual lines, as well as upon the variations of load which are peculiar to each line, it is obviously impracticable to adjust the voltage at the bus-bars or generator terminals so as to compensate for line drop and to maintain approximately constant voltage at the service end of every line. One of the following remedies must be applied individually to each line or feeder, according to its requirements:

(*a*) Make the line of such conductivity and spacing that the resistance and reactance are low enough to bring the impedance drop on the line within the limits of voltage variation permitted between full load and zero load, at the given power-factor.

(*b*) Equip each individual transmission line or feeder with a **"feeder voltage-regulator"** to boost the voltage on that feeder by an amount equal to the impedance drop on the feeder. This regulator is usually furnished with means for keeping it automatically adjusted at all loads so that the service voltage at the end of the feeder is nearly constant. The regulator may be located in the generating station, in the substation, or on the pole or in the manhole at the service point, as is most convenient.

Often, and particularly in the case of the longer lines or feeders, a determination of the most economical size of wire (see Art. 66, page 323) will show that the power can be transmitted more cheaply by **installing the most economical size of wire and adding a feeder voltage regulator to compensate excessive voltage drop** than by installing a wire large enough to keep the voltage regulation at the service point within the same limits without the feeder regulator. These regulators have therefore become very common and practically essential in transmissions having considerable power capacity or length.

Feeder Voltage Regulators are of two types, which may be distinguished as the **Induction type** and the **Compensator type.** The operating principle of the former is shown in Fig. 169. A primary coil *PP* is tapped across the bus-bars or the transmission line, and produces alternating flux in the iron core *cc*. This flux induces an e.m.f. in a secondary coil *SS* (which is placed at an angle with *PP* corresponding to 90 degrees), provided the core *cc* does not happen to lie in the same plane with this secondary coil. In the latter event, no e.m.f. is induced in *SS* because the flux is parallel to the turns of *SS* and does not link with them. The secondary coil is in series with the transmission line or feeder. If the core *cc* is inclined in one direction through the secondary coil *SS* the e.m.f. induced therein is in the same direction as the primary e.m.f., and the voltage on the load side of the regulator is greater than the voltage on the generator side by an amount equal to the voltage E_s which is being induced in the secondary coil. If the core *cc* be turned slightly so as to incline oppositely with respect to the plane of *SS*, while still approximately perpendicular to *PP* as before, the e.m.f. E_s induced in *SS* is in the opposite direction, and the feeder voltage becomes $(E_p - E_s)$. With E_p and primary current growing in direction shown by the arrows, it is as if we thrust a north pole (shown by arrows on *cc*) upward through *SS* in the first case, and downward in the second case. In either case E_s is in phase with E_p, or at 180 electrical degrees to it, and the load voltage $(E_p \pm E_s)$ is the arithmetical sum or difference of E_p and E_s depending on the angular position of *cc*. The value of E_s will vary with this angle, as more or less of the primary flux is made to link with the secondary.

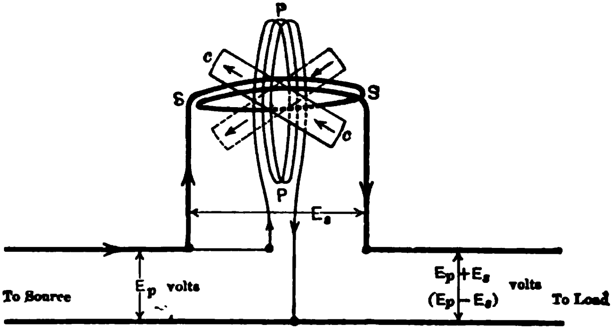


FIG. 169. Diagram of a voltage regulator; induction type, single-phase.

The **compensator type** of feeder voltage regulator is shown in Fig. 170, where we have a transformer with its primary *PP* connected across the transmission line through a reversing-switch *a*, and taps brought out from various points along its low-voltage secondary coil *SS* to contact blocks on a switch b_1b_2 along which slides a contact arm *c*. This contact arm is connected to the feeder in such manner that the voltage E_s induced in the secondary coil between the points b_1 and c is either added to or subtracted from the primary voltage, depending upon the position of the reversing switch a. Thus, if E_s' be the maximum voltage that can be had from the secondary with c moved up to b_2, the limits of load voltage are $(E_p \pm E_s')$, depending upon which way the switch a is thrown.

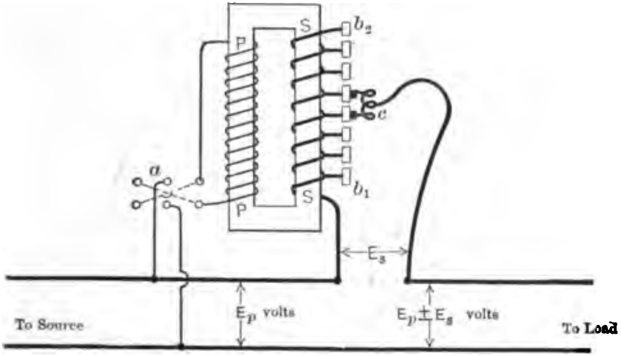


FIG. 170. Diagram of a feeder voltage regulator; compensator type, single-phase.

To avoid interruption of the current supply to the load (which might cause severe and destructive sparking or arcing at the switch contacts), the contact on the arm c is made broad enough to span two blocks on the switch b_1b_2. This would result in damage to the secondary SS due to short-circuiting some of its turns in moving from one block to another, if some preventive were not employed. One method is to snap the contact c from one block to the next by means of a spring so quickly that no arc can be formed and no flicker noticed in the load current. Another method, shown in Fig. 170, is to split the contact c into two parts which are connected together through a coil, which has sufficient reactance to limit the short-circuit current to a small value, and sufficient carrying capacity not to be injured by passage of the main load current through it. The load circuit is tapped from the middle of this coil, and the result is that the useful load current flows in equal amount but in opposite directions through the two halves of the coil, so that no flux and no reactance drop are produced in it by the load currents. This device is convenient to use wherever we desire to change from one tap to another on any transformer without disturbing the circuit.

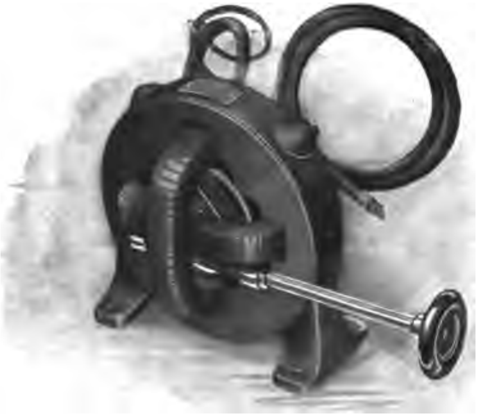


FIG. 171. Feeder voltage regulator to be operated by hand. *The General Electric Co.*

Fig. 171 is a photograph of a two-pole single-phase induction-type regulator, similar to that shown in Fig. 169, and arranged to be operated by hand. Fig. 172 shows a similar regulator arranged to be automatically controlled, and to be hung on a pole or placed in a manhole at the load-center or at the point where the feeder connects to the consumer's service taps. A contact-making voltmeter automatically closes a control circuit when the voltage on the load side of the regulator reaches a value approximately one-half of one per cent above or below the value which it is adjusted to maintain. This control circuit operates a small motor in such direction as to move the core *c* (Fig. 169) or the contact *c* (Fig. 170), and restore the load-voltage to the predetermined value.

Fig. 173 shows the core and windings of a fairly large single-phase two-pole induction-type regulator. The moving element carries the primary coil, because this consists of a large number of turns of fine wire receiving full line voltage and the connections to it are therefore light and flexible. With this rotor in the position shown, no e.m.f. is induced in the secondary coil; but if the voltmeter and control motor move the core clockwise the feeder e.m.f. is raised, let us say, and if counter-clockwise it is lowered. We may easily see, however, that when the rotor is in the neutral position as here shown, the secondary coil surrounded completely by iron forms a choking coil through which the load currents must pass, and the voltage regulation of the circuit is caused to be very bad. To prevent this difficulty a coil of heavy wire short-circuited on itself is mounted on the drum and fixed at an angle of 90 electrical degrees with the primary. When the primary is in neutral position, the secondary induces a sufficient current in this short-circuited coil to reduce to practically zero value the flux and reactance due to the load currents in the secondary.

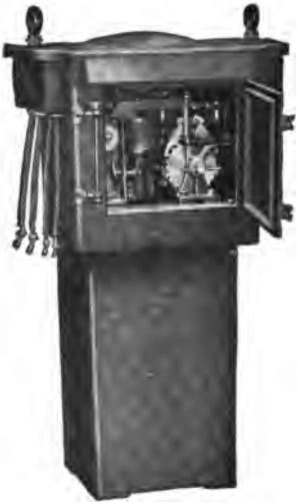


FIG. 172. An automatic feeder voltage regulator. Pole type. *The General Electric Co.*

Fig. 174 shows an installation of large stationary induction-type feeder regulators in a substation. Fig. 175 shows the internal connections of a commercial regulator of the compensator type, in which the contact blocks *AAA* on the dial switch correspond to the blocks on the switch b_1b_2 of Fig. 170.

Each of two adjacent blocks is touched by a number of contact fingers which differ but slightly in length. Each contact finger is attached to a collector ring on the movable

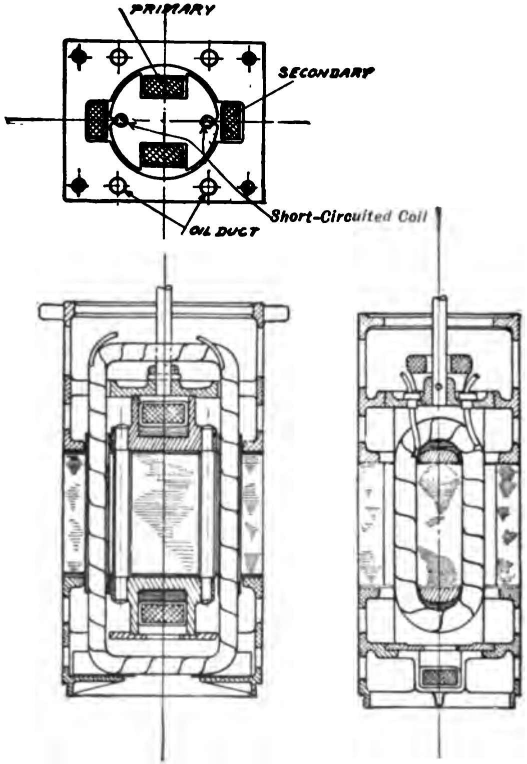


FIG. 173. Section of winding and core of single-phase induction regulator. *The General Electric Co.*

drum, and these rings are all connected in parallel through individual preventive-resistances to the feeder circuit as shown. When the contacts are at the upper middle position in the upper part of Fig. 175, the feeder voltage is exactly the same as the generator voltage. Moving the contacts clockwise will raise the feeder voltage, and moving them counter-clockwise will lower the feeder voltage by causing the e.m.f. in the left half of the secondary, in series with the feeder, to oppose the generator e.m.f. The preventive resistances serve the same purpose as the coil *c* in Fig. 170. They do not waste appreciable power due to passage of load current through them because, as may be seen in lower Fig. 175, there are four of them in parallel connected to each contact-block.

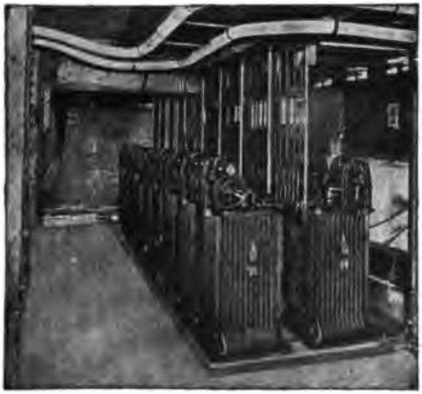


Fig. 174. Installation of General Electric automatic induction feeder regulators at the Philadelphia Electric Co.

The compensator type of regulator has the advantage over the induction type that the moving element is much easier to move and can therefore be moved more quickly, resulting in closer regulation or more nearly constant feeder voltage. There is a large torque, due to magnetism, resisting movement of the rotor in the induction type, whereas in the former the resisting torque is due only to friction. The efficiency of the compensator type may also be greater due to the fact that load currents traverse only as much of the secondary winding as may be necessary, instead of all of it; and the power-factor of the induction type may be lower on account of the air gap in the magnetic circuit, rqeuiring a greater magnetizing component of current.

The compensator type of regulator is usually built for single-phase circuits only, and must be applied individually to the phases of a three-phase circuit. The induction regulator can easily be built for polyphase circuits merely by mounting upon both rotor and stator three windings with proper angle between them (see Chap. IX, First Course), the rotor windings being connected together in Δ or Y to the line wires, and each stator winding being connected in series with the corresponding line wire. The internal actions of a

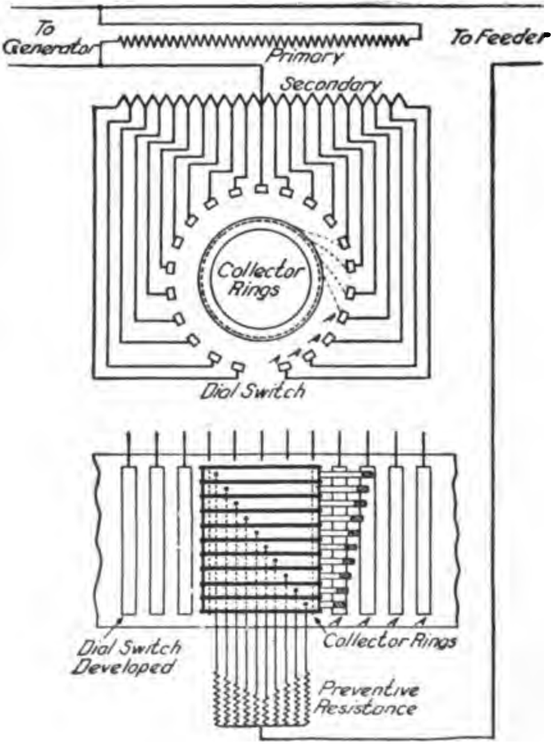


FIG. 175. Internal connections of compensator type regulator. *The General Electric Co.*

polyphase induction regulator differ in detail from those of the single-phase induction regulator. In the single-phase type the magnetic flux is fixed in direction and varies harmonically in value with time, while the e.m.f. induced in each phase of the secondary is in phase with the e.m.f. of the corresponding phase of the primary. In the polyphase type the flux rotates around the axis of the rotor and the stator at an angular velocity depending upon the frequency of the primary e.m.f. and upon the number of poles for which the windings are arrañged. The value of the flux is fixed, and the e.m.f. added by the secondary winding to each phase of the feeder circuit has a phase relation to the impressed e.m.f. which depends upon the angular position of the rotor with respect to the stator. It is like a wound-rotor induction motor (which see, in Chap. VII of this book), in which the rotor is held stationary against the torque which tends to rotate it, and each phase of the stator winding is connected in series with one of the line wires while the corresponding phase of the rotor winding is connected across the same phase of the line.

Prob. 62–4. Unless specified otherwise, feeder voltage regulators are usually designed to raise or lower the line voltage by 10 per cent. Their rating in volt-amperes is equal to the product of the current they can deliver times the amount by which they can raise or lower the voltage of this current above or below its normal value. Under these conditions, what would be the rating of a single-phase regulator for a 100-kv-a. 2300-volt feeder?

Prob. 63–4. Under the conditions of Prob. 62, what would be the kv-a. rating of a three-phase induction regulator for a three-wire feeder delivering 100 amperes per wire with 2300 volts between line wires?

Prob. 64–4. Under the conditions of Prob. 62, what would be the limiting values of load voltage and current supplied through a single-phase induction-type regulator rated 2.3 kv-a. for a 2300-volt circuit?

Prob. 65–4. By use of recording voltmeters attached to the lines, the following were observed to be the limiting values between which the voltage fluctuated:

(*a*) Generator voltage (104 to 119) times 20.

(*b*) Feeder voltage beyond induction-type regulator (110.5 to 115) times 20.

(*c*) Feeder voltage beyond compensator-type regulator fed from same generator at approximately the same time (114 to 116) times 20.

If the candle power of tungsten lamps varies as the 3.63 power of the applied voltage, determine the ratio between maximum and minimum candle power of a given lamp attached, under the conditions given above, to:

(*a*) the generator directly,
(*b*) the induction regulator,
(*c*) the compensator regulator.

Prob. 66–4. Would it be preferable to use a polyphase regulator or three single-phase regulators under each of the following conditions, and why?

(*a*) Three-phase balanced circuit feeding polyphase motors principally.

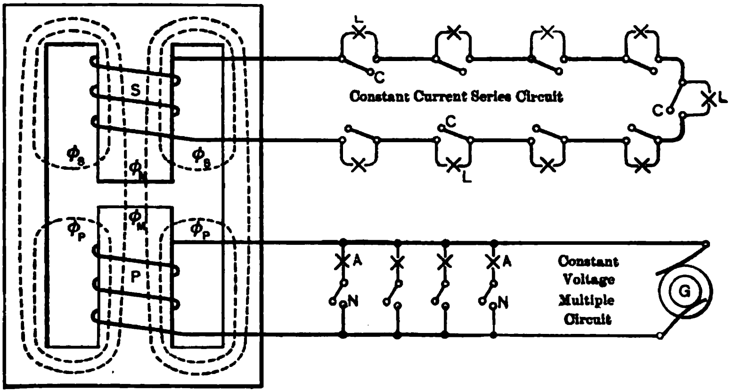
(*b*) Three-phase unbalanced circuit feeding lights principally.

64. The Constant-current Transformer.

In supplying power to electric lamps for street lighting, it is much more economical or less expensive to connect the lamps in series than to connect them in multiple or parallel. As in the **multiple system** it is necessary to maintain **constant voltage** across the circuit in order that the addition or extinction of lamps shall not affect the current supplied to every other lamp in the system, so in the **series system** it is necessary to maintain the **current constant** in order that the insertion of more lamps into the series or the extinction of some lamps by short-circuiting them shall not affect the operation of the other lamps.

In the multiple system the voltage across the circuit is maintained constant at a value suitable to a single lamp, and the copper wire must be heavy enough to carry the sum of currents required by all the lamps, with a voltage drop and a power loss within limits prescribed in fairly definite manner by the quality of service demanded and by the requirement of economy in transmission. In the series system the current in the circuit is maintained constant at a value suitable to the individual lamp, and the copper wire need be only large enough to carry this much current over the given route with a power loss not to exceed the economical limit, it being unnecessary to consider the voltage drop in the wires as a limiting condition in this case. The investment in copper is therefore very much less in the series system than in the multiple system to supply the same loads, due to the higher voltage and lower current.

Inasmuch as electric power is usually generated and transmitted at approximately constant voltage, for practically every kind of load except street-lighting systems, it is necessary to have some device to put in the central station or in a conveniently located substation which shall be able to take power at constant voltage (variable current) and deliver it at constant current (variable voltage). Such a device is called a "constant-current transformer," and is to-day a very common piece of equipment. The principle of operation is shown in Fig. 176, where we have a primary coil *P*, connected to a constant-voltage a-c. generator *G*, and mounted on the same iron core with a secondary coil *S* which is connected to a series circuit of lamps *L*. The magnetic circuit has an air



L
C
Constant Current Series Circuit
C
L
S
ϕ_S
ϕ_S
ϕ_M
ϕ_M
ϕ_P
ϕ_P
P
A
N
A
N
Constant Voltage Multiple Circuit
G

FIG. 176. Diagram showing the principle of the constant current transformer. Note the air gap in the transformer core.

gap in it between P and S, as indicated on the middle limb or core of the transformer. As we vary the impedance of the external part of the secondary circuit by opening or closing the switches C which short-circuit the lamps L, we find that the relative variation of current in the secondary is very much less than it would be if there were no air gap in the magnetic circuit of the transformer (i.e., if it were like an ordinary constant-potential transformer).

The reason why an air gap interposed in the magnetic circuit between the primary and secondary coils causes the transformer to be self-regulating for approximately constant current rather than for approximately constant voltage should be fairly clear after a careful study of Article 41, page 164. For the total flux generated by the primary current there are two paths which are magnetically in parallel, namely, the path of the mutual flux ϕ_M which links both P and S, and the path of the primary leakage flux ϕ_P which links with all or part of the primary without linking the secondary. There is a similar parallel path for the secondary leakage flux ϕ_S. Now, in the ordinary transformer the reluctance of the all-iron path of the mutual flux ϕ_M is so low in comparison with the reluctance of the part-air path of the leakage flux ϕ_P that most of the flux which threads the primary coil and induces the counter e.m.f. in it also threads the secondary coil and induces e.m.f. in it; that is, ϕ_M is a very large part of the total primary flux $(\phi_M + \phi_P)$ when there is no air gap in the magnetic circuit. But when we introduce such an air gap, the reluctance of the path of ϕ_M becomes more nearly equal to the reluctance of the parallel path of ϕ_P, and the same total flux $(\phi_M + \phi_P)$ redistributes itself so as to make ϕ_P larger and ϕ_M smaller. As the value of e.m.f. induced in the secondary coil depends directly upon the amount of ϕ_M, we see that the presence of the air gap results in a very poor voltage regulation at the secondary terminals. That is, a given reduction of impedance in the secondary circuit produces a much less increase of secondary current than before the introduction of the air gap, because of the greater decrease of secondary voltage due to the gap.

Apparently, therefore, a transformer must be designed to have large leakage reactance and bad voltage regulation in order to regulate itself for approximately constant current. This is, in fact, true for generators also. Fig. 177 illustrates the general relations. The ordinates of curve E_1 represent terminal volts and abscissas represent corresponding amperes output of a generator or transformer having zero per cent voltage regulation (absolutely constant voltage). The ordinates of curve Z_1 represent impedances corresponding to various values of current (abscissas) and are obtained by dividing the ordinate on E_1 (volts) by the corresponding abscissa (amperes). Similarly the ordinates of curve Z_2 represent impedances of the external secondary circuit of a transformer which has rather poor voltage regulation as

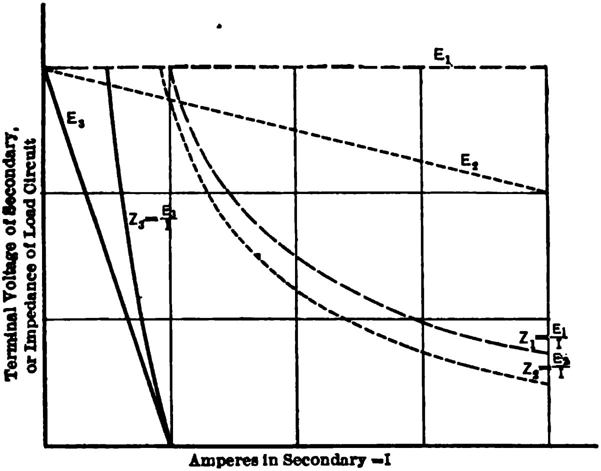


FIG. 177. Curves showing practically constant-current regulation of transformer which has bad voltage regulation. Note that whatever the impedance Z_3 may be, the current I remains practically the same when the voltage has the characteristics of curve E_3.

shown by the pronounced slope of the voltage curve E_2, and the ordinates of curve Z_3 represent external impedances for a transformer having extremely bad voltage regulation as shown by curve E_3. From the curves of Fig. 177 we find that while Z is increasing over the interval from 1 to 3 units in all cases (it is immaterial what scales we choose for ordinates and abscissas, for volts or ohms and for amperes), I varies from a value of 3 to a value of 1, or over a range of $\frac{3-1}{3}$ or 67 per cent in the case of the transformer (E_1) having perfect voltage regulation; I varies from 2.45 to 0.95, or over a range of 61.2 per cent in the case of a transformer (E_2) having somewhat poorer voltage regulation; and I varies from 0.75 to 0.5 or over a range of 33 per cent in the case of a transformer (E_3) having very bad voltage regulation.

In the common commercial types of constant-current transformer the method for obtaining constant-current regulation is somewhat different. Thus, in Fig. 178 the lower (primary) coil is stationary, while the upper (secondary) coil is free to move along the iron core, which has no air gap in it. The secondary is suspended from counterweights by cords which run over wheels or grooved sectors as shown in Fig. 178. These sectors are so shaped or adjusted that with the secondary coil in any position along the core the difference between the weight of the suspended coils and the lifting effort exerted on them by the counterweights is exactly equal to the magnetic repulsion between the primary coil and the secondary coil when carrying the current which the transformer is adjusted to maintain. If the impedance of the external circuit be reduced by short-circuiting any part (or even all) of the load, the resulting momentary rise of current will increase the repulsive force between coils and lift the secondary. As the secondary moves away from the primary the reluctance of the leakage paths decreases and the leakage flux increases, thereby reducing the mutual flux and the secondary induced voltage corresponding to constant primary impressed or counter e.m.f. The secondary coil will move upward and the secondary voltage will be thus reduced at the same time until the coil finds itself in a position where there is again equilibrium between the forces acting, which will be when the current is restored to its former value. The current for which the

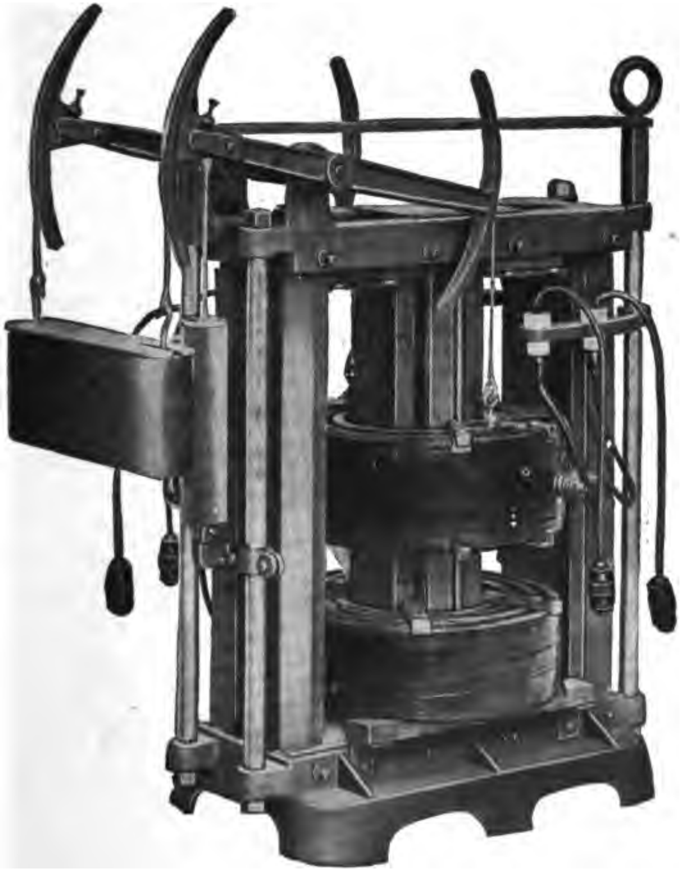


FIG. 178. A 60-cycle, 20 KV-A., 2200-volt, 5.5-amp., constant-current transformer of the commercial type. Note that instead of the air gap in the core, the secondary coil is free to move along the core, and thus increase or decrease the leakage flux. *The General Electric Co.*

transformer regulates itself may be adjusted over a certain range by changing the amount of counterweight. Manufacturers commonly guarantee these transformers to regulate within 1 per cent above or below rated current, from no load to full load.

The constant-current regulation is thus obtained by automatically increasing the leakage reactance in just sufficient amount to compensate any decrease of external (secondary) impedance, or vice versa. As the secondary ampere-turns are maintained constant, the primary ampere-turns and amperes will also be constant except for the change in the magnetizing component of current. The primary impressed voltage being maintained constant, it follows that the power-factor of the primary will vary in direct proportion to the total watts output of the secondary. Heavier load means larger impedance in the secondary external circuit, higher voltage at the secondary terminals, *P* and *S* coils closer together, and higher power-factor in the primary circuit, with practically no change in the copper loss and a slight increase of the iron loss (due to reduction of leakage flux and corresponding increase of mutual flux in the all-iron magnetic circuit).

TABLE VI

PERFORMANCE DATA ON CONSTANT-CURRENT TRANSFORMERS WITH INCANDESCENT LAMP LOAD

From Bulletin of Adams-Bagnall Electric Co., on "Street-lighting"

Size of transformer, kv-a.	Efficiency, in per cent.				Power-factor, in per cent.				Weight of transformer, pounds.
	Full load.	¾ load.	½ load.	¼ load.	Full load.	¾ load.	½ load.	¼ load.	
4	95	93	90	83	86	66	45	24	625
8	96	95	92	87	87.2	66.2	45.2	24.2	900
12	96.2	95.5	93	87.5	87.9	66.6	45.2	24.3	1200
16	96.5	96	94	88	88.5	67	45.3	24.5	1650
22	97.2	96.2	94.4	89.4	88.6	67.1	45.4	24.5	1800
32	97.4	96.4	95	90.5	88.8	67.2	45.5	24.5	2100

Note. Full load means rated kv-a. output.

Fig. 180 represents the simplest possible connections of a constant-current transformer to the power supply and to the load circuit. Lightning arresters (see Fig. 175 in First Course, and Art. 89 of this book) are considered to be an essential part of the equipment because the external circuit is out of doors. Instrument cases are grounded to avoid potentials on them which might endanger the life of the station attendant who touches them. When the secondary circuit contains a very large number of lamps it is sometimes considered preferable to divide it into two circuits, and the primary and secondary coils may also be divided into two parts each. This is shown in Fig. 179, which corresponds to Fig. 178. Either external circuit may be "killed" by inserting the plug in the switch which short-circuits that series

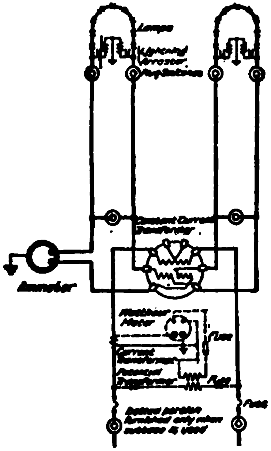


FIG. 179. The connections for a constant-current transformer like that in Fig. 178, having double coils and supplying two series circuits. *The General Electric Co.*

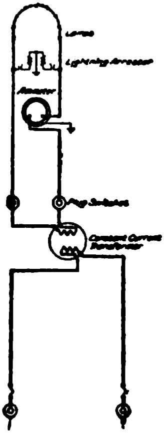


FIG. 180. Simple connections for a constant-current transformer with a single coil and supplying but one circuit. *The General Electric Co.*

and then removing the plugs from the switches which disconnect the line wires from the short-circuited secondary.

Prob. 67–4. How many series incandescent lamps, each rated 250 candle-power, 170 watts, 25.7 volts, 6.6 amperes, spaced 200 feet apart and connected by No. 6 copper wire, can be supplied by a 16-kv-a. transformer whose characteristics are as given in Table VI? Consider the lamps and line to be non-inductive. Frequency 60 cycles.

Prob. 68–4. If the transformer of Prob. 67 is fed from 2300-volt constant potential mains, calculate: (*a*) Secondary terminal voltage at full load (rated size being considered to refer to kv-a. output). (*b*) Primary amperes input at full load.

Prob. 69–4. Assuming the load of the 4-kv-a. transformer of Table VI to be entirely non-inductive, draw curves having watts in external circuit as abscissas, and as ordinates the following: (*a*) Secondary terminal voltage at 4 amperes; (*b*) magnetizing component of primary current, in amperes. Primary voltage is 2300.

Prob. 70–4. From the data given for the 12-kv-a. transformer in Table VI, assuming the load to have unity power-factor, calculate the following: (*a*) total watts loss at 1, $\frac{3}{4}$, $\frac{1}{2}$ and $\frac{1}{4}$ times rated load; (*b*) constant or copper loss; (*c*) variable or iron loss at 1, $\frac{3}{4}$, $\frac{1}{2}$ and $\frac{1}{4}$ times rated load.

Prob. 71–4. From data given in Table VI for the 8-kv-a. transformer, assuming it to be rated and adjusted for 4 amperes secondary and 2200 volts primary, calculate the equivalent primary reactance due to magnetic leakage at 1, $\frac{3}{4}$, $\frac{1}{2}$ and $\frac{1}{4}$ times rated load. Power-factor of load is unity.

65. Instrument Transformers. Series Transformers. In alternating-current systems the measuring instruments (as voltmeters, ammeters, wattmeters, watt-hour meters, power-factor meters) and the limit devices (as relays for operating circuit breakers to guard against overload, low voltage and reverse current or power) are not usually connected directly to the power circuit, but are linked thereto by "instrument transformers," more commonly known as "potential transformers" and "current transformers." The most important function of these transformers is to insulate electrically from the high-tension circuit the measuring instrument and the relays, which are on the switchboard and may be touched by the attendants. Aside from consideration of the personal safety of the operators, it would be too expensive and usually impracticable to insulate the parts of the instruments for high voltage. It is preferable to put the insulation into instrument transformers and to use instruments insulated for low voltage only. Furthermore, by use of instrument transformers of various ratios we are enabled to standardize our equipment of instruments, using generally 110-volt voltmeters, and 5-ampere ammeters or wattmeters, through transformers of suitable ratio, to read pressures, currents, or powers of any magnitude.

The method of using such transformers is illustrated in Fig. 179. A potential transformer has its high-tension coil shunted across the primary mains at, say, 2300 volts. The low-tension coil, probably 110 volts, connects to the pressure coil of the watt-hour meter. (We might also connect **in parallel** to the same transformer a voltmeter and the pressure coil of a wattmeter or a power-factor meter.) In series with the supply mains shown in Fig. 179 is a current transformer, having its secondary connected to the current coil of the watt-hour meter (we might also connect, **in series** to the same current-transformer secondary, an ammeter and the current coil of a wattmeter or of a power-factor meter). Notice that the low-tension circuits of instrument transformers, as well as the cases of instruments, are usually furnished with a good electrical connection to earth.

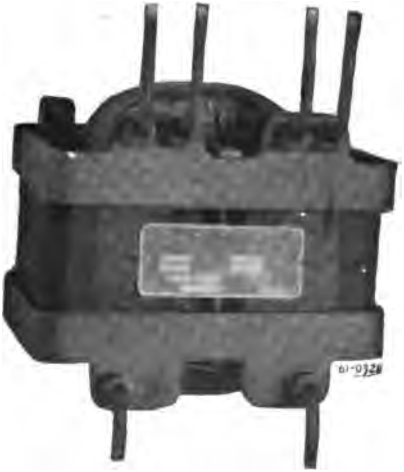


FIG. 181. 200-watt potential transformer. *Wagner Electric Co.*

Instrument transformers are of small power capacity,— from 15 to 200 volt-amperes, depending upon how many instruments are to be operated or upon the amount of power required to operate relays. The voltage ratio of potential transformers, or the current ratio of current transformers, usually remains practically constant for all voltages or currents, respectively, under the rated value, and for outputs less than rated wattage of the transformer, regardless of what particular instruments may be used on the low-tension side. It is preferable, however, to have each instrument or group of instruments calibrated and used in connection with a particular transformer, if greatest accuracy is desired. Fig. 181 shows a potential transformer of rather large size (200 volt-amperes) wound for 110 volts low-tension with either 1100 or 2200 volts high-tension, depending on whether the two high-tension coils are connected in parallel or in series. It is intended to be bolted to some convenient point of the framework which supports the main conductors.

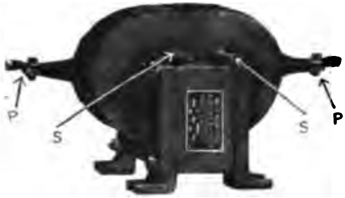


Fig. 182. Stationary current transformer. *Westinghouse Electric and Mfg. Co.*

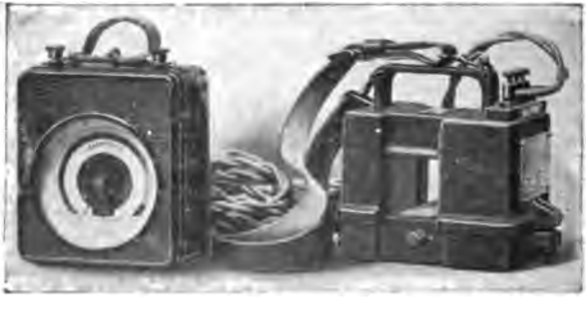


Fig. 183a. Portable series or current transformer. *Westinghouse Electric and Mfg. Co.*

Fig. 182 shows a current transformer, of which the primary *PP* is connected in series with the high-tension conductors and the secondary terminals *SS* are connected to ammeters, relays and current coils of wattmeters, all in series with one another (**never** in parallel). Fig. 183a and 183b shows a portable testing outfit consisting of a "split-type" current transformer connected to an ammeter. A part of the magnetic circuit is cut out and attached to the remainder by a hinge, so that the transformer can be opened and clamped around any cable or bus bar, permitting the current to be measured without disturbing the circuit in any way. If the primary circuit passes once through the hole in the core, it is equivalent to a single turn starting from the generator or bus bar and returning thereto; if the conductor passes through the core twice, it is equivalent to a primary of two turns, and so on. Although convenient to use, the split type has poor characteristics.

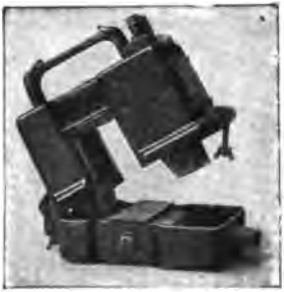


FIG. 183b. Transformer of Fig. 183a shown ready for inserting cable.

When we are measuring only amperes or volts, the accuracy depends principally upon the constancy of the **ratio** of the current transformer or of the potential transformer. When these transformers actuate **wattmeters,** however, a factor of even greater importance than the ratio is the **phase difference** between primary and secondary terminal voltages of the potential transformer, or between primary and secondary currents in the current transformer. The calibration curves of Fig. 184 refer to the type of current transformer illustrated in Fig. 182 and show that the variations from the ideal condition of constant ratio and zero (or 180°) phase displacement, though small, are quite appreciable. The transformers are often "compensated" (adjusted to give exact ratio and 180° phase displacement between primary and secondary) at about 65 per cent of their rated capacity. In the case of the current transformer, particularly, the ratio of transformation for currents is not equal to the ratio of turns in the primary and secondary coils.

It is important to know how the current transformer adjusts its secondary current to the current flowing in the primary or main circuit. Necessarily it is designed so that the resistances and reactances of primary and secondary coils are very low; thus, when the secondary is connected through meters and relays whose impedances are low, the voltage drop across the primary is so small that it has no effect upon the primary current flowing, the value and wave form of which depend, therefore, upon the load itself and upon the generator. A given number of amperes flowing in the line will produce a certain number of ampere-turns of m.m.f. acting in the primary of the transformer. A flux will be produced, generating an e.m.f. in the secondary coil, which produces a current in the secondary circuit if it is closed, as it **always** should be. As in the secondary current increases, the m.m.f. due to it increases, and as this m.m.f. is opposed to the primary ampere-turns, only the **vector difference** between the primary and secondary ampere-turns is available to excite or magnetize the core. For any given primary current, therefore, the secondary current will increase only to such value as can be maintained by the secondary e.m.f. induced by the flux produced by the vector difference between the primary ampere-turns and the

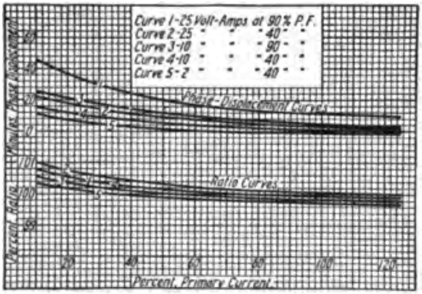


Fig. 184. Characteristic curves of current transformers. The power-factor here referred to is that of the load on the current transformer itself. *Westinghouse Electric and Mfg. Co.*

secondary ampere-turns. When this value has been reached there will be no further tendency for the secondary current to rise, and we shall have equilibrium.

From the foregoing it may be seen that the value of secondary current corresponding to a given value of primary current, or the current ratio of the series transformer, will depend upon:

(*a*) **The impedance of the secondary circuit.** If this impedance be increased, the secondary current cannot increase to as large value as formerly for the same primary current, because a larger secondary e.m.f. must be induced per ampere of secondary current on account of the larger impedance, and therefore there must be a larger flux and larger magnetizing component of primary ampere-turns per ampere of secondary current. The same total primary ampere-turns cannot therefore produce as many secondary ampere-turns as formerly. Incidentally, the flux density in the core must increase as the secondary impedance increases, therefore the core losses will be greater and the temperature of the transformer will rise; also, there will be a greater voltage drop across the primary coil, and, as this is in series with the line, the voltage regulation of the load may be appreciably poorer.

(*b*) **The design of the magnetic circuit.** The flux density must not be allowed to reach excessive values under operating conditions. The cross-section of iron must be relatively larger than demanded by good practice in other types of transformer, to allow for relatively wide variations without approaching the knee of the saturation curve; and the number of turns must be large enough in both primary and secondary so that a small vector difference between the m.m.f.'s due to the currents in these coils will produce relatively large net magnetizing force. If these principles are violated, the current ratio will vary greatly as the primary current varies.

(*c*) **The power-factor of the secondary circuit,** or the ratio between reactance and resistance of instrument coils or other loads connected thereto. The flux, secondary induced e.m.f. and secondary current are produced by the vector difference between the primary ampere-turns and the secondary ampere-turns, and as the ratio of X to R in the secondary circuit increases, the load component and the exciting component in the primary come more nearly into phase with the total primary current, or more nearly into phase with each other. This results in a smaller secondary current for the same total primary current, and therefore in a larger current ratio for the transformer (the ratio of an instrument transformer being understood always to mean the number by which the instrument indication is to be multiplied in order to find the corresponding value for the primary or high-tension circuit).

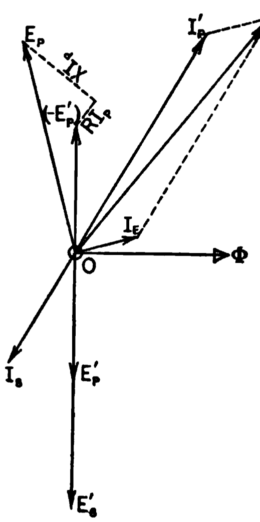


FIG. 185. Current, flux and voltage relations in a 2:1 current transformer.

If the student is unable to construct for himself vector diagrams to illustrate the deductions stated above, Fig. 185 to 188 will assist. Fig. 185 shows the ordinary relations for a current transformer having ratio of 2 : 1 — that is, such that the mains carry 2 times the current that flows through the secondary or the instruments connected in series thereto. When a current I_P amperes is forced through the primary, the flux ϕ automatically adjusts itself as to value and phase (in a way which depends upon the design of the magnetic and electric circuits) in such manner as to induce a secondary e.m.f. E'_S sufficient to force a current I_S against the total resistance and reactance of the secondary circuit. I_S must be such that when the corresponding (equal opposing) ampere-turns (I'_P times number of turns in primary) are subtracted vectorially from the total primary ampere-turns due to I_P, the difference (I_E times number of primary turns) is just sufficient to produce the flux ϕ, according to the design and operating characteristics of the magnetic circuit.

When Fig. 186 is compared with Fig. 185, we see the effects of changing the current I_P in the primary circuit to half of its former value, keeping the secondary circuit unchanged. As the power component of I_E decreases approximately as the square of ϕ or E'_S, while the magnetizing component of I_E (in phase with ϕ) decreases in nearly direct proportion to ϕ, we reason that I'_P will be greater than half its former value, and I_P will be more nearly in phase with ϕ and therefore further out of phase with I_S because I_E is more nearly in phase with ϕ. We see therefore why it is that in Fig. 184 a decrease in primary current increases the current ratio (of I_P to I_S, or of I_P to I'_P) and also increases the phase displacement of I_S (or of I'_P) with respect to I_P.

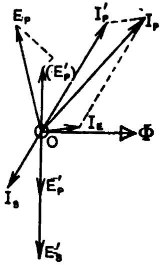


FIG. 186. Current, flux and voltage relations in the current transformer of Fig. 185, when the primary current is halved.

Comparing now Fig. 187 with Fig. 185, we see the effect of increasing the total impedance of the secondary circuit by approximately 50 per cent, with the same ratio of X to R in the secondary and the same current (I_P amperes) in the primary. E'_S must be considerably increased notwithstanding I_S is slightly reduced. While ϕ must be increased in proportion to E'_S, the magnetizing component of I_E must increase approximately in simple proportion to ϕ, and the power component of I_E approximately as the square of ϕ (see Art. 38); therefore I_E will increase relatively somewhat more than ϕ and will be further out of phase with ϕ. As a result, I'_P and I_S will be somewhat less than before for the same value of I_P; the current ratio of the transformer ($I_P \div I_S$) will be greater, and the phase displacement between I_P and I_S will probably be somewhat reduced.

Comparing Fig. 188 with Fig. 185, we see the effect of increasing the ratio of X to R, or lowering the power-factor, in the secondary circuit, while keeping unchanged the total secondary impedance and the primary current I_P. In general the result is that I_S and I'_P are brought into closer phase correspondence with I_E and therefore with I_P, causing the ratio $(I_P \div I_S)$ to be increased and the phase displacement to be reduced. These facts also are verified by the characteristic curves in Fig. 184.

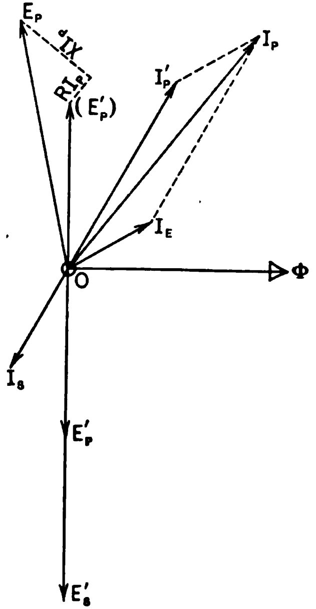


FIG. 187. Current, flux and voltage relations in the current transformer of Fig. 185 when the impedance of the secondary circuit is increased about 50%.

The secondary circuit of a series transformer, especially of an instrument transformer of series type, should never be opened. If changes of connections must be made, first place a short circuit across the secondary terminals. The transformer usually is arranged to give a small value of secondary current for a very much larger value of primary current, — say, in the ratio of 5 amperes to 500 amperes. In this case, the secondary must have nearly (though not exactly) 100 times as many turns as the primary. When the secondary circuit is opened there is no I_S and no counter m.m.f. in the magnetic circuit; consequently, all of the ampere-turns due to I_P go to magnetize the core. Thus, in Fig. 185, I_S and I'_P would be reduced to zero, and I_E would become equal to I_P. The flux would thereby be increased enormously, and as a result of this the secondary induced e.m.f. E_S would likewise be increased enormously — probably to a value great enough to kill the person who handled the secondary. We may also consider that the primary acts merely as a choke coil when the secondary is opened and the counter m.m.f. is reduced to zero; the voltage drop E_P across the primary is increased to a considerable value on account of the increase of ϕ, and for every single volt increase of E'_P, we have 100 volts (in this case) increase of E_S. Moreover, the core losses go up very much faster than the flux and flux density; consequently, the transformer very soon becomes overheated.

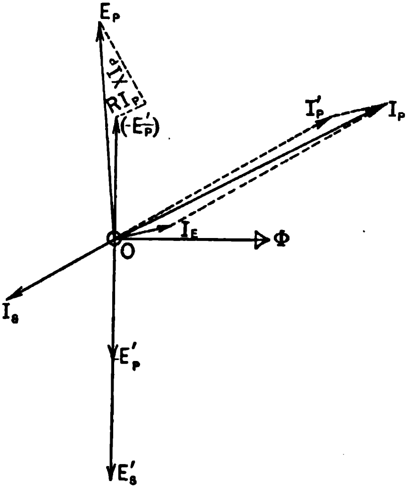


FIG. 188. Current, flux and voltage relations in the current transformer of Fig. 185, when the power-factor of the secondary circuit has been lowered.

Prob. 72–4. A split-type current transformer like that shown in Fig. 183 has a rating of 400 amperes to 5 amperes, when the primary circuit passes once straight through the hole in the core. Approximately what will be the reading on an ammeter with 5-ampere range connected to the secondary terminals, when a primary circuit carrying 80 amperes passes 4 times through the core?

Prob. 73–4. How many times should the primary circuit be passed through the core in the transformer of Prob. 72, in order that an ammeter of 5 amperes range may indicate in the middle of the range when used to measure a current whose value is 100 amperes?

Prob. 74–4. A current transformer of 20 : 1 ratio and a potential transformer of 20 : 1 ratio are used with a wattmeter rated 5 amperes, 150 volts, 750 watts. (*a*) What are the largest permissible values of current and e.m.f. in the primary circuit? (*b*) Neglecting phase displacements due to the transformers, calculate what number of kilowatts will be indicated by the wattmeter when the primary circuit delivers 80 amperes at 2200 volts (single-phase) and 80 per cent power-factor? (*c*) What power in the primary circuit will be thus represented?

Prob. 75–4. A polyphase wattmeter consisting of two distinct single-phase wattmeters combined in one case and using a single moving system is connected to a three-phase three-wire circuit according to the two-wattmeter method for measuring power (see Art. 41, First Course), using instrument transformers. If each of the two pressure coils in the wattmeter is rated 150 volts, and each of the two current coils is rated 5 amperes, what should be the ratio (nearest larger multiple of 5) of each potential transformer and of each current transformer to measure 200 kw. with 6600 volts between line wires, at a power-factor of 85 per cent?

Prob. 76–4. (*a*) If we desire to mark a new scale showing primary kilowatts over the natural or true scale of the wattmeter of Prob. 75, what factor relates the two scales when using these instrument transformers?

(*b*) If the phase displacement in the current transformer is 30 minutes (of electrical or time angle) while the ratio is correct as given in Prob. 75, what would be the **apparent** kw. in the primary circuits when the power delivered is really 200 kw.? Assume the errors of the potential transformer to be negligible.

66. Series Transformer for Lighting Circuits. The number of lamps in a single series street-lighting circuit is commonly made large enough to require 5000 to 7500 volts. Many mishaps have demonstrated that such circuits must be kept well out of the way so that there shall be no likelihood of people touching them. Thus, it is bad practice to bring an underground series circuit up into the base of a lamp-post, where the cut-out block may be touched, or to light a fire-alarm box, a police signal box, or a letter box from a series circuit. Sometimes, also, it is desired to place a few incandescent lights in a building where the nearest available power supply is a series street-lighting circuit. For all such cases it is possible to obtain a low-voltage series circuit taking power from the high-voltage series circuit although electrically insulated therefrom, by means of a specially designed type of series transformer.

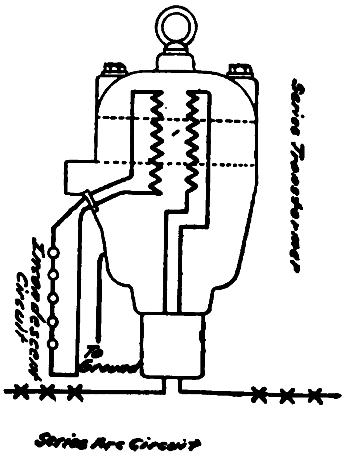


FIG. 189. Series transformer for street lighting. A secondary series current of low and safe voltage is obtained and regulated to constant current by the main or primary circuit although it is electrically insulated therefrom. *The General Electric Co.*

Such series transformers are manufactured in sizes from 40 to 2000 watts to change from one to another of any of the standard values of current for series circuits. Fig. 189 shows an outline of one of these transformers and the electrical connections. The secondary current is maintained in very nearly fixed ratio to the primary current, in the manner explained in Art. 65 for instrument transformers. Consequently the secondary current is maintained as nearly constant as the primary current by the same regulating mechanism, such as a constant-current transformer. However, the secondary circuit is electrically insulated from the primary, and no injury can result to a person touching the secondary, regardless of what conditions may exist on the primary circuit. If the secondary circuit is opened, the voltage is prevented from rising beyond a safe value by reason of the peculiar construction of the magnetic circuit. The sectional area of the core is contracted at several points so that when the counter m.m.f. due to secondary current is reduced to zero, the primary ampere-turns cannot produce an excessive amount of flux on account of saturation of the core at these contracted parts. Thus, the curve in Fig. 190 shows that although the transformer regulates to within about 2 per cent of constant rated current up to normal full load or 100 per cent of rated secondary voltage, the open-circuit voltage of the secondary is not over 155 per cent of normal or full-load voltage.

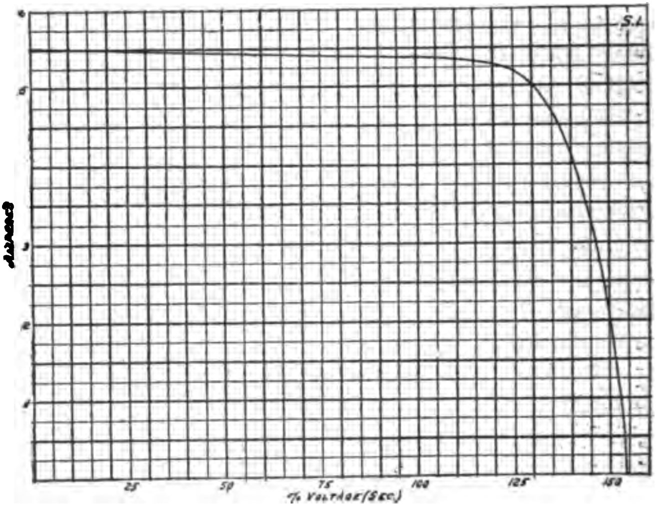


Fig. 190. External characteristics of the transformer of Fig. 189, showing that it regulates within 2 per cent of constant current between full load and no load although the voltage is not over 150 per cent of normal when the secondary circuit is opened. Full-load secondary voltage is taken as normal voltage.

Prob. 77–4. A series arc-lighting circuit is fed from a constant-current transformer the primary of which has one-fifth as many turns as the secondary, and takes power from 2300-volt constant-potential single-phase 60-cycle mains. If you were to break the secondary circuit at any place and hold the broken ends in your hands, what e.m.f. would act upon your body?

Prob. 78–4. A 22-kv-a. constant-current transformer with characteristics as given in Table VI delivers its rated full load at 4 amperes to a single series incandescent-lamp circuit. Considering the secondary circuit to be non-inductive, calculate: (*a*)

Terminal voltage of secondary circuit. (*b*) What e.m.f. would act upon the body of a man who while standing on ground touched a point distant from one end of the line $\frac{1}{8}$ of the total number of lamps, while the line is grounded by rubbing against the limb of a tree at a point distant from the other end of the line $\frac{1}{8}$ of the total number of lamps? The resistance of the latter ground connection is 1000 ohms and the resistance of the man's person from hand to ground is 15,000 ohms.

Prob. 79–4. Assuming that the curve of Fig. 190 represents the performance of a 1000-watt transformer, and that the load is non-inductive, draw to scale curves having as abscissas the wattage in secondary circuit and as ordinates the following: (*a*) Voltage across secondary terminals. (*b*) Resistance of secondary external circuit.

Prob. 80–4. If the primary current of the transformer in Prob. 79 is the same as the normal secondary current, namely 5.5 amperes, calculate what must be the approximate equivalent impedance of the transformer when the secondary circuit is opened.

SUMMARY OF CHAPTER IV

TRANSFORMERS ARE SAID TO BE "BANKED" OR IN PARALLEL when their primaries are connected in parallel to the same line and their secondaries are connected in parallel to the same bus-bars. Lower total kilovolt-ampere capacity is required for the same load and better all-day efficiency is secured than when transformers are operated independently. The cost of low-tension transmission system and the characteristics of the connected apparatus limit the extent to which this banking can be successfully and economically carried. A disadvantage is that an accident to one transformer will generally interrupt the service from the others in the bank.

PROPER CONDITIONS FOR PARALLELING TRANSFORMERS.

(*a*) The ratio of primary to secondary voltage should be the same for all transformers in the bank and the terminals of similar polarity only should be connected together. Otherwise, large local currents will flow whether or not the transformers are connected to a load, and the load will be poorly distributed among the transformers.

(*b*) The percentage of impedance should be approximately the same for all the transformers. Otherwise, the transformers with the lower impedance will be compelled to carry more than their share of the load.

(*c*) The ratio of the resistance to the impedance should be the same for each transformer. Otherwise, a larger transformer kv-a. capacity than that of the load will be necessary. The distribution of a load between two transformers A and B is according to the equation

$$\frac{\text{Kv-a. in } B}{\text{Kv-a. in } A} = \frac{\%\text{ impedance of } A}{\%\text{ impedance of } B} \times \frac{\text{Rated kv-a. of } B}{\text{Rated kv-a. of } A}.$$

THE HIGH-TENSION TERMINALS OF A TRANSFORMER ARE MARKED A and B. The low-tension terminals are marked X and Y with the positive direction from X to Y if the positive direction in the high-tension side is from A to B.

TO TEST POLARITY OF TRANSFORMER TERMINALS connect a low-tension coil in series with a high-tension coil and note by voltmeter reading whether the voltage across the combination is lower or higher than the voltage across the high-tension coil alone. If higher, unlike poles (*B* and *X*, or *A* and *Y*) are connected.

AN AUTOTRANSFORMER has the secondary winding partly in series with the primary winding. Part of the load to the receiving circuit is supplied directly from the supply circuit, the remainder is supplied indirectly through the secondary windings. The ratio of the voltage between the high-tension terminals to the voltage between the low-tension terminals is approximately equal to the ratio of turns in the windings between the respective terminals. In the form of an equation

$$\frac{E_H}{E_L} = \frac{N_H}{N_L}.$$

The ratio of the current in the high-tension coil to the current in the low-tension coil equals the ratio of the low-tension voltage to the high-tension voltage minus the low-tension voltage.

The advantage of autotransformers lies in the greater efficiency and lower cost for the same capacity. These advantages are very marked if the ratio approaches unity.

The chief objection to autotransformers is the fact that greater danger to life and property is incurred because the low-tension coil is electrically connected to the high-tension coil.

TRANSFORMERS ON POLYPHASE SYSTEMS present the following problems:

(*a*) Transformation from 2 or 3 phases at a given voltage into the same number of phases at some other voltage.

(*b*) Transformation from 2 or 3 phases at a given voltage into a different number of phases at the same voltage.

(*c*) Transformation from 2 or 3 phases at a given voltage into a different number of phases at a different voltage.

(*d*) Polyphase transforming systems which cannot be operated in parallel.

(*e*) Systems in which all phases are housed in a single " polyphase transformer " as distinguished from polyphase transforming systems consisting of aggregates of separate single-phase transformers.

TO CHANGE THREE-PHASE AT ONE VOLTAGE INTO THREE-PHASE AT ANOTHER VOLTAGE by the use of three separate transformers, four combinations are possible.

(*a*) High-tension coils *Y*-connected; low-tension coils *Y*-connected.

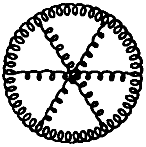
(*b*) High-tension coils *Y*-connected; low-tension coils Δ-connected.

(*c*) High-tension coils Δ-connected; low-tension coils *Y*-connected.

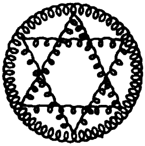
(*d*) High-tension coils Δ-connected; low-tension coils Δ-connected.

It is possible to transform from one three-phase voltage to another three-phase voltage by the use of but two transformers. This is called an OPEN-DELTA OR A *V*-CONNECTION. Each transformer of a *V*-connected group must have 57.7 per cent of the total capacity of the three-phase load when balanced.

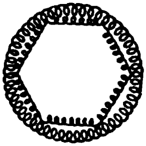
TO TRANSFORM THREE PHASES TO SIX PHASES. Three methods are possible. The first two are in common use for supplying six-ring synchronous converters with power from a three-phase line.



(1) Diametral arrangement.

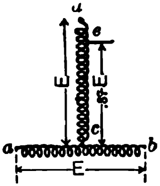


(2) Double-delta.



(3) Ring connection.

TO TRANSFORM TWO-PHASE TO THREE-PHASE or vice versa, two similar transformers with a Scott or T-connection are used. If this diagram represents autotransformers, two-phase power is put in at *ab* and *cd*, while three-phase power is taken out at *ab*, *be* and *ea;* or vice versa.



Scott connection.

The diagram may also represent secondaries of ordinary two-coil transformers.

In the latter case, the transformer *ab* must have capacity equal to 57.7 per cent of the total kv-a. of three-phase power delivered, and the other transformer 50 per cent, total 107.7 per cent.

IF SEVERAL BANKS OF TRANSFORMERS ON THE SAME SYSTEM ARE CONNECTED IN PARALLEL on one side, then to connect the other sides in parallel the connections must be such that the voltage between any two lines on this side will have the same phase in all the banks. From this relation result the following rules:

(*a*) With YY on one bank, the other must be YY or $\Delta\Delta$.
(*b*) With $Y\Delta$ on one bank, the other bank must be $Y\Delta$ or ΔY.
(*c*) With ΔY on one bank, the other bank must be ΔY or $Y\Delta$.
(*d*) With $\Delta\Delta$ on one bank, the other bank must be $\Delta\Delta$ or YY.

Even when these relations are satisfied a short-circuit will result unless the three phases of each bank are connected in the proper sequence. (American Handbook for Electrical Engineers.)

THREE-PHASE TRANSFORMERS ARE CONSTRUCTED by combining parts of the magnetic circuits of three single-phase transformers so as to form a single structure. A saving of approximately 16 per cent in the amount of iron required per kilovolt-ampere of three-phase power transformed and an increase in efficiency of from 0.15 to 0.40 per cent, as well as a very substantial reduction in the floor space required per kilovolt-ampere of transformer capacity, may be accomplished.

There are three types of three-phase transformers, — core-type, shell-type and hexagonal type.

FEEDER VOLTAGE REGULATORS are used to keep the voltage constant within ½ per cent at any one point in a line over a wide range of loads. There are two forms of this device, — the INDUCTION TYPE and the COMPENSATOR TYPE.

In the induction regulator, the current from the line is used for setting up an alternating flux in a movable core. This core can be swung either manually or automatically into such a position that the flux in it induces an alternating e.m.f. in a secondary coil which is in series with the line. This induced e.m.f. can be made to "boost" or "buck" the line voltage sufficiently to keep the terminal voltage constant.

The compensator regulator is merely an autotransformer with the secondary in series with the line. Taps from various points on the secondary allow the amount of "boosting" or "bucking" to be controlled by means of a special switch with sliding contact. This type is more rapid in action and more efficient than the induction type.

A CONSTANT-CURRENT TRANSFORMER attached to a constant-voltage system automatically supplies from its secondary a constant current to a series lighting system, the impedance of which varies from time to time over a wide range. This transformer is so constructed that any increase in current in the load circuit to which it is attached will reduce the mutual flux in the core and thus reduce the secondary voltage and restore the current to practically its former value.

INSTRUMENT TRANSFORMERS are small transformers for stepping down the voltage or the current from the line or bus bars to such values that the power may be used to actuate meters and "limit" devices. A POTENTIAL TRANSFORMER is of the same type as a constant-voltage power transformer and is used to step down the voltage. A CURRENT OR SERIES TRANSFORMER is used to reduce the current. This saves expense in the cost of the measuring instruments, especially in the insulation required in them. The danger to operators is likewise lessened.

The "ratio" of a current transformer is the ratio of primary current to secondary current. Similarly, the "ratio" of a potential transformer is the ratio of primary terminal volts to secondary terminal volts. In either case, the ratio of turns is somewhat different.

The ratio of a current transformer generally decreases as the load (or current) increases. The secondary current is generally out of phase with the primary current by a fraction of a degree, this amount becoming greater as the load decreases, and as the power-factor of the instrument circuit increases. These variations affect principally the accuracy of wattmeter indications. In general the instruments and transformers should be calibrated while connected together as used.

CAUTION. NEVER OPEN THE SECONDARY OF A SERIES TRANSFORMER. IF IT IS NECESSARY TO CHANGE CONNECTIONS, FIRST SHORT-CIRCUIT THE SECONDARY TERMINALS.

SPECIAL SERIES TRANSFORMERS ARE USED on series lighting circuits to lessen the danger from contact with high voltage. By contracting the core at one or two places, it is possible to construct transformers of this type, the secondary of which can be opened without a dangerous rise in the secondary voltage.

PROBLEMS ON CHAPTER IV

Prob. 81–4. A 10-kv-a. and a 20-kv-a. transformer, each of which has ratio (of turns) 5 : 1, are connected in parallel to a motor which takes 30 kv-a. at 440 volts and 0.87 power-factor. The first transformer has resistance 1.0 per cent and impedance 4 per cent; the second transformer has resistance 0.75 per cent and impedance 2 per cent. What kv-a. and current does each transformer deliver, single-phase?

Prob. 82–4. Three single-phase transformers with equal ratios of 10 : 1 are connected in Δ on the high-tension side to a 2300-volt three-wire three-phase line. Each phase of the low-tension windings supplies one of the following loads: (1) 90 kw. at unity power-factor, (2) 60 kv-a. at 0.8 power-factor, (3) 30 kw. at 60 per cent power-factor. Neglecting losses in the transformers, what is the current in each high-tension line wire?

Prob. 83–4. It is desired to transform 200 kv-a. from two-phase at 2300 volts to three-phase at 230 volts by Scott-connected transformers. What should be the current and voltage rating and the ratio of transformation of each transformer?

Prob. 84–4. If three autotransformers be connected in delta to a three-phase line with 2300 volts between line wires, what is the lowest three-phase voltage that may be obtained with symmetrical loading of all wires, and where should each autotransformer be tapped in order to obtain it?

Prob. 85–4. Given three exactly similar autotransformers as follows: A_1B_1 with tap at C_1; A_2B_2 with tap at C_2; A_3B_3 with tap at C_3. Connect A_2 to C_1, A_3 to C_2 and A_1 to C_3. What must be the AC voltage as a percentage of the AB voltage, in order that 440 volts three-phase may be obtained from the junction points A_2C_1, A_3C_2, A_1C_3, when the ends B_1, B_2, B_3 are connected to a 2300-volt three-phase line?

Prob. 86–4. Two identical transformers each rated 50 kv-a. 2300/230 volts and having 2 per cent resistance and 4 per cent impedance, each have additional turns and taps in the secondary coils to give 5 per cent and 10 per cent more than the rated voltage (i.e., 241.5 and 253 volts). In paralleling these transformers, the 105 per cent terminals of the first were accidentally connected to the 110 per cent terminals of the second. Calculate items requested in Prob. 7–4.

Prob. 87-4. Perform the calculations of Prob. 10-4 on the basis of data given in Prob. 86-4.

Prob. 88-4. From Fig. 110, state the conditions which must be fulfilled to bring the terminal voltage E_t into phase with the induced voltages E_1 and E_2.

Prob. 89-4. The 1000-kv-a. transformer of Example 7 (Chap. III) is connected in parallel with another transformer, rated 500 kv-a., 60 cycles, 110,000/22,000 volts. The constants of the latter transformer, obtained from test, are as follows: Impedance volts, 2.5 per cent; total equivalent IR at full load, 0.5 per cent; impedance watts, 3000. What will be the kv-a. load on the former when the latter is delivering its rated load?

Prob. 90-4. How many henrys of inductance must be connected in series with the low-tension coil of each transformer in Example 1, Chap. IV, in order that no part of the kv-a. capacity of any transformer shall be wasted, or that the full kv-a. capacity of all transformers shall be available at secondary mains?

Prob. 91-4. The fuse on transformer A of Example 1 blows out at 25 per cent overload. At this time what is the total kv-a. taken from secondary mains, and what per cent of its rated load will each of the remaining transformers immediately be called upon to carry?

Prob. 92-4. The transformer of Prob. 24-4 is connected as follows: A_1 to H_1, B_1 to A_2, B_2 to X_1 and L_1, Y_1 to Y_2, X_2 to L_2 and H_2. Answer the questions of Prob. 25-4, assuming the same core flux.

Prob. 93-4. The transformer of Prob. 24-4 is connected as follows: A_1 to H_1, B_1 to A_2, B_2 to X_1, Y_1 to X_2 and L_1 and H_2, Y_2 to L_2. Answer the questions of Prob. 25-4, assuming the same core flux.

Prob. 94-4. The transformer of Prob. 24-4 is connected as follows: A_1 to H_1, B_1 to X_1 and L_1, Y_1 to X_2, Y_2 to L_2 and A_2, B_2 to H_2. Answer the questions of Prob. 25-4, assuming the same core flux.

Prob. 95-4. The transformer of Prob. 24-4 is connected as follows: A_1 to H_1, B_1 to X_1 and X_2 and L_1, Y_1 and Y_2 to L_2 and A_2, B_2 to H_2. Answer the questions of Prob. 25-4, assuming the same core flux.

Prob. 96-4. The transformer of Prob. 24-4 is connected as follows: A_1 and A_2 to H_1, B_1 and B_2 to L_1 and X_1, Y_1 to X_2, Y_2 to L_2 and H_2. Answer the questions of Prob. 25-4, assuming the same core flux.

Prob. 97–4. State all the voltage ratios that are possible to obtain with the transformer of Prob. 24–4 and the corresponding largest kv-a. output that may be delivered to secondary mains without endangering the transformer in any way. Do not include bucking combinations.

Prob. 98–4. Draw vector diagrams in accordance with the conventions indicated in Fig. 118 to 126, corresponding to the following connections between three exactly similar single-phase transformers (H_1, H_2 and H_3 represent high-tension line wires; L_1, L_2, L_3 represent low-tension mains): B_1 to B_2 to B_3; A_1 to H_1, A_2 to H_2, A_3 to H_3; X_1 to X_2 to Y_3; Y_1 to L_1, Y_2 to L_2, X_3 to L_3.

There being 2300 volts between any two high-tension line wires, and 10 : 1 ratio in each transformer, calculate the voltages L_1 to L_2, L_2 to L_3, L_3 to L_1.

Prob. 99–4. Give answers requested in Prob. 98–4, but on basis of the following connections: A_1 to A_2 to B_3; X_1 to X_2 to X_3; B_1 to H_1, B_2 to H_2, A_3 to H_3; Y_1 to L_1, Y_2 to L_2, Y_3 to L_3.

Prob. 100–4. Give answers requested in Prob. 98–4, but on basis of the following connections: B_1 to B_2 to H_2; A_2 to A_3 to H_3; B_3 to A_1 to H_1; Y_1 to Y_2 to Y_3; X_1 to L_1, X_2 to L_2; X_3 to L_3.

Prob. 101–4. Give answers requested in Prob. 98–4, but on basis of the following connections: B_1 to A_3 to H_1; B_3 to A_2 to H_2; B_2 to A_1 to H_3; X_1 to Y_2 to X_3; Y_1 to L_1, X_2 to L_2; Y_3 to L_3.

Prob. 102–4. The high-tension coils of the transformers specified in Prob. 98–4 are connected as follows to the three-phase 2300-volt line: B_1 to B_3 to H_1; A_3 to A_2 to H_2; B_2 to A_1 to H_3. Specify two proper methods for connecting the secondaries in Δ, and draw the corresponding vector diagrams according to the conventions indicated in Fig. 122 to 126.

Prob. 103–4. Specify two proper methods for connecting in Y the secondaries of three transformers whose primaries are connected as in Prob. 102–4 and draw the corresponding vector diagrams.

Prob. 104–4. The transformers of Fig. 137 take power from a 13,000-volt three-phase line to drive a six-ring converter delivering 600 volts at d-c. terminals, zero load. Calculate e.m.f. across each high-tension and each low-tension coil.

Prob. 105–4. Answer the questions of Prob. 104 with relation to the double-delta connection of Fig. 138 for the same converter.

Prob. 106–4. Draw a vector diagram to illustrate the e.m.f. relations that would be obtained if, with connections completed only to L_1, L_3, L_5, X_2Y_4 were to be connected directly to L_4 in Fig. 138b. Calculate the values that would then be obtained for the e.m.f.'s from X_4Y_6 and X_6Y_2 to converter rings L_2 and L_6.

Prob. 107–4. If in Fig. 138, with connections completed properly only to L_1, L_3, L_5, X_4Y_6 is connected to L_6, calculate the e.m.f.'s from X_2Y_4 and X_6Y_2 to rings L_2 and L_4.

Prob. 108–4. If the connections of the second delta to the converter are begun incorrectly as represented at A in Fig. 139, what will be the e.m.f. (expressed as percentage of the delta voltage) existing between the remaining points which are supposed to be connected together, namely L_6 to X_2Y_4 and L_4 to X_4Y_6?

Prob. 109–4. Six unmarked wires which are supposed to belong to a six-phase system, come to you through a conduit. Describe how you would proceed to determine, by means of voltmeter only, whether a correct six-phase system can be obtained from these wires, and how each wire should be numbered so that the consecutive phases should be between consecutively numbered wires.

Prob. 110–4. Having determined that six-phase can be obtained from the six wires of Prob. 109–4, and having tagged these wires with numbers 1, 2, 3, 4, 5 and 6 in proper sequence, describe how you would proceed to connect them to the six rings of a converter so as to avoid possibility of mishap.

Prob. 111–4. Find whether it is possible to connect together the six secondary coils of Fig. 133 and 134 in such manner as to obtain a two-phase system — i.e., two equal voltages with 90° phase difference between them. If it can be done, specify the connection and draw the corresponding vector diagram.

NOTE. Each transformer in Prob. 112, 113, 114, 115, 116, 117 and 118 has separate coils for primary and secondary. Not autotransformers.

Prob. 112–4. We desire to draw 150 kv-a. three-phase at 220 volts from a two-phase 2200-volt line by means of two single-phase transformers, with secondaries tapped for T-connection as shown in Fig. 140. Assuming that two (2) volts are induced in each turn of every coil, draw a complete sketch of connections and mark the number of turns in each coil or part of coil.

Prob. 113–4. Calculate: (*a*) the number of amperes flowing in each coil or part of coil in Prob. 112–4, (*b*) the kv-a. rating of each transformer, and (*c*) total kv-a. rating of transformers required.

Prob. 114–4. If coils *ab* and *cd* in Fig. 140 are the secondaries of transformers each of which has a rating of 100 kv-a., how many kv-a. of single-phase load can be taken in phase A_3, phases B_3 and C_3 being unloaded?

Prob. 115–4. If coils *ab* and *cd* in Fig. 140 are the secondaries of transformers each of which has a rating of 100 kv-a., how many kv-a. of single-phase load can be taken in phase B_3, phases A_3 and C_3 being unloaded?

Prob. 116–4. If coils *ab* and *cd* in Fig. 140 are the secondaries of transformers each of which has a rating of 100 kv-a., what total number of kv-a. equally divided between phases *be* and *ea* may be taken from the three-phase terminals, phase *ab* being unloaded? Power-factor of loads is 100 per cent.

Prob. 117–4. Solve Prob. 116–4 on the assumption that the total output is equally divided between phases *ab* and *be*, at unity power-factor, the third phase *ea* being unloaded.

Prob. 118–4. The three-phase terminals of the transformers of Prob. 116–4 are loaded as follows: Phase *ab*, 50 kv-a. at 87 per cent power-factor; Phase *be*, 50 kw. at 80 per cent power-factor. How many kw. at 60 per cent power-factor can be taken from phase *ea* without causing the current to be excessive in any part of either transformer?

Prob. 119–4. The loads on the system of Fig. 147, using transformers as specified in Prob. 46–4 and 47–4, are as follows: $I_1 = 2$ kv-a. at 90 per cent power-factor, $I_2 = 2$ kv-a. at 80 per cent power-factor. How many kv-a. at 60 per cent power-factor may be taken at I_3?

Prob. 120–4. Using the data of Prob. 47–4 under the conditions of Prob. 119–4 calculate the voltages H_1H_2, H_2H_3 and H_3H_1 respectively.

Prob. 121–4. The loads on the system of Fig. 147, using transformers as specified in Prob. 46–4 and 47–4, are as follows: $I_1 = 2$ kv-a. at 90 per cent power-factor, $I_2 =$ zero. Assume 2300 volts on each phase, high-tension. (*a*) How many kv-a. may be taken at I_3 at 90 per cent power-factor? (*b*) At 60 per cent power-factor?

Prob. 122–4. Calculate the voltages E_1, E_2, E_3 between secondary mains under the conditions of part (*a*) of Prob. 121–4.

Prob. 123–4. Calculate the voltages E_1, E_2, E_3 between secondary mains under the conditions of part (*b*) of Prob. 121–4.

Prob. 124–4. Three-phase core-type transformers cannot be operated three-phase with a short-circuit on any phase. Explain:

(*a*) What would happen if it were so operated. (*b*) The reason for the difference of behavior between core-type and shell-type three-phase transformers in this respect.

Prob. 125–4. In assembling the transformer of Fig. 163, phase No. 3 is accidentally reversed, B_3 being connected to neutral with A_1 and B_2. What fluxes will thereby be changed in value, and by what percentage of their normal values respectively?

Prob. 126–4. In assembling the transformer of Fig. 160, phase No. 3 is accidentally reversed, B_3 being connected to neutral with A_1 and A_2. What fluxes will be thereby changed in value, and by what percentage of their normal values respectively?

Prob. 127–4. On account of failure in phase No. 3 of Fig. 160 this coil (NB_3) is disconnected and short-circuited upon itself, while the other connections and the line voltages remain unchanged. Describe, quantitatively where possible, any changes which will occur in fluxes or exciting currents.

CHAPTER V

SHORT TRANSMISSION AND DISTRIBUTING LINES

The fundamental ideas in the transmission and distribution of electrical power may best be understood by considering an actual installation. Thus let us suppose that a town can utilize 1200 kw. at 0.80 power-factor for 3000 hours per year, and that it is situated ten miles from water power. Naturally, at this source of power is the most economic place to locate the generating station. Our problem then is: What is the most practicable electric transmission system to install and what are the main characteristics and peculiarities of the system?

66. Most Economical Size of Wire, Single-phase Line.

1. Voltage. The choice of voltage is more or less arbitrary, — the engineer always trying to use as high a voltage as conditions permit. Better methods of insulating both line and machines are continually raising the voltage at which power may be most economically transmitted over given distances. The expense of insulating the line and the apparatus connected to it increases rapidly as we choose higher voltages. Of course, the cost of the copper in the circuit goes down rapidly at the same time, as the high voltages enable us to transmit the same power with the same loss over smaller wires. Above a certain voltage, the difficulty and expense of insulating increases faster than the cost of the copper decreases, so that the total cost of transmission would be increased by raising the voltage any further. The limiting pressure at which this occurs is being continually raised by improvements in the manufacture of insulation, which make good insulation cheaper, or insulation of a given cost much stronger. At the present time this economical limit is about 140,000 volts, but there are prospects that it will go higher.

An old "thumb rule" which may be used for moderate distances is "**1000 volts to the mile.**"[*] That is, a two-mile line would be constructed to operate at approximately 2000 volts, a ten-mile line at 10,000 volts, etc. The following voltages have become standardized by practice:

2200 to 2400, 6000 to 6900, 11,000, 13,200 to 13,800, 22,000 to 24,000, 33,000, 44,000, 66,000, 88,000, 110,000, 140,000 to 150,000.

Accordingly, we may choose 11,000 volts as a practicable pressure at which to transmit power over the 10 miles required by this problem.

2. Alternating or Direct Current. In America there are practically no industrial direct-current systems of a voltage higher than 600 volts. This fact leads to the choice of alternating current.

3. Frequency. There are at present two standard frequencies in this country, 25 cycles and 60 cycles. Installations of 60 cycles are more numerous, but the tendency is toward the lower frequency on account of the following advantages:

(1) Lower iron losses in the generating machines as pointed out in Art. 4 and 38 of this book.

(2) Less "charging current" taken by the transmission line as will be explained in Chapter VI.

[*] This rule apparently is based on the fact that 1000 volts per mile is the most economical voltage, allowing 2 per cent line drop, 10 per cent interest and depreciation, 3000 hours use per year, and line of copper wire at a cost per pound of 14 times the cost of power per kw-hr. A voltage-distance table of actual installations taken at random is given below.

Distance, miles.	Voltage.	Distance, miles.	Voltage.	Distance, miles.	Voltage.
7	11,000	8	11,000	6	11,000
44	50,000	29	50,000	84	50,000
144	100,000	108	100,000		

For a full discussion of the choice of voltage see Still, "Overhead Electric Power Transmission."

(3) Lower speed for machines of the same number of poles as explained in Art. 65. (First Course.)

However, because at present it is the more common frequency for electric-lighting loads on account of the freedom from flickering, we will select 60 cycles. We must then employ at the receiving end the frequency of the central station, unless we wish to use an auxiliary machine known as a "frequency changer."

4. Size of Line Wire. Single-phase. Having decided to use an alternating-current system of 60 cycles and 11,000 volts, the size of the conductors will depend upon whether we employ a single-phase or a three-phase system. It will be shown later that the three-phase system possesses a great advantage over the single-phase, and this would probably be chosen. However, for the sake of completeness, we will consider the single-phase first, and then take up the three-phase system and compare the two.

The choice of voltage, frequency and size of wire is determined primarily by the amount of capital that must be invested in the plant to install and to operate it. This is most clearly seen in the method in common use to determine the size of wire. **The conductor should always be of such a size that it results in the lowest total annual expense.** This annual expense consists of two items:

(1) **Fixed charges,** which include

(*a*) Interest on the money invested in the line.

(*b*) Taxes and depreciation in the value of the line.

(2) **Value of Energy lost in line.**

Perhaps the clearest plan is to tabulate, as in Tables I, II and III, the cost of transmitting by several sizes of wire, under identical conditions, and to pick out the size showing the lowest total cost.

Our problem, then, is to select the most economical size of copper wire to transmit 1200 kw., single-phase, 80 per cent power-factor, at 11,000 volts at the receiving end. This power is to be delivered steadily for 3000 hours per year. These figures are fairly representative of modern practice.

The length of the line is 10 miles, the length of wire required, 20 miles.

Let us start with a No. 000 gauge, stranded copper wire, and compute the cost. According to the Wire Table I, Appendix B, 20 miles of this wire would weigh

$$20 \times 2740 = 54{,}800 \text{ lbs.}$$

The cost of wire for a transmission line (except for very large sizes) is proportional to the weight of the conductor. The cost per pound for installed wire would depend upon market price of metals, which fluctuate widely, and also upon cost of transportation and of labor. Assume that conditions are such that 20 cents per pound installed would be a fair average price.

The cost of No. 000 conductor would then be

$$54{,}800 \times \$0.20 = \$10{,}960.$$

On this sum we must allow yearly interest at 5 per cent; in addition, the annual taxes and depreciation would approximate 3.5 per cent.

The **fixed charges** would then amount to $3.5 + 5 = 8.5$ per cent.

$$0.085 \times \$10{,}960 = \$932.00.$$

The power loss in the line would be the I^2R loss of the line.

The apparent power delivered would be

$$\frac{1200}{0.8} = 1500 \text{ kv-a.}$$

The current (with pressure of 11,000 volts at the load end of the line) would be:

$$\frac{1{,}500{,}000}{11{,}000} = 136.4 \text{ amp.}$$

The resistance (from Table I, Appendix B) of 20 miles of No. 000 wire

$$= 20 \times 0.328 = 6.56 \text{ ohms.}$$

$$\begin{aligned}\text{Power loss} &= I^2R \\ &= 136.4^2 \times 6.56 \\ &= 121{,}900 \text{ watts} \\ &= 121.9 \text{ kw.}\end{aligned}$$

The total energy lost per year would then amount to

$$121.9 \times 3000 = 366{,}000 \text{ kw-hr.}$$

At a conservative estimate of 1 cent per kw-hr, the energy lost in the line per year would cost

$$366{,}000 \times \$0.01 = \$3660.00.$$

The total cost per year due to the transmission line would be the sum of the fixed charges and the line loss.

$$\$932.00 + 3660 = \$4592.00.$$

If we compute in the same way the annual cost of a No. 0000 transmission line, we find (see Table I) that, while the cable costs more and thus the interest, taxes and depreciation are more every year, the cost of the power lost in the line is so much less that the total annual cost falls to $4069.

TABLE I

RELATIVE COSTS OF TRANSMITTING POWER OVER CONDUCTORS OF DIFFERENT SIZES, OTHER CONDITIONS BEING THE SAME

	No. 000 stranded.	No. 0000 stranded.	250,000 cir. mils.	300,000 cir. mils.	350,000 cir. mils.	400,000 cir. mils.	450,000 cir. mils.
	$	$	$	$	$	$	$
Cost of 20 miles of wire at 20¢ per lb.	10,960	13,880	16,360	19,560	22,960	26,280	29,880
Annual fixed charges at 8.5% of wire cost	931	1,179	1,390	1,663	1,950	2,233	2,540
Annual cost of energy lost in line at 1¢ per kw-hr.	3,660	2,890	2,416	2,064	1,774	1,533	1,365
Total annual cost of transmission	4,591	4,069	3,806	3,727	3,724	3,766	3,905

As we continue to increase the size of the wire, the fixed charges continue to increase and the cost of power lost in the line continues to decrease. However, the fixed charges increase more rapidly than the cost of line loss decreases, so that the total expense does not continue to decrease indefinitely, but reaches a minimum value at a definite size of wire. The total annual cost of any wire smaller than this size will be greater.

TABLE II

Relative Costs of Transmitting Power, Using Conductors of Different Sizes

Cost of copper being double that of Table I

	No. 000 stranded.	No. 0000 stranded.	250,000 cir. mils.	300,000 cir. mils.	350,000 cir. mils.	400,000 cir. mils.	450,000 cir. mils.
	$	$	$	$	$	$	$
Cost of 20 miles of wire at 40¢ per lb.........	21,920	27,760	32,720	39,120	45,920	52,560	59,760
Annual fixed charges at 8.5% of wire cost..	1,862	2,358	2,780	3,325	3,900	4,466	5,080
Annual cost of energy lost in line at 1¢ per kw-hr...............	3,660	2,890	2,416	2,064	1,770	1,533	1,365
Total annual cost of transmission........	5,522	5,248	**5,196**	5,389	5,670	5,999	6,445

TABLE III

Relative Cost of Transmitting Power, Using Conductors of Different Sizes

Cost of energy double that of Table I

	No. 000 stranded.	No. 0000 stranded.	250,000 cir. mils.	300,000 cir. mils.	350,000 cir. mils.
	$	$	$	$	$
Cost of 20 mi. of wire at 20¢ per lb...............	10,960	13,880	16,360	19,560	22,960
Annual fixed charge at 8.5% of wire cost........	931	1,179	1,390	1,663	1,950
Annual cost of energy lost in line at 2¢ per kw-hr...	7,320	5,780	4,832	4,128	3,540
Total annual cost of transmission	8,251	6,959	6,222	5,791	5,490

	400,000 cir. mils.	450,000 cir. mils.	500,000 cir. mils.	550,000 cir. mils.
	$	$	$	$
Cost of 20 mi. of wire at 20¢ per lb.	26,280	29,880	32,840	36,080
Annual fixed charges at 8.5% of wire cost......................	2,233	2,540	2,790	3,070
Annual cost of energy lost in line at 2¢ per kw-hr..................	3,066	2,730	2,478	2,230
Total annual cost of transmission .	5,299	5,270	**5,268**	5,300

According to Table I, the wire which shows the smallest annual cost, $3724, is one of 350,000 cir. mils area. The next smaller, 300,000 cir. mils, would cost $3727 per year, while the next size larger, 400,000 cir. mils, would cost $3766 per year.

It will also be seen from a study of the three tables that the lowest total annual expense always occurs when the fixed charges on the line become most nearly equal to the yearly cost of energy lost in the line. It will be noted that the fixed charges exceed the cost of energy lost for all sizes larger than the most economical; but for all sizes smaller than the most economical, the cost of lost energy exceeds the fixed charges. By making use of the fact that the fixed charges should about equal the cost of lost energy we may arrive at the most economical size without many trials.

Note from a comparison of Tables I and II and Curves I and II in Fig. 191 drawn from the data in the tables, that a higher price for copper causes it to be more economical to use a smaller wire. In this case, Table II is for copper wire at double the price used in Table I. Doubling the price of copper makes it more economical to use a 250,000 cir-mil wire, instead of a 350,000 cir-mil wire. The total annual expense, however, is higher than when copper is cheaper.

Note from a comparison of Tables I and III and Curves I and III, that a higher price per kilowatt-hour for energy makes it more economical to use a conductor of larger size, — doubling the energy cost makes it more economical to use a wire of 500,000 cir. mils rather than the one of 350,000 cir. mils. Here again the lowest annual expense is greater than when power is cheaper.

The Tables and the Curves of Fig. 191 show that there is no practical need of very precise computations for the determination of the most economical size of wire. The curves are all very flat in the region of the most economical size, and a choice of any one of two or three sizes in the vicinity of the minimum cost will not result in any appreciable

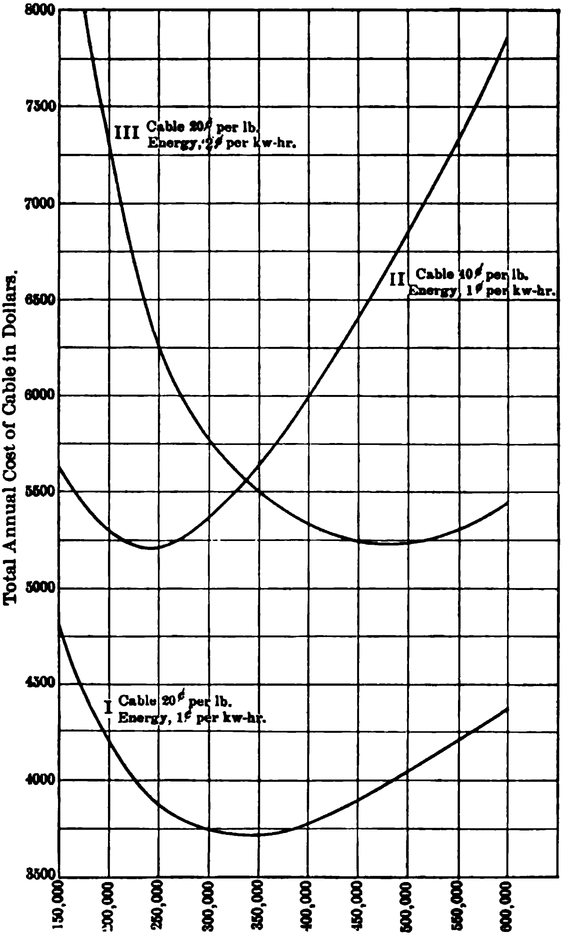


FIG. 191. Relation between the size of the cable and the total annual cost charged to the transmission line. Note the flatness of the curves at the lowest points, and the increased steepness of the parts farther away from the lowest points.

difference in the total cost. However, as we go farther away on either side of the most economical size, the cost begins to increase rapidly. It is worth while, therefore, to make the above approximate calculations. Furthermore, the above solution, however precisely the mathematics may be done, is only approximate, there being other items of cost to be considered in the final selection of size of wire besides the cost of installed cable and the cost of lost energy. There are also the annual charges on the additional cost of generating and transforming equipment required to handle the power that will be lost in the line, and the fixed charges on the additional cost of poles and insulators due to change in the size of wires. The cost per pound of the wire itself often becomes more for sizes larger than 500,000 cir. mils because each cable is split up into two or more parallel cables for convenience in handling and because heavier pole construction may be required.

The **form of the load curve** also should be taken into account when computing the line loss. The energy annually lost in the line is proportional to the "square root of the mean square" of current delivered throughout the year. For the same average power delivered, therefore, the line loss may vary between wide limits, as we change the daily and annual load curves or schedules.*

Prob. 1-5. If it were decided to transmit the power in the above example at 22,000 volts, what would be the most economical size of wire? Copper conductor in place costs 20 cents per pound. Energy costs 1 cent per kw-hr. and all other data the same as in the above example.

Prob. 2-5. If the line in Prob. 1 were to be constructed of aluminum, what would be the most economical size? Compare total annual cost of transmission by the aluminum line, with the corresponding cost by the copper line of Prob. 1. Aluminum weighs 0.304 as much as copper, has 1.61 times the mil-foot resistance and costs 1.7 times as much, say 35 cents per pound, installed.

---

* For a method of determining the annual line loss, see "Standard Handbook for Electrical Engineers," Sec. 12-234.

Prob. 3-5. (*a*) What per cent of the power transmitted is lost in the line in the example in the text, using the most economical size of conductor?

(*b*) In Prob. 1?

(*c*) In Prob. 2?

67. Voltage Drop in Line. Line Regulation. Before we can definitely decide to install a line with conductors of a given size, it is always well to see that the voltage variation at the receiving end is not excessive between a no-load and a full-load condition. When a load is put on a line the voltage across the receiving end falls, on account of the pressure consumed in overcoming the voltage reactions along the line due to the current. It is necessary to define some standard way of stating the magnitude of this loss of voltage. The method is to determine the voltage at the receiving end with non-inductive full load on the line. Then determine the voltage when the load is removed, meanwhile keeping the impressed voltage constant at the sending end of the line. The difference between these voltages is called the **regulation of the line.** The percentage regulation is the percentage which the change in voltage is of the normal rated voltage at the receiving end.

Thus, per cent line regulation

$$= \frac{(\text{No-load volts}) - (\text{full-load volts})}{(\text{Full-load volts})} \times 100 \text{ per cent}$$

(at unity power-factor).

Unless otherwise stated, the **load must be non-inductive** and the **voltage at the sending end** of the line must **remain unchanged.**

Good regulation for a power load ranges between 5 and 10 per cent. For a lighting load it should never exceed 5 per cent and does not usually exceed 3 per cent.

Let us test the regulation of the 10-mile line of Example 1, using the most economical copper wire, — 350,000 cir. mils.

If the power were direct-current, the process would be simple, as follows:

Voltage at full load = 11,000 volts.

Full-load current $\frac{1,200,000}{11,000}$ = 109 amperes.

Resistance of line = 3.18 ohms = 20 × 0.159. (See Table I, Appendix B.)

Line drop = 3.18 × 109.
= 347 volts.

Volts at sending end = 11,000 + 347 = 11,347 volts.

Volts at load end, at no load = 11,347.

Line regulation for direct current

$$= \frac{11,347 - 11,000}{11,000} = 3.2\%.$$

This line, we see, would have a sufficiently good regulation for a direct-current system.

We will now determine the regulation of the same line when it carries alternating current. Alternating current has to overcome, not only the resistance of the line wires, but also their inductive reactance. Thus the line drop in a circuit carrying alternating current is usually greater than if the same circuit were carrying direct current, and therefore the regulation would be poorer. It would be good practice to string the wires of a 11,000-volt circuit about 30 inches apart.* The reactance of the circuit at 60 cycles may be found from the equation given in Chapter V, First Course. It is usually taken, however, from tables computed by means of this equation. From Table III, Appendix B, we find that for 350,000 cir-mil cables strung 30 inches apart:

The reactance per mile of single wire = 0.591 ohms.

The reactance of 20 miles of cable = 20 × 0.591
= 11.82 ohms.

The current for a non-inductive full load $= \frac{1,200,000}{11,000}$
= 109 amperes.

---

* See Curve (I) in Appendix B for good practice as to distance between wires.

$$
\begin{aligned}
\text{The reactance drop then} &= E_x \\
&= 11.82 \times 109 \\
&= 1290 \text{ volts.}
\end{aligned}
$$

There would thus be a line drop of 347 volts, as in direct-current power, due to the resistance of the wires, and a line drop of 1290 volts peculiar to alternating-current power, due to the reactance of the line. The sending voltage must therefore be great enough to supply these line drops and leave 11,000 volts at the load end. Since the reactance drop differs in phase from the resistance drop by 90°, it is necessary to add vectorially the load voltage, the resistance drop, and the reactance drop of the line, to find what the sending voltage must be in order to supply them.

Construct the topographic vector diagram of Fig. 192.

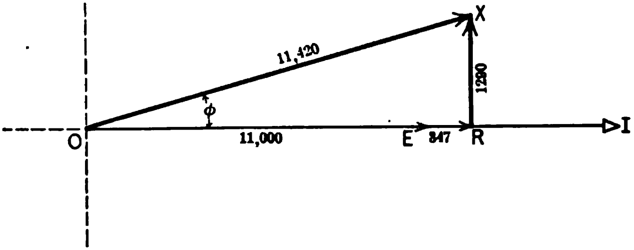


FIG. 192. Topographic diagram for finding the generator voltage *OX* to supply a load voltage of *OE* over a line having a resistance drop of *ER* and a reactance drop of *RX*. The load has unity power-factor.

The voltage vector *OE* of the load will lie along the current vector *OI*, since the voltage and current of the load must be in phase, the power-factor being unity. The vector of the resistance drop, *ER*, must also lie along the current vector *OI*, as resistance drop is always in phase with the current. The vector *RX* of the e.m.f. required to overcome the reactance voltage, must lead the current by 90°. The vector sum of these voltages is the line *OX*, and can be computed from the equation,

$$\begin{aligned}\overline{OX} &= \sqrt{(\overline{OE}+\overline{ER})^2+\overline{RX}^2}\\ &= \sqrt{11{,}347^2+1290^2}\\ &= 11{,}420 \text{ volts.}\end{aligned}$$

The voltage at the sending end must therefore be 11,420 volts and the voltage at the receiving end would of course rise to this value when the load was taken off, there being, at that time, no current and therefore no reacting voltages in the line.

$$\begin{aligned}\text{The regulation} &= \frac{11{,}420-11{,}000}{11{,}000}\\ &= 3.82 \text{ per cent.}\end{aligned}$$

This is slightly higher than the value 3.2 per cent, which we obtained for regulation when the line carried direct current, but it is very good regulation for a line carrying alternating current.

68. Line Regulation at a Power-factor Less than Unity. But the load which we specified for this line in Example 1 had a power-factor of 0.80 which is about the usual power-factor of an alternating-current load used for industrial purposes. (See Art. 6.)

To find the voltage variation and what is sometimes called the "regulation at 0.80 power-factor" we proceed as follows:

$$\text{The apparent power} = \frac{1200}{0.80} = 1500 \text{ kv-a.}$$

$$\text{The current (single phase)} = \frac{1{,}500{,}000}{11{,}000} = 136.4 \text{ amp.}$$

The resistance drop in the line $= 136.4 \times 3.18 = 434$ volts.

The reactance drop in the line $= 136.4 \times 11.82 = 1610$ volts.

Construct the topographic vector diagram Fig. 193.

To represent the 11,000 volts at the load draw the vector *OE* at 37° lead to the current, because on an inductive load of 80 per cent power-factor the voltage leads the current 37°.

To represent the 434 volts which the sending end must supply in order to overcome the resistance of the line, draw the vector *ER* parallel to the current *I*, since the voltage consumed in overcoming resistance only is always in phase with the current.

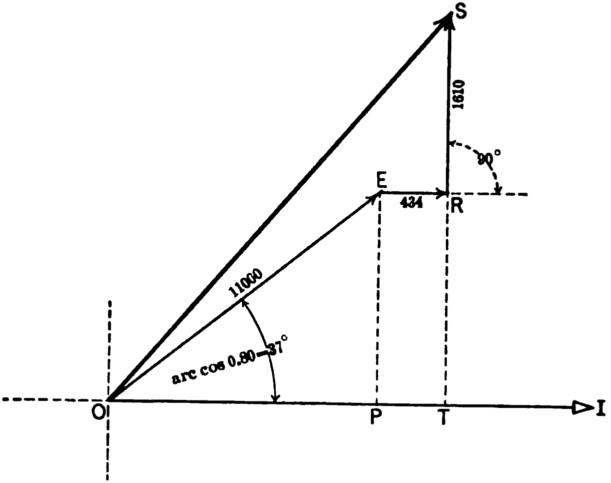


FIG. 193. The vector *OS* represents the sending voltage for a load voltage *OE*, over a line having a resistance voltage of *ER* and a reactance voltage *RS*, when the power-factor of the load is 80 per cent.

To represent the 1610 volts which the sending end must supply in order to overcome the reactance of the line, draw the vector *RS* 90° ahead of *ER*, because the voltage consumed in overcoming reactance is always 90° ahead of the current.

The total voltage at the sending end of the line must therefore be *OS*, the vector sum of *OE* (the voltage at the load), *ER* (the resistance drop) and *RS* (the reactance drop).

When the load is thrown off, the voltage at the receiving end will rise to the same value as the voltage at the sending end.

Thus *OS* also represents the no-load voltage at the receiving end. To find the value of *OS*, draw the construction lines *EP* and *RT*.

$$OP = 11{,}000 \cos 37^\circ = 8800$$
$$RT = 11{,}000 \sin 37^\circ = 6600.$$
$$\begin{aligned} OS &= \sqrt{(OP + 434)^2 + (RT + 1610)^2} \\ &= \sqrt{152{,}570{,}000} \\ &= 12{,}351 \text{ volts.} \end{aligned}$$

The no-load voltage thus equals 12,351 volts.

$$\text{The regulation at 0.80 power-factor} = \frac{12{,}351 - 11{,}000}{11{,}000} = 12.3 \text{ per cent.}$$

Note that the regulation of the line when the load had the commercial power-factor of 0.80 was very much poorer, being more than three times as great as the regulation at unity power-factor. There are two reasons for this. First, in order to transmit at the same voltage the same quantity of power at a low power-factor as at unity power-factor, a large current must flow. This means both greater resistance drop and greater reactance drop. In this case the resistance drop was increased from 347 to 434 volts, and the reactance drop from 1290 to 1610 volts.

In the second place, it will be seen from a comparison of Fig. 192 and 193 that the reactance drop and the voltage of the load are more nearly in phase and therefore their vector sum is more nearly equal to their arithmetical sum when the power-factor is lower. Thus, not only is the reactance drop greater at a low power-factor, but a greater fraction of it is in phase with the line voltage and, therefore, tends to increase it much more than at unity power-factor.

This alone shows the desirability of having a load in which the current lags as little as possible behind the voltage especially when the reactance of the line is greater than its resistance. In fact, a slightly leading current is generally advantageous.

Prob. 4-5. Find the regulation of the line in the above example when the power-factor of the load is 70 per cent with the same kilowatt load.

Prob. 5-5. What would be the line regulation of Prob. 4 if the size of the line wire were increased to 450,000 cir. mils?

Prob. 6–5. What would be the line regulation of Prob. 4 if we decreased the size of the conductor to 250,000 cir. mils?

69. Three-phase, Three-wire System. Cost. Let us now consider how the cost and the regulation of this 10-mile line would be affected if we installed a three-phase, three-wire system instead of a single-phase system with its two conductors. We will, of course, compute the line cost and regulation on the same basis as for the single-phase line, that is, the transmission of 1200 kw., to a distance of 10 miles, with 11,000 volts between wires and a power-factor of 80 per cent at the end of the line with 3000 hr. of use at full load annually.

Apparent power as in single-phase system

$$= \frac{1200}{0.80} = 1500 \text{ kv-a.}$$

Apparent power in three-phase system is found according to the equation

$$P_a = \sqrt{3}\,EI;$$

where P_a = apparent power in volt-amperes.

E = effective voltage between conductors, in volts.

I = effective current along each conductor, in amperes.

Therefore $1{,}500{,}000 = \sqrt{3} \times 11{,}000 \times I.$

$$I = \frac{1{,}500{,}000}{1.73 \times 11{,}000}$$

$$= 78.8 \text{ amperes.}$$

Or we may use the equation

$$P = \sqrt{3}\,EI\cos\theta;$$

where P = effective power, in watts.

E = effective voltage between conductors, in volts.

I = effective current along each conductor, in amperes.

$\cos\theta$ = power-factor of the load.

Thus $$1{,}200{,}000 = \sqrt{3} \times 11{,}000 \times I \times 0.80.$$

$$I = \frac{1{,}200{,}000}{1.73 \times 11{,}000 \times 0.80}$$

$$= 78.8 \text{ amperes.}$$

We now compute, as in the case of a single-phase line, the total annual cost of the line when constructed of conductors of different sizes. Starting with No. 0 stranded copper conductor, we find the cost of line as follows:

One mile of No. 0 weighs 1730 lb.

A ten-mile three-wire system would weigh

$$30 \times 1730 = 51{,}900 \text{ lb.}$$

At 20 cents per pound, the line would cost

$$\$0.20 \times 51{,}900 = \$10{,}380.$$

Annual fixed charges at 8.5 per cent would amount to

$$\$10{,}380 \times 0.085 = \$882.$$

The resistance of one conductor of No. 0 wire, as per Table I, Appendix B, would be

$$10 \times 0.518 = 5.18 \text{ ohms.}$$

Each line wire in the three-phase system carries 78.8 amp., therefore the I^2R loss per conductor would be

$$78.8 \times 78.8 \times 5.18 = 32{,}200 \text{ watts.}$$

$$= 32.2 \text{ kw.}$$

The loss in the three wires would be

$$3 \times 32.2 = 96.6 \text{ kw.}$$

For a year of 3000 hours, the total energy loss would be

$$3000 \times 96.6 = 289{,}800 \text{ kw-hr.}$$

At 1 cent per kw-hr., this would cost

$$289{,}800 \times 0.01 = \$2898.$$

Therefore the total yearly cost of the transmission equals

$$\$2898 + \$882 = \$3780.$$

Since the cost of lost energy is greater than the fixed charges, we compute the cost of larger wires in order to find the size of conductor which produces the lowest total annual cost. Setting these down as in Table IV we see that the

TABLE IV

	No. 0 stranded.	No. 00.	No. 000.	No. 0000.	250,000 cir. mils.	300,000 cir. mils.	350,000 cir. mils.
	$	$	$	$	$	$	$
Cost of 30 miles of wire at 20¢ per lb........	10,380	13,140	16,440	20,820	24,560	29,350	34,440
Fixed annual charges at 8.5% of wire cost..	882	1,117	1,398	1,770	2,090	2,490	2,930
Annual cost of energy lost in line at 1¢ per kw-hr...............	2,898	2,299	1,834	1,450	1,213	1,033	886
Total annual cost of transmission........	3,780	3,416	3,232	3,220	3,303	3,523	3,816

most economical conductor would be No. 0000. The most economical size for a two-wire system under the same conditions was found to be 350,000 cir. mils. (See Table I.)

Three conductors of No. 0000 wire at an annual expense of $3220 will conduct as much power as two conductors of 350,000 cir. mils at a yearly expense of $3724. There is then a decided yearly saving in using a three-phase system instead of a single-phase.

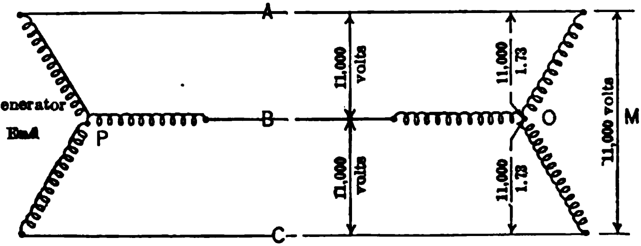


FIG. 194. Diagram of a star-connected generator station sending power over a three-wire line to a star-connected load.

Prob. 7–5. What would be the most economical size of conductor to use in the three-phase system of the above example if the price of copper were 40 cents per pound installed?

Prob. 8–5. If the cost of energy were 2 cents per kw-hr., what would be the most economical size of conductor to use in the above example?

Prob. 9–5. If a three-phase system were installed in Prob. 1–5, what would be the most economical size of conductor?

Prob. 10–5. Compute Prob. 2–5, using a three-wire three-phase system.

70. Regulation of a Three-wire Three-phase System. We should always compute the regulation of a proposed three-phase installation, just as in the case of a single-phase line, in order to see whether or not the most economical wire produces too great a voltage variation at the different loads. For this purpose, it is much simpler always to consider the loads at the receiving end to be star-connected. If the loads are delta-connected, we have merely to consider the voltage between any conductor and an imaginary neutral as explained below. At present let us consider the load as star-connected at *M*, as in Fig. 194. The scheme is to compare the change in voltage across one phase (say *OA*) from no load to full load with the voltage at full load across the same phase (*OA*).

The full-load voltage between the line wires at the receiving end being 11,000 volts, the full-load voltage across any phase or coil of a star-connected load would be $\frac{11,000}{1.73} = 6360$ volts. Thus the full-load voltage across each of the coils *OA*, *OB*, and *OC* at the receiving end *M* would be 6360 volts.

Let us consider coil *OA* only. We found that the full-load line current at 80 per cent power-factor must be 78.8 amperes. In order to force this full-load current of 78.8 amperes through the coil *OA*, the voltage across the corresponding coil *PA* of the generator must be great enough to overcome the resistance and the reactance of the line wire *A*, and supply the 6360 volts across the coil *OA*. The

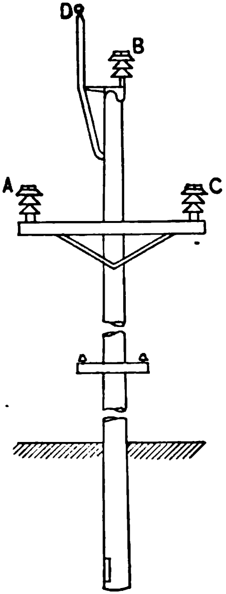


FIG. 195. The conductors *A*, *B*, and *C* are placed equidistant from one another. The wire *D*, usually of iron, is placed above *B* in order to afford protection against lightning.

voltage across the generator coil *PA* is thus the resultant of the voltage across the load coil *OA*, and the resistance voltage and the reactance voltage of the line-wire *A*. The resistance of the 10-mile line-wire *A*, size 0000 B. & S. gauge (see Table I, Appendix B), is 2.59 ohms. The pressure necessary to overcome the resistance of the line is equal to

$$78.8 \times 2.59 = 204 \text{ volts.}$$

The reactance * of the 10-mile line-wire *A* with the wires spaced equidistant and 30 inches from one another as in Fig. 195 is equal to

$$10 \times 0.621 = 6.21. \quad \text{(See Table III, Appendix B.)}$$

The pressure consumed in overcoming the reactance is

$$78.8 \times 6.21 = 489 \text{ volts.}$$

Construct Fig. 196 similar to Fig. 193, with the exception that the voltages in Fig. 196 are those across coils rather than between line wires.

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* When the wires of a system are not spaced equidistant from one another the reactance of the middle wire will differ from that of the other two. To avoid this unbalanced reactance, the wires are usually **transposed** every five miles on a short line or every 10 to 40 miles on a long line, so that when the whole length of the line is considered the average distance between them is the same. This average value is used when computing the reactance. Thus three wires might be strung one directly over the other with 36 inches separating each outside wire from the middle one. The average distance would then be

$$\frac{36 + 36 + 72}{3} = 48 \text{ inches,}$$

and this would be the value used in the table or formula for finding the reactance.

When the wires are not transposed the **equivalent distance** between them is used. Equivalent distance $= \sqrt[3]{\text{product of the three distances.}}$ The equivalent distance in this case would equal $\sqrt[3]{36 \times 36 \times 72}$ or 45. **Unless it is otherwise stated, the lines mentioned in this book are to be considered as transposed.**

$$OS = E_{PA} = \sqrt{(VN + 489)^2 + (OT + 204)^2}.$$
$$VN = 6360 \sin 37^\circ$$
$$= 3816.$$
$$OT = 6360 \cos 37^\circ$$
$$= 5088.$$
$$E_{PA} = \sqrt{4305^2 + 5292}$$
$$= 6823.$$

Therefore the voltage across the coil *PA* of the generator would have to be 6822 volts when the full load is taken from the receiver end. When the load decreases to zero, the voltage across one phase or coil (as *OA*) at the receiving end (*M*) would equal the voltage across the corresponding coil of the generator. The no-load voltage across one phase thus would equal 6822 volts.

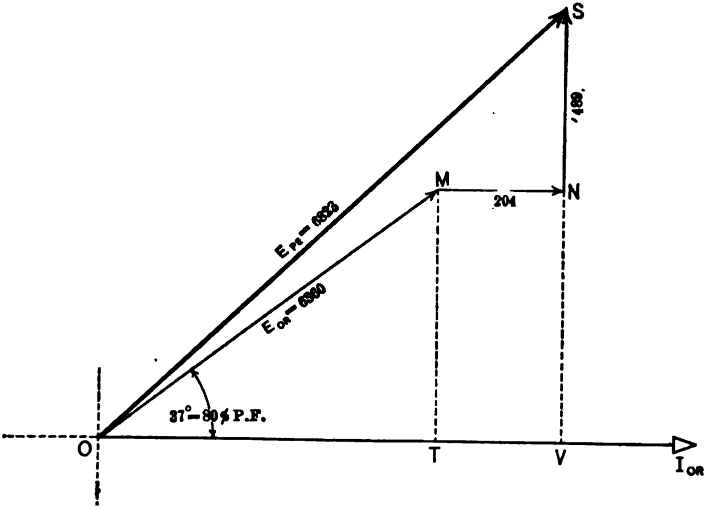

Fig. 196. Vector *OS* represents the voltage across one coil at the sending end of line in Fig. 194; *OM*, the voltage across one coil of the load; *MN*, the resistance drop of one wire; and *NS*, the reactance drop of one wire.

The change in voltage is equal to

$$6822 - 6360 = 462 \text{ volts.}$$

The regulation at 80 per cent power-factor

$$= \frac{462}{6360} = 7.27 \text{ per cent.}$$

Since the load is assumed to be balanced, the same change takes place across each coil of the load. The change across the line wires will thus be in the same ratio. Consequently the voltage regulation of this three-wire three-phase system is approximately 7 per cent, which is very satisfactory for a load of only 80 per cent power-factor. It will be remembered that the voltage regulation for a single-phase line under exactly similar conditions was 12.7 per cent. Thus the three-phase system is not only more economical but also has better regulation.

If the receiving or the sending end of the line is delta-connected, the method of computing the voltage regulation is similar to the above. Fig. 197 represents such a system, in which both the receiving end *M* and the sending end *G* are delta-connected. We may consider each coil of each end to be made up of two coils of a star connection. Thus the coil *TV* of the receiving end may be imagined to be made up of the two coils *TO* and *OV* connected in star at the neutral point *O*. The voltage across either of these imaginary coils would be $\frac{11{,}000}{1.73} = 6360$ volts. This is called the "**voltage to**

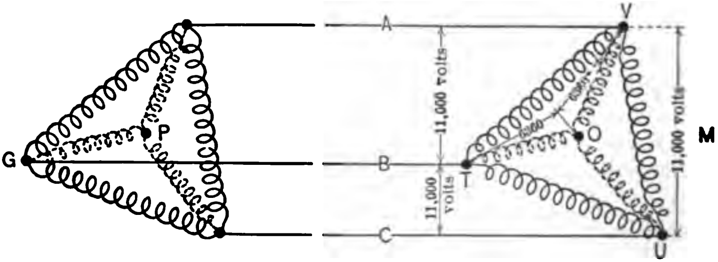

FIG. 197. Phase *TV* of the delta-connected load *U* may be considered to be made up of coils *TO* and *OV*. The phase *VU* may be considered to be made up of *VO* and *OU*, etc. The points *P* and *O* are thus the neutral points of star-connected coils.

neutral" in a delta-connected arrangement. In determining the voltage regulation of a delta-connected system, the amount which this "voltage to neutral" changes when the load changes from full load to no load is found, and compared with the "voltage to neutral" at full load, just as in the case of a star-connected arrangement.

The use of the Mershon diagram will save much mathematical work in computing the regulation of transmission lines. This diagram is equivalent to a large number of vector diagrams like those of Fig. 193 and 196. See Croft's "American Electrician's Handbook," page 143.

Prob. 11–5. Compute the voltage regulation of the three-phase system of Prob. 7–5, assuming the load to be star-connected.

Prob. 12–5. If the load on the system of Prob. 7–5 were delta-connected, what would the regulation be?

Prob. 13–5. Three-phase power is to be transmitted 14 miles. Power to be delivered, 4200 kw. Voltage at load, 33,000 volts; wires arranged in vertical plane, 50 inches apart; frequency, 25 cycles; power-factor of load, 85 per cent; size of wire, No. 0, B. & S. Compute the regulation if the load is delta-connected.

71. To Compute the Voltage at the Load. When an induction motor is started, it usually takes a much larger current than the full-load current, and always at a low power-factor. The voltage at the receiving end of a transmission line drops considerably under these conditions. This variation in terminal voltage can be determined as follows:

Example 2. In a three-phase transmission line each wire of which has 2 ohms resistance and 3 ohms reactance, the full load is 2000 kw. at 85 per cent power-factor. The full-load voltage at the receiving end is 6600 volts. Find the voltage of the load when several of the induction motors, which constitute part of the load, are starting simultaneously, and lower the power-factor to 75 per cent, at the same time increasing the load to 2100 kw.

The first step is to find the voltage at the sending end from data of normal conditions. This is done as in all previous examples of this chapter.

$$P = \sqrt{3}\,EI\cos\theta.$$

$$I = \frac{2{,}000{,}000}{1.73 \times 6600 \times 0.85}$$

$$= 206 \text{ amp. per line-wire.}$$

$$\text{Resistance drop} = 2 \times 206 = 412 \text{ volts.}$$

$$\text{Reactance drop} = 3 \times 206 = 618 \text{ volts.}$$

$$\text{Voltage to neutral} = \frac{6600}{1.73} = 3810 \text{ volts.}$$

Construct a diagram as in Fig. 193 and 196 and solve for OX, the voltage to neutral at the sending end.

$$OX = 4490 \text{ volts.}$$

With the usual method of operation, a voltage to neutral of approximately 4490 volts at the sending end would be maintained under all conditions (unless a feeder voltage regulator is used; see Art. 63 and 72).

The second step is to find what the voltage to neutral at the sending end would have to be, if the voltage at the receiving end were to remain 6600 volts, or 3810 volts to neutral, under the new conditions of load and power-factor.

$$P = \sqrt{3}\,EI\cos\theta.$$

$$I = \frac{2{,}100{,}000}{1.73 \times 6600 \times 0.75}$$

$$= 246 \text{ amperes.}$$

$$\text{Resistance drop} = 2 \times 246 = 492 \text{ volts.}$$

$$\text{Reactance drop} = 3 \times 246 = 738 \text{ volts.}$$

Construct a diagram as in Fig. 193 and 196 and find the value of OX, the voltage to neutral at the sending end.

$$OX = 4670 \text{ volts.}$$

Thus the voltage to neutral at the sending end would have to rise to 5030 volts in order to keep the voltage of the load up to 6600 volts (3810 volts to neutral) when the extra load at a low power-factor was thrown on.

But the conditions at the sending end are such that the voltage to neutral remains practically constant, 4490 volts at all loads. Therefore, if the sending voltage remains constant, the voltage at the receiving end must fall on account of the extra line drop between the generator and the load.

As a third step, we may then consider that the voltage to neutral of the sending end drops from 4670 volts to 4490 and compute the corresponding drop in the load voltage, from 6600 to ($\frac{4490}{4670}$ of 6600), or 6360 volts. As a check on this value we have merely to compute what the generator voltage would have to be in order to maintain a voltage of 6360 volts (3670 volts to neutral) at the receiving end when loaded with 2100 kw. at 75 per cent power-factor. The check value is 4560 volts to neutral, showing an error of about 1.5 per cent.

The error in this method lies in the fact that in the second step we have used too small a value for the line current. We may, therefore, better our result by repeating the second and third steps using a value for the line current, which we now know is more precise, as found by the equation

$$I = \frac{2,100,000}{1.73 \times 6360 \times 0.75}$$
$$= 255 \text{ amperes.}$$

By using 255 instead of 246 amperes for the line current, we find that it would require a generator voltage to neutral of 4710 volts to maintain 6600 volts between terminals at the load. With the generator voltage to neutral remaining 4490, the load voltage would be $\frac{4490}{4710} \times 6600$, or 6290 volts, which checks to within less than 1 per cent.

By repeating steps two and three a number of times, each time using a more precise value for the line current, it is possible to obtain the load voltage to any desired degree of precision. For most practical work one such repetition is sufficient.

Prob. 14–5. A 200-h.p. 2300-volt three-phase induction motor of 93 per cent efficiency has a power-factor of 90 per cent at full load. The distributing circuit has a resistance of 0.4 ohm and a reactance of 0.32 ohm per wire. (*a*) What must be the voltage at the generator end of the distributing line to operate the motor at rated load and at its rated voltage?

(*b*) What is the voltage regulation, assuming a constant voltage at the generator end of the distributing line?

Prob. 15–5. In starting, the induction motor of Prob. 14 takes three times normal current and the power-factor drops to 60 per cent. Assuming that the voltage at the generator end of the distributing line remains constant, what is the voltage across the motor on starting?

Prob. 16–5. If the generator voltage in Prob. 13–5 remains constant, what will the voltage at the load become when the load consists of 2000 kw. at 95 per cent power-factor? Check and show per cent error.

Prob. 17–5. What will be the voltage of the load in line of Prob. 13–5 when only 500 kw. are being delivered at unity power-factor? Compute per cent error due to this method.

72. Feeder Voltage Regulators and Line-drop Compensators. We have already shown that it is desirable that the voltage at the load end of a transmission line or feeder and the voltage at the generator end of the line shall both be maintained approximately constant (see Art. 7). We have seen that this may be accomplished by applying to each individual feeder a special type of transformer known as a Feeder Voltage Regulator, whose primary (high-tension) coil is shunted across the feeder lines and whose secondary is in series with the feeder. (See Art. 63.) It is necessary to readjust the voltage regulator for each change of current or of power-factor in the feeder, inasmuch as such changes tend to alter the impedance drop on the feeder or the voltage at the load end of the feeder. The adjustment of the regulator may be accomplished by hand if the load conditions change slowly, or automatically by a small alternating-current motor actuated by a "contact-making voltmeter," if the load conditions change rapidly.

Whenever the regulation of voltage at the load end of a feeder must be accomplished by adjustments (manual, or automatic by regulators) at the generating station or at any point distant from the load, it becomes necessary to have means of measuring the load voltage from the distant point. One method, of running small pressure wires parallel to the feeder to connect the voltmeter with the distant terminals, becomes too expensive when the feeder is long. In such cases, a "line-drop compensator" is used. The purpose of this device is to reproduce to a definite scale, within the local circuit of a voltmeter attached to the feeder at its station end, all of the reacting voltages produced in the line by the currents flowing, so that the voltmeter receives a pressure representing what is left at the load end of the line rather than the impressed voltage at the place where the voltmeter is attached.

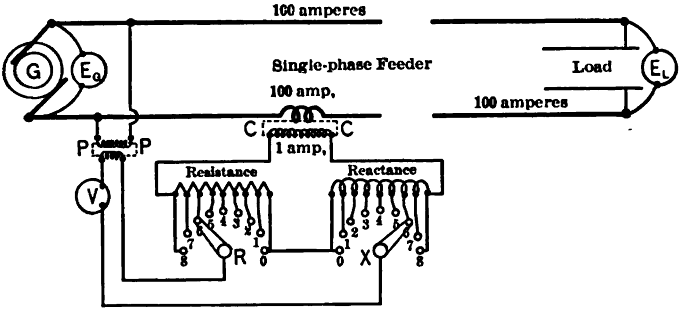

FIG. 198. Diagram of a "line-drop compensator." Voltmeter *V* installed in the generating station indicates the Voltage E_L at the load, and not the voltage E_G of generating station. The voltmeter *V* really measures the e.m.f. across the secondary of the potential instrument transformer *P*, as affected by the resistance drop and the reactance drop of *R* and *X*, which depend upon the line current.

Thus, in Fig. 198, the voltmeter V attached to this single-phase feeder does not indicate the pressure E_G at the generator or at the place where the voltmeter is attached, but rather the pressure E_L at the load end of the feeder, provided the contact-arms R and X are set properly. A current-transformer CC in series with the

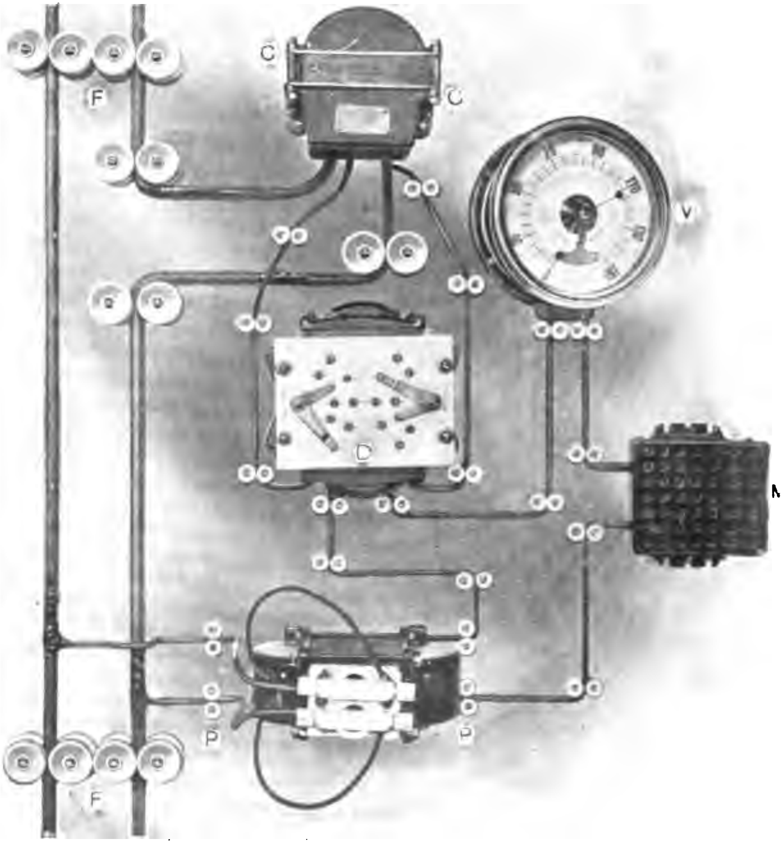

FIG. 199. Diagram of connections for a compensator on a single-phase circuit. *Westinghouse Electric & Mfg. Co.*

feeder has its secondary connected to resistance and reactance in series. These are therefore traversed by a current which is always directly proportional to the current which flows in the feeder itself. By means of the contact-arms *R* and *X* various portions of the resistance drop and reactance drop may be inserted in circuit with the voltmeter *V*, which is also connected to the generator end of the feeder through the potential transformer *PP*.

Now, if the contact-arms *R* and *X* are set so that the values of resistance and of reactance included between them bear the proper relation to each other and to the current ratio of the series transformer *CC*, the e.m.f.'s from *R* and *X* diminish the e.m.f. from *PP* in exactly the same way and by exactly the same percentage that the back e.m.f.'s or voltage reactions in the main line diminish the voltage from the generator. Of course, if the connections of *R* and *X* to *V* and *P* are reversed, the line drop is added to instead of subtracted from the generator voltage, and no adjustment of *R* and *X* can be found which will make the indications of *V* proportional to the voltage at the load end of the feeder.

Figure 199 illustrates the actual wiring of one of these line-drop compensators (*D*) in a single-phase feeder (*F*). The voltmeter *V* connects through the multiplier *M* (usually external with stationary types of voltmeter, though internal with portable types) to the low-tension side of the potential-transformer *PP* and the taps from the resistance and reactance in *D*. The current through this resistance and reactance is obtained from the secondary of the current-transformer *CC* in series with the feeder. When the contact-arms on the compensator are set so that the relation of reactance drop and of resistance drop between them to the low-tension e.m.f. of *PP* is exactly the same as the relation of reactance drop and of resistance drop in the feeder to the voltage of the generator, then the indications of *V* multiplied by a constant factor (ratio of *P*) give the voltage E_L but not the voltage E_G. In the case of polyphase feeders the connections are more complicated, although the same principles apply. In this way, it is possible to know at the generating station what adjustment must be made at the generator in order to keep the voltage at the load constant.

Note: In the following problems, neglect the resistance and reactance of the instrument transformers.

Prob. 18–5. In Fig. 198, the current ratio of *CC* is 100 : 1, and the voltage ratio of *PP* is 20 : 1. The single-phase feeder has altogether 10 ohms reactance and 2 ohms resistance. (*a*) What should be the amount of resistance and of reactance, in ohms, between R and X in order that the voltmeter V shall indicate in fixed ratio to the load voltage E_L? (*b*) What factor must the actual voltages at V be multiplied by in order to obtain the correct value of E_L volts?

Prob. 19–5. The single-phase feeder of Fig. 198 is designed to deliver 400 kw. at 0.80 power-factor and 2300 volts ($= E_L$), with a voltage regulation (at this power-factor) of 10 per cent and a transmission efficiency of 95 per cent. If the ratio of CC is 100 : 1, and of PP is 20 : 1, (*a*) what must be the values, in ohms, of resistance and reactance included between the contact-arms R and X, and (*b*) by what ratio must the voltage V be multiplied to obtain the voltage E_L?

Prob. 20–5. The feeder of Fig. 198 supplies load at a distance of 5 miles over No. 00 wires spaced 36 inches apart. $E_G = 6600$ volts and the ratio of PP is 60 : 1. What must be the ratio of CC (nearest larger multiple of 5) in order that we may use as much of each dial as possible on a line-drop compensator having a maximum range of 10 ohms resistance and 10 ohms reactance, to indicate the voltage E_L when the load has its maximum value of 500 kw. at 0.80 power-factor?

Prob. 21–5. (*a*) Assuming that we select for CC in Prob. 20 a transformer having a ratio which is the nearest larger multiple of 5 to the ratio which we theoretically require, calculate how many ohms of resistance R and reactance X must be tapped off in the compensator to make the reading of V as nearly as possible proportional to E_L. (*b*) What then will be the ratio between the actual voltages E_L and V?

Prob. 22–5. If each dial R and X in Prob. 21 has only ten equal steps of resistance or of reactance, 1 ohm each, calculate:

(*a*) The setting of each dial which will make the indications of V as nearly as possible proportional to E_L.

(*b*) The ratio of E_L volts to V volts for this setting of dial R and X, which will make V represent E_L with full load delivered from the feeder.

(*c*) The percentage error involved when the product of the ratio of (*b*) times the voltage V is assumed to represent the voltage E_L at zero load.

SUMMARY OF CHAPTER V

THE VOLTAGE at which electrical power is transmitted over short distances is usually about 1000 volts per mile.

IN AMERICA, ALTERNATING CURRENT is used in practically all industrial installations, the potential of which is above 550 volts.

THE FREQUENCIES in greatest use are 25 cycles for railway and power work and 60 cycles for lighting.

THE MOST ECONOMICAL SIZE OF TRANSMISSION WIRE is that size which results in the smallest sum total of annual fixed charges (such as interest in money invested, taxes, repairs and depreciation) and annual cost of energy lost in the line. The sum of these items becomes a minimum when the fixed charges equal the cost of lost energy.

THE VOLTAGE REGULATION OF THE LINE must also be taken into consideration when determining the size of wire to be used for transmission unless proper regulation is accomplished by feeder voltage regulators. The voltage regulation of a line equals

$$\frac{(\text{No-load voltage}) - (\text{Full-load voltage})}{(\text{Full-load voltage})}.$$

These voltages must be measured at the load end of the line. Good regulation for power service ranges between 5 and 10 per cent. For lighting service it should never exceed 5 per cent.

THE INDUCTIVE REACTANCE of a line causes voltage to be consumed in the line when an alternating current is sent over the line. This reactance drop leads the resistance drop by 90°.

THE GENERATOR VOLTAGE equals the no-load voltage at the load end of a short line, and can be found by adding vectorially the reactance drop and the resistance drop to the voltage at the load end.

A THREE-PHASE TRANSMISSION LINE is more economical than a single-phase line for transmitting a given amount of power to a given distance, and has better regulation under the same conditions. In a three-phase system the voltage to neutral and the resistance and reactance drop in one line conductor are always used in computing the regulation.

TO COMPUTE THE VOLTAGE AT THE LOAD END of a transmission line corresponding to any amount of load and power-factor:

FIRST STEP. Compute the voltage at the generator end of line from data under normal conditions.

SECOND STEP. Compute what the voltage at the generator end would have to be in order to maintain the same constant voltage at load end under new conditions of load.

THIRD STEP. The voltage at the load end under new conditions is practically the same fraction of the voltage under normal conditions, that the generator voltage computed by the "first step" is of that computed by the "second step." This method is an approximation which is precise enough for all practical conditions.

FEEDER VOLTAGE REGULATORS are used to maintain a constant voltage at the load when power-factor or power is varied within prescribed limits. Such regulators may be operated automatically by a motor controlled by a contact-making voltmeter.

LINE-DROP COMPENSATORS are combinations of adjustable resistance and adjustable reactance intended to be connected to the feeder and to the voltmeter through proper instrument transformers suitably arranged. When properly adjusted they cause the voltmeter at the station to indicate the voltage at the distant end of the feeder for all values of power and power-factor at the load.

PROBLEMS ON CHAPTER V

Prob. 23–5. The 11000-volt 3-phase 8-mile pole line of the Southern Power Co., running from Catawba to Pineville, is strung with aluminum stranded cable, the resistance of which is equivalent to the resistance of No. 2 copper wire. The load at Pineville consists of three 37.5-kv-a. and three 125-kv-a. transformers. Power-factor of load equals 80 per cent; frequency, 60 cycles. Compute the voltage regulation of this line. (Elec. Jour., Vol. VIII.)

Prob. 24–5. Assuming the voltage at Catawba to remain constant at the value found in Prob. 23, compute the voltage at Pineville when only 150 kw. at 95 per cent power-factor are being used there.

Prob. 25–5. Calculate the most economical size of copper wire for the three-phase distributing system of Prob. 23. Estimate copper conductor at 19 cents per pound; fixed charges at 9 per cent of line cost; electric energy at 4 mills per kw-hr., 3000 hr. at full load per year.

Prob. 26–5. A distributing system arranged as in Fig. 200 is delivering 400 kw. at 6600 volts and 80 per cent power-factor to transformer at sub-station A, and 250 kw. with 90 per cent power-factor to the sub-station *B*, which is near the main line. The conductors from *A* to *B* are No. 1 solid copper, and from *B* to the generating station *S* are stranded copper No. 00. The conductors are arranged as in Fig. 195, 30-inch spacing throughout. (*a*) Compute the voltage at *B* and at the station *S*. (*b*) What is the line regulation at *A* and at *B*?

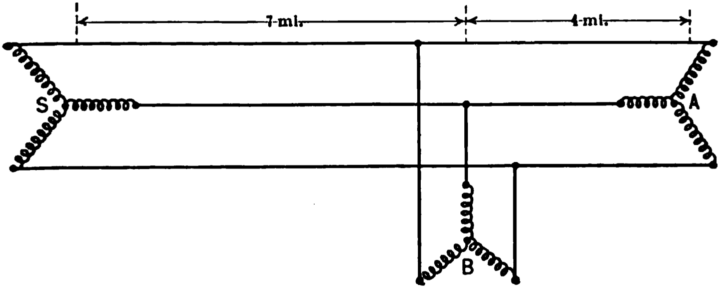

FIG. 200. Power is distributed over the three-wire line to sub-station *A* and to sub-station *B* which is on a spur line.

Prob. 27–5. If the station at B is on a spur line 5 miles from the main line, compute (*a*) and (*b*) of Prob. 26. The wires from main line to B are of No. 2 solid copper.

Prob. 28–5. Assuming the rate of interest and depreciation, the cost per pound of installed copper cable, hours of use, etc., to be as in Example 1, deduce an expression relating the total annual fixed charges in dollars, to the size of conductor denoted by the symbol r, representing the resistance in ohms per mile of conductor. At 20° C. commercial annealed copper wire has a weight of 0.3195 pounds per cubic inch and resistivity of 872.5 ohms per mile-pound.

Prob. 29–5. Assuming data as in Example 1 and Prob. 28, deduce an expression relating the annual value (dollars) of energy lost in the line conductors, to the size of conductor denoted by symbol r representing its resistance in ohms per mile.

Prob. 30–5. If the variation of all items in the first cost of a transmission line which depends upon the size of conductor were to be in direct proportion to the weight per mile of the conductor, the lowest total annual cost of transmission would be attained when the conductor is of such size that the annual fixed charges are exactly equal to the annual value of energy lost in the line. Assuming this to be true (which it is, nearly enough for rough, practical calculations), calculate the exact size in ohms per mile and in circular mils of the most economical conductor in Example 1. (*a*) For Table I. (*b*) For Table II. (*c*) For Table III.

Prob. 31–5. To be ready for growth of load expected in the near future, the size of wire installed in a transmission line may be made larger than that calculated to be the most economical size for the present loading. Under the conditions of Example 1 and by the methods outlined in Prob. 28, 29 and 30, calculate how much (per cent) greater than the least total cost of transmission, the annual cost would be if the line wire were made larger than the most economical size by (*a*) 50 per cent; (*b*) 100 per cent; (*c*) 200 per cent.

Prob. 32–5. The transmission specified in Example 1 is to be installed under the condition that it is for temporary service only, and will be dismantled at the end of four years with a scrap value equal to 40 per cent of the initial cost. Assume that the money put aside for depreciation charges does not earn interest, and that the tax rate, cost of copper and value of energy, etc., are as in Example 1. Calculate the most economic size of conductor in this case: (*a*) In ohms per mile. (*b*) In circular mils.

Prob. 33–5. Other conditions being as specified in Example 1, calculate what percentage decrease in the value per kw-hr. of energy would justify a saving of 25 per cent in the amount of conductor material used.

Prob. 34–5. A rule for rough calculations of transmission line is to allow in the conductors a power loss equal to approximately 10 per cent of the power delivered. Other conditions being as in Example 1, calculate what relation of the cost per pound of copper installed to the value of a kilowatt-hour will make it permissible to use this rule. The equations and methods suggested in Prob. 28, 29 and 30 may be used here to advantage.

Prob. 35–5. Good voltage regulation (without the aid of voltage regulators) on the short transmission in Example 1, Table I, demands that the resistance drop be not greater than 3 per cent of the voltage delivered at the load (11,000 volts). Calculate how much greater than the least value must be the annual cost of this transmission, in order to accomplish such regulation without a feeder voltage regulator.

Prob. 36–5. Show, for the general case, that when the most economical size of conductor is chosen, the voltage drop per mile of conductor **due to resistance** is dependent only upon the material and cost per pound of conductor, the percentage of fixed charges on this cost, hours of use, and the value of a kilowatt-hour saved from the line losses; that it does not depend upon the amount of power transmitted, the total distance or length of transmission line, voltage between conductors, or any other factor.

Prob. 37–5. Show that the most economical size of conductor under any given conditions requires the line to be proportioned on the basis of a certain number of circular mils of sectional area per ampere of current transmitted, and that this number depends upon the same factors as stated in Prob. 36 for the resistance drop per mile of conductor with most economical size.

Prob. 38–5. Calculate the percentage regulation at unity power-factor for the line of Example 1, when the frequency is (*a*) 25 cycles per second. (*b*) 133 cycles per second.

Prob. 39–5. Calculate the percentage regulation at 80 per cent power-factor for the line of Example 1, when the frequency is (*a*) 25 cycles per second. (*b*) 133 cycles per second.

Prob. 40–5. From the data in Table III, Appendix B, draw a a curve using as abscissas the distance between wires (range 1 inch to 120 inches), and as ordinates the inductive reactance (ohms per mile of single conductor). Consider only a 350,000 cir-mil conductor. Explain reasons for the form of this curve.

Prob. 41–5. From the data in Table III, Appendix B, draw a curve using as abscissas the size of wire (cir. mils, from one million to No. 6 B. & S. gauge), and as ordinates the inductive reactance (ohms per mile of single conductor). Consider only a spacing of 30 inches. Explain reasons for the form of this curve.

Prob. 42–5. Calculate the voltage regulation of the line specified in Example 1 (with power-factor of 0.8 at receiving end) corresponding to spacings of 2, 4, 6, 8 and 10 feet, and using a 350,000 cir-mil conductor. Draw a curve between spacing (in feet) as abscissas, and percentage regulation as ordinates. Explain reasons for the form of this curve.

Prob. 43–5. (*a*) With a power-factor of 80 per cent at the load in Fig. 193, what will be the power-factor at the generator terminals?

(*b*) What should be the ratio of reactance to resistance of the line to which Fig. 193 refers, in order that the power-factors at load and at generator terminals may be equal?

(*c*) If the size of wire be chosen so that the resistance drop is equal to the reactance drop in Fig. 193, what should be the voltage regulation?

Prob. 44–5. Calculate the percentage regulation on basis of data given in Example 1, with load having 0.80 power-factor, the spacing and size of wire being adjusted to give the following constants for the line: (*a*) Resistance of line = 2 ohms, reactance = 4 ohms; (*b*) Resistance of line = 4 ohms, reactance = 2 ohms. Compare and discuss these results.

Prob. 45–5. If the frequency were reduced from 60 cycles to 25 cycles per second in Prob. 44–5, what would the answers to questions (*a*) and (*b*) become? Compare these values with each other and with corresponding values from Prob. 44–5.

Prob. 46–5. Required, to calculate what size conductor is required for the transmission specified in Example 1, with a spacing of 30 inches, in order that the voltage regulation of the line shall not exceed 8 per cent. Power-factor of load assumed 100 per cent. Proceed as follows:

(*a*) Find tentative value for size of line (B. & S. gauge number, or circular mils), on the assumption that entire change of voltage is due to resistance only.

(*b*) Using reactance taken from tables for this size and the given spacing, calculate the regulation of the line.

(*c*) If the regulation obtained in (*b*) differs appreciably from the specified value, repeat the calculation for the next larger or smaller sizes of conductor with same spacing, and so on until the proper size is found.

Prob. 47–5. Deduce an expression or equation to show the relation of the most economical size of conductor (circular mils) to the factors upon which it depends, such as amount of power to be delivered (P watts), pressure at load (E volts between line wires), power-factor ($\cos \theta$), cost per pound of installed conductor (c dollars), fixed charges (p per cent), value of one kilowatt-hour saved (h dollars), and equivalent number of hours used per year of rated load (y hours). Assume three-phase three-wire lines of annealed copper.

Prob. 48–5. Solve Prob. 47–5 on basis of a three-phase three-wire line of aluminum cable. See Prob. 2–5 for data on aluminum.

Prob. 49–5. Solve Prob. 47–5 on basis of a single-phase two-wire line of copper conductors.

Prob. 50–5. Solve Prob. 47–5 on basis of a four-wire two-phase line of copper conductors.

Prob. 51–5. Using the method of procedure outlined in Prob. 46–5 and the data given in Prob. 13–5, calculate what size of conductor (B. & S. gauge, or circular mils, nearest standard size) should be used to keep the voltage regulation of this three-phase line within 8 per cent.

Prob. 52–5. If the load at B in Prob. 26–5 consisted of over-excited synchronous motors, so that the 90 per cent power-factor was due to a leading current, what would be the values of (*a*) in Prob. 20–5?

Prob. 53–5. The voltage of a central station is 13,000 volts, 60 cycles, three-phase. How far can this station transmit 1000 kw. at 85 per cent power-factor with a line regulation of 15 per cent? Conductors are No. 000 stranded copper spaced 36 inches equidistant.

Prob. 54–5. If the power-factor of the load in Prob. 53–5 were raised to unity, how much farther could the power be transmitted with the same regulation?

Prob. 55–5. An eleven-mile three-phase line with 16,000 volts, 60 cycles, at the sending end is to supply power at 92 per cent power-factor and 8 per cent regulation. The line consists of stranded aluminum cables spaced 36 inches equidistant and of a size equivalent in resistance to 250,000 cir-mil copper. How much power can it deliver under these specifications?

Prob. 56–5. In the system shown in Fig. 201, the distance between the generating station *G* and the receiving end is 8 miles.

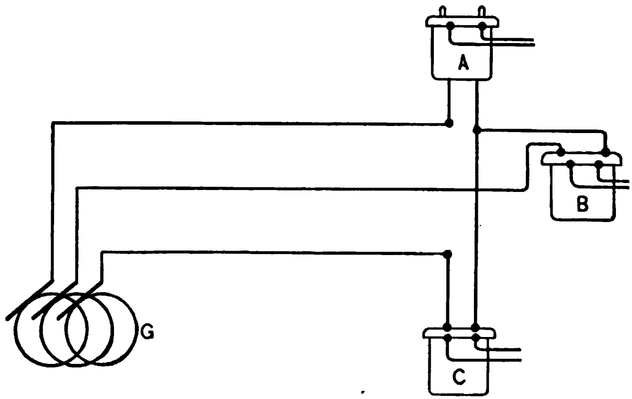

FIG. 201. Diagram of a system for transmitting the power of the station *G* to the three single-phase transformers *A*, *B* and *C*.

The voltage at the receiving end is 11,000 volts, 60 cycles, when each transformer has its full load of 600 kw. at 90 per cent power-factor. The line wires consist of 400,000 cir-mil stranded copper spaced 3 feet apart, as in Fig. 195. Compute the voltage at the generating station.

Prob. 57–5. The transformers of Prob. 56 become loaded as follows: Each transformer has a load of 500 kw. at 80 per cent power-factor. Assuming that the voltage at generator station remains as in Prob. 56, what will the voltage across each load transformer become? Check your computed voltages and state per cent error.

Prob. 58–5. What e.m.f. will be obtained between line wires at the load end of a three-phase three-wire line 10 miles long, of No. 000 copper with wires spaced as in Fig. 195 and 30 inches apart (constants as in Tables I and III of Appendix B), carrying a balanced 60-cycle load of 80 amperes per wire from a generator whose e.m.f. is 11,000 volts between any two terminals? Power-factor at generator is 80 per cent.

Prob. 59–5. Calculate the voltage at the load end of the line in Prob. 58 when the current is 40 amperes and when it is 120 amperes per wire, while the voltage at the generator terminals and the power-factor at the load remain constant. Draw a curve to suitable scale, using kv-a. delivered as abscissas, and load voltage as ordinates (voltage characteristic of the line). Discuss the form of this curve.

Prob. 60–5. While the line specified in Prob. 58 is delivering 60 amperes per wire at 87 per cent power-factor to a balanced load at the end of the line, another balanced three-phase load of 40 amperes per wire at 50 per cent power-factor is tapped from the middle point of the line. Calculate the voltage between line wires and the kilovolt-amperes and kilowatt output at each load. The station pressure is 11,000 volts.

Prob. 61–5. Calculate the amperes per wire and the power-factor at the station for Prob. 60. Calculate also the total kilovolt-amperes and kilowatt output at the station, and the efficiency of transmission.

Prob. 62–5. If the voltage impressed upon the sending (station) end of the line of Prob. 58 is 11,000, what will be the voltage between wires at the receiving end where a balanced load of 870 kw. at 87 per cent power-factor is being consumed? What will be the current per wire and the station power-factor under this condition?

CHAPTER VI

LONG TRANSMISSION LINES. CAPACITY REACTANCE

THE city of Oakland, Cal., is supplied with electric power from the Big Bend power plant 154 miles away. A map showing the location of the line is shown in Fig. 202. Two three-phase lines are run on the same towers in the manner shown in Fig. 203. Each conductor is a seven-strand copper cable, No. 000 B. & S. gauge. A three-phase generator by means of transformers can supply 10,000 kv-a. at 100,000 volts, 60 cycles, to the power-plant end of the line.

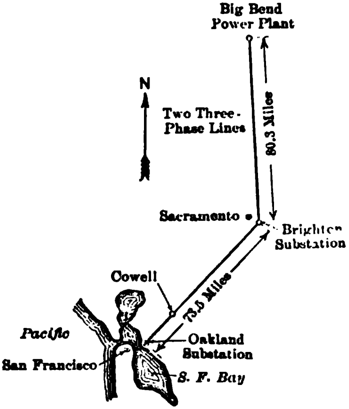

FIG. 202. Map showing the route of the 100,000-volt line from Big Bend to Oakland, Cal. *Faccioli in the Trans. A.I.E.E.*

Here is a long high-voltage transmission line, the characteristics of which are essentially different from the short lines which we studied in the previous chapter. Many facts come to light which are startling, when we first perceive them. For instance, when the Oakland end of the line was open, an ammeter inserted in the line wire near the power plant showed that a current of 48 amperes was flowing along the conductors at the power-house end. The voltage between the conductors at the open Oakland end was found to be 111,000 volts, while at the power plant end it was only 89,600 volts.

Why should this large current flow into the line conductors when the receiving end is open? Why, under these conditions, should the voltage at the receiving end be higher than the voltage at the sending end?

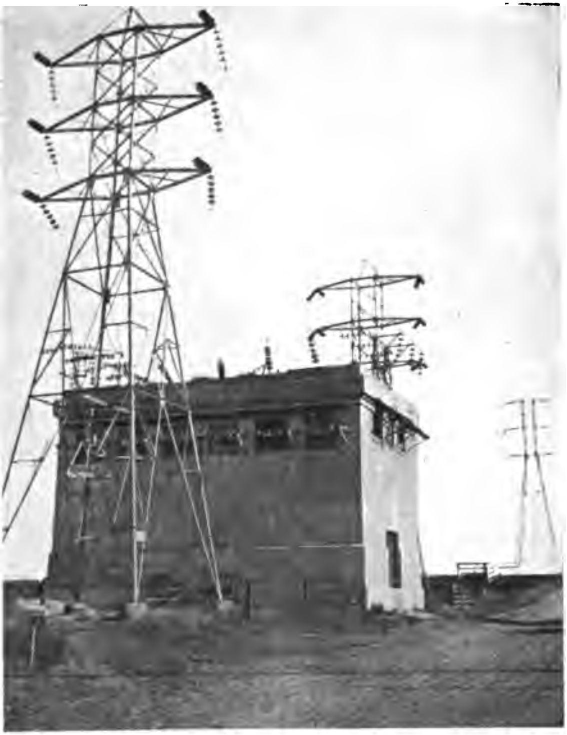

Fig. 203. Type of transmission lines used on the Big Bend-Oakland Line. *Faccioli in the Trans. A.I.E.E.*

73. Capacitance. The answer to these questions involves a study of the characteristics peculiar to long transmission lines. The line current of 48 amperes, for instance, is due to the fact that these wires, 154 miles long, offer a large surface which must be covered by the electric charge every time the voltage changes. Thus when the voltage is rising to its maximum positive value, a charge of electricity is forced out over the line wire to fill it up with electricity. Then as the pressure dies out, this charge flows back, there being no pressure to keep it forced out along the wire.

It is as though we were forcing water through an **elastic** pipe. Even though the far end of the pipe were plugged, the walls would stretch and allow a quantity of water to flow into the pipe. This quantity would depend upon how elastic the pipe was and upon how great the pressure was. As soon as the pressure on the pipe was removed, the elasticity of the pipe would cause the water to flow back again. Thus if the pressure were intermittent or alternating, there would be an intermittent or an alternating surge of water through the sending end of the pipe.

Similarly the electric line possesses a sort of **elasticity** to the electricity. **Electric elasticity is called the Capacitance of the line.**

This electric elasticity, or capacitance, allows an electric pressure to send a charging current into the line in order to fill it. This sets up a sort of electric strain along the wire, so that when the applied pressure is removed, the back-pressure due to this electric strain forces back the electricity, just as the pressure due to the strained condition of the water pipe forces back the water as soon as the applied water pressure is removed. Accordingly, if the electric pressure is continually alternating, the electricity will be forced out and back along the line, producing an alternating current of electricity along the conductor.

We have said that the amount of the water forced in and out of a water pipe depends upon how great the elasticity of the pipe line is and upon how large the applied pressure is. Similarly the amount of electricity forced out and back along a conductor depends upon how great the elasticity (or capacitance) of the line is, and upon how much pressure is applied to it. In fact, we measure the capacitance of a conductor by the quantity of electricity, in ampere-seconds, which one volt pressure can supply to a conductor when applied to it.

If the elasticity (or capacitance) of the conductor is so large that a pressure of one volt causes one ampere-second (one coulomb) of electricity to be supplied to the conductor, we say that the conductor has unit capacitance.

This unit of capacitance of one ampere-second per volt is called a Farad. But no conductor has such a large capacitance that one volt pressure can force anywhere near an ampere-second on it, so we commonly use one-millionth of the farad as a unit capacitance and call it a **Microfarad.**

Thus a conductor has a microfarad capacitance when the application of one volt supplies one-millionth of an ampere-second to the conductor. This 154-mile conductor has a capacitance of only 2.2 mf. and one volt pressure would supply only 0.0000022 amp-sec. to it.

Thus when there is no pressure between the ground and a conductor of 2.2 mf. capacity, there is no electric charge upon it, but when there exists a steady pressure of one volt between the ground and the conductor there is a charge of 2.2 millionths of an ampere-second upon the conductor.

If the voltage between the ground and the conductor were raised to a steady value of 2 volts, there would be an electric charge of 4.4 millionths of an ampere-second on the conductor. A steady value of 100,000 volts between the conductor and the ground would put a charge of 0.22 amp-sec. upon the conductor.

74. Condensers. The combination of the ground and the wire is said to constitute a **condenser.** Any two conductors separated by a non-conductor, such as air, mica, glass, etc., make a condenser.

The capacitance of a condenser depends upon how much surface the two conductors have, how far apart the conductors are, and what kind of material is between them. It does not depend upon the kind of material in the conductors.

If the surfaces of the two conductors are large, if the distance between them is very small, and if the material in this separating space is glass or mica, the capacitance of the condenser will be comparatively large. Commercial condensers for use with induction coils are made by putting thin strips of mica or oiled paper between sheets of tin or lead foil. The sheets of foil constitute the conductors and the mica acts as an insulator. Mica thus used is called the **dielectric.**

Fig. 204 shows the conventional diagram of such a condenser joined to a battery cell. Note that alternate plates are joined to one side of the line and the remaining plates to the other battery terminal. This gives a large area to each conductor, and at the same time places the conductors very near to each other. The dielectric is not represented in the diagram. The conventional form for illustrating capacitance is the symbol of Fig. 205.

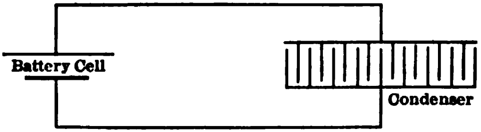

FIG. 204. The conventional diagram of a condenser connected to a battery cell.

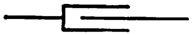

FIG. 205. The conventional symbol used to represent capacitance.

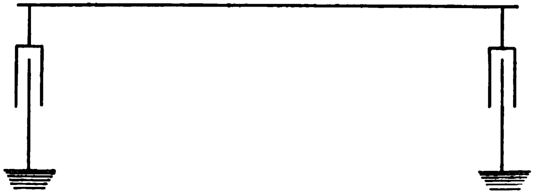

FIG. 206. Conventional diagram of a line wire, the capacitance (to ground) of which is represented by two condensers, one at each end.

Thus Fig. 206 represents a transmission wire, with the capacitance to ground considered as concentrated in a condenser at each end. This is much simpler in computations than considering the line to have its elasticity or capacitance distributed along the line. The transmission line can then be likened to a non-elastic pipe line with an air chamber at each end.

Example 1. The ground and one wire of the Big Bend-Oakland line constitute a condenser of 2.2 mf. capacity. If a pressure of 110 volts, direct current, is applied between the wire and the ground, how much electricity will be forced on the wire?

1 volt will force 0.0000022 amp-sec. on the wire.
110 volts will force

$$110 \times 0.0000022 = 0.000242 \text{ amp-sec.}$$

Or in the form of an equation

$$Q = CE$$

when Q = quantity of electricity in **amp-sec.**
C = capacitance of condenser in **farads.**
E = pressure between conductors in **volts.**

In this case,

$$\begin{aligned} Q &= 0.0000022 \times 110 \\ &= 0.000242 \text{ amp-sec.} \end{aligned}$$

Example 2. If the electricity takes 0.002 sec. to fill up the line in Example 1, what average current will flow into the line during the time of charging?

$$\begin{aligned} \text{Av. current} &= \frac{\text{Quantity}}{\text{Time}} = \frac{\text{amp-sec.}}{\text{sec.}} \\ &= \frac{\text{amperes} \times \text{seconds}}{\text{seconds}} = \text{amperes} \\ &= \frac{0.000242}{0.002} \\ &= 0.121 \text{ amp.} \end{aligned}$$

Prob. 1-6. What is the capacitance of a condenser that holds 0.012 ampere-second under a pressure of 110 volts?

Prob. 2-6. How many volts would be required to put 0.07 ampere-second into the condenser of Prob. 1?

Prob. 3-6. The capacitance of a condenser is 30 mf. How many ampere-seconds will it hold when the pressure is 220 volts?

Prob. 4-6. What is the average charging current if it takes 0.05 sec. for the condenser in Prob. 3 to become fully charged on a 110-volt circuit?

Prob. 5–6. If the condenser in Prob. 4 were placed on a 240-volt circuit and it took same length of time to charge, what would be the average current?

75. Cause of Condenser Action. Condenser action is best explained by referring to the generally accepted theory that electricity consists of two kinds, — one called positive electricity and the other, negative electricity.

Thus, by this theory, a battery or a generator sends out two streams of electricity, one flowing around the circuit in one direction, the other in the other direction. In Fig. 207, the battery cell may be thought of as sending out a stream of positive charges to the plate *A*, and of negative charges to the plate *B*, since *A* is connected to the positive terminal and *B* to the negative terminal of the battery. According to this theory one-half of the work in an electric circuit is done by the positive charges flowing in one direction, and the other half by the negative charges flowing in the other direction. This part of the theory need not confuse us, since the work done is the same whether it is all done by the negative or all by the positive or is divided between the two.

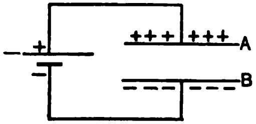

FIG. 207. The battery may be considered to send out positive charges of electricity to plate *A* and negative charges to plate *B*.

The part of the theory that particularly interests us is that which deals with these two kinds of electricity, negative and positive, in their effects upon each other.

It has been found by experiment that the laws for bodies so charged are similar to the laws for bodies magnetized. It must not be inferred from this statement that electricity and magnetism are the same. They are not the same, though they exhibit peculiar interrelations and analogies.

Electrified bodies resemble magnetized bodies in that bodies charged with the same kind of electricity repel one another, while bodies charged with unlike kinds attract one another. This is usually illustrated as follows:

If we bring near together two objects which have unlike charges, they attract each other with considerable force: and the charge on one seems to attract the charge on the other.

Thus *A*, in Fig. 208, represents an insolated wire which has a positive charge of electricity. The charge is distributed practically uniformly around the wire. If a wire *B* similarly charged were brought near *A*, the charges would appear to repel each other and the distribution would be somewhat as in Fig. 209. If, however, a wire *C* negatively charged were brought near *A*, the charges would appear to attract each other and the distribution would be as in Fig. 210. The two charges in this case are said to bind each other.

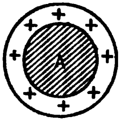

FIG. 208. The electric charge on an isolated wire would be distributed practically uniformly around the wire.

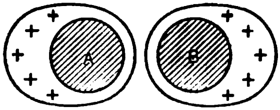

FIG. 209. Two like charges would repel each other and cause this unequal distribution on the conductors placed near each other.

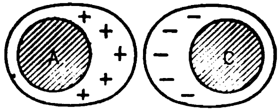

FIG. 210. Two unlike charges attract each other.

Now suppose wire *A* were connected to a source of supply of positive electricity as, for example, to the positive terminal of a battery cell or a generator, and wire *C* were connected to a source of supply of negative electricity, for example, the negative terminal of a cell or a generator. Fig. 207 represents such a case. It is easy to imagine now, how the positive charge on *A* draws a much larger negative charge from the battery cell out to *B* than if *A* were not near *B*. Also the negative charge on *B*, in turn, attracts a much larger charge of positive electricity from the battery cell out to *A*. The nearer *A* and *B* are together, the greater this attraction will be, and the greater the charge each will have. In this way the binding effect of one charge on another is very evident. The result is that two plates will hold a larger charge, with the same difference of potential or voltage between them, if they are near together, than if they are separated by a large distance. They will thus have a **greater capacitance** according to our definition of capacitance. Of course the larger the plates, the greater the capacitance also.

76. A Transmission Line as a Condenser. Now a conductor of a transmission line acts as one plate of a condenser, and the ground as the other plate. When the wire is charged with positive electricity, it tends to repel the positive electricity from that part of the ground nearest the wire, and to attract or "bind" the negative charge in the earth near the wire.* The binding effect of these two charges upon each other produces the condenser-effect of the line. Two wires near each other may act as the two plates, and the air between as the dielectric. In underground cables the cable acts as one plate of a condenser, the protective sheath of lead acts as the other plate of the condenser, and the insulating material as the dielectric. Due to the nearness of the ground or sheath to the wires in underground cables, the capacitance of such lines is very large in comparison with the capacitance of the usual overhead lines.

Since most transmission lines consist of at least two conductors, it is more convenient, in computing the capacity of a line, to consider one wire as one plate of a condenser and the return wire as the other plate and to neglect the effect of the ground. The error involved in this method is very slight indeed, as will be seen from a study of the diagram of Fig. 211. If conductor *A* at any instant were positive, the return conductor *B* would at that instant be negative, thus forming the positive and negative plates of a condenser with an air dielectric. The conductor *A* would tend also to bind a negative charge on that portion of the ground nearest the wire, and *B* would tend to bind a positive charge on that portion of the ground nearest the wire *B*. But both wires are so near to each other and so far from the ground that one wire affects any point on the ground with almost the same strength as the other wire, and thus one just about neutralizes the other's effect, as far as binding a charge on the ground is concerned. Thus the ground, not being oppositely charged with respect to either wire, does not act to either as the plate of a condenser. The same is true of the sheath of a cable containing more than one conductor.

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* Experience seems to indicate, in terms of the theory just stated, that when a conductor is apparently not charged either positively or negatively, it in reality contains equal charges of negative and positive electricity which exactly neutralize each other. A charge on a conductor thus means an **excess** of positive or of negative electricity.

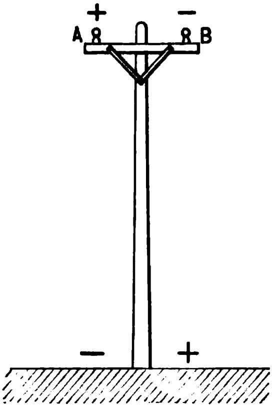

FIG. 211. The conductors *A* and *B* cause about equal amounts of positive and negative charges to be bound on the ground at the base of the pole. Thus one wire neutralizes the other as far as the capacitance to ground.

77. Condensers in Parallel and in Series. In order to understand the application of the formula for the capacitance of transmission lines it is necessary to note the effect of joining a number of condensers in **Parallel** and in **Series.**

CONDENSERS IN PARALLEL. Consider two condensers I and II, Fig. 212, joined in parallel across the mains, the voltage of which

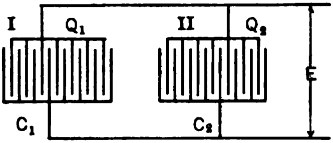

FIG. 212. The two condensers I and II are joined in parallel across a line having a voltage of E. The capacitance of the two parallel condensers equals $C_1 + C_2$.

is E. The capacitance of condenser I is C_1; of condenser II, C_2. Find the combined capacitance of the two when so joined.

Let C be the combined capacitance;
Q_1 = quantity of charge in condenser I;
Q_2 = quantity of charge in condenser II.

Then total quantity in both condensers $= Q_1 + Q_2$.

$$(1) \qquad Q_1 + Q_2 = CE.$$

(Quantity of total charge equals total voltage times total capacitance.)

But $$Q_1 = C_1E$$
and $$Q_2 = C_2E.$$
Therefore (2) $$Q_1 + Q_2 = (C_1 + C_2)E.$$

From (1) and (2) we have
$$CE = (C_1 + C_2)E,$$
or
$$C = C_1 + C_2.$$

Thus the capacitance of condensers joined in parallel equals the sum of the capacitances of the separate condensers. Joining condensers in parallel is merely adding the plate area of one to that of the other.

Example 3. What is the capacitance of 4 condensers of 3, 0.2, 7 and 2.5 microfarads respectively, when joined in parallel?

$$C = C_1 + C_2 + \ldots .$$
$$C = 3 + 0.2 + 7 + 2.5 = 12.7 \text{ microfarads.}$$

Prob. 6-6. What charge is sent into condenser I, Fig. 212, when $E = 120$ volts? Capacitance of I = 6 mfs.

Prob. 7-6. (*a*) What charge is sent into condenser II, Fig. 212, when $E = 120$ volts? (*b*) What charge goes to combination? Capacitance of II = 4 mfs.

CONDENSERS IN SERIES. Condensers in series present a peculiar phenomenon. Let condenser I of capacitance C_1, Fig. 213, be joined in series with condenser II of capacitance C_2. Let E be voltage across combination, E_1 across condenser I, and E_2 across condenser II.

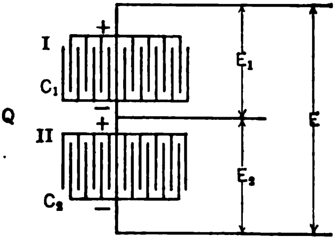

FIG. 213. The condensers I and II are joined in series across the line having a voltage of E. The capacitance of the series combinations of condensers equals $\frac{1}{C_1} + \frac{1}{C_2}$

The charge Q is sent into the condensers under the action of the voltage E. Since the two condensers are in series the same charge must be sent into each, just as the same current is sent through resistances in series. Thus the charge sent into each is Q, and the charge sent into the combination is also the same Q.

Let C = combined capacitance of C_1 and C_2.

(1)
$$E = \frac{Q}{C};$$
$$E_1 = \frac{Q}{C_1};$$
$$E_2 = \frac{Q}{C_2}.$$

But

(2)
$$E = E_1 + E_2.$$

Therefore, from (1) and (2),

$$\frac{Q}{C} = \frac{Q}{C_1} + \frac{Q}{C_2};$$

and

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}.$$

Thus the reciprocal of the combined capacitance of condensers in series equals the sum of the reciprocals of the capacitances of the separate condensers.

Example 4. If the capacitance of condenser I in Fig. 213 is 2 mf. and that of condenser II, 5 mf., what is the capacitance of the two condensers joined in series?

$$\frac{1}{C}=\frac{1}{C_1}+\frac{1}{C_2};$$

$$\frac{1}{C}=\frac{1}{2}+\frac{1}{5}=\frac{7}{10};$$

$$C=\frac{10}{7}=1.43 \text{ mf.}$$

Prob. 8–6. What charge is sent into the condensers, of above example, if the voltage across the combination is 120 volts?

Prob. 9–6. What is the voltage across condenser I and across condenser II, Fig. 213, if voltage across the two in series is 120 volts? Capacitance of I = 8 mf., of II = 3 mf.

For the purpose of calculating line capacitance it is merely necessary to know that:

Joining in parallel two condensers of equal capacitance produces a combination having double the capacitance of each condenser.

Joining these condensers in series produces a combination having half the capacitance of each.

78. Formula for Capacitance of Overhead Cables. Two-Wire.

If we had to work with two-wire lines only, the simplest formula for capacitance would be

$$C_m = \frac{0.0194^*}{\log_{10}\frac{s}{r}}$$

in which

C_m = capacitance, in **microfarads,** of one mile of line (2 wires).

s = distance between wire centers in **inches.**

r = radius of each wire in **inches.**

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* For the derivation of this formula, see Pender's "Principles of Electrical Engineering," page 271.

Example 5. What is the capacitance of a 40-mile two-wire line, if the conductors are No. 00 solid wires and spaced 4 feet apart?

By the formula, the capacitance per mile:

$$C_m = \frac{0.0194}{\log \frac{s}{r}}.$$

$s = 48$ inches.

$d =$ diameter of No. 00 wire $= 0.3648$ inch.

$r = 0.182$.

$$C_m = \frac{0.0194}{\log \frac{48}{0.182}}$$

$$= 0.00802 \text{ mf.}$$

Fig. 214 shows the distribution of capacitance in a 4-mile line. The capacitance of each mile of the line is represented by a condenser placed between the line wires. Note that the four condensers, each representing the capacity of a mile length of wire, are in parallel. The capacitance of a four-mile line is therefore equal to four times the capacitance of a one-mile length of the line.

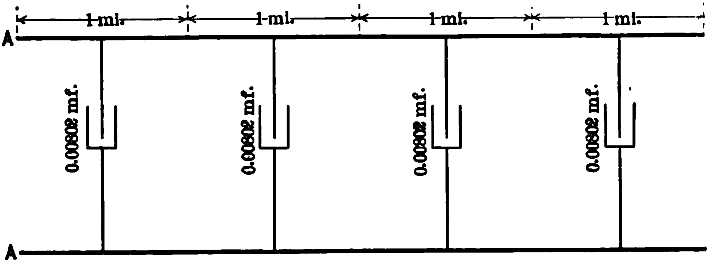

FIG. 214. The capacitance of a 4-mile line equals the capacitance of four condensers (each being the capacitance of one mile) joined in parallel.

Similarly the capacitance of 40 miles = 40 × 0.00802 = 0.321 mf. This 0.321 mf. is then the capacitance of the line consisting of two wires.

79. General Formula for Capacitance of Overhead Lines. Since we have many three-wire lines, and in order to make one table of values apply to both two- and three-wire lines, we do not commonly use this method of computing the capacitance, nor this way of stating the capacitance of the line.

Instead of computing the capacitance of the **line**, we compute the capacitance of **one wire** only of the line. We consider every line to be made up of at least three wires, one of which may be an **imaginary neutral** wire. Consider the two-wire line of Fig. 215. The lines A and B represent the two real wires; the line C, midway between A and B, represents an imaginary wire which is the neutral. The capacitance of wire A, considered with relation to the imaginary neutral wire C, would be represented by the condenser C_A. The capacitance of wire B considered with relation to the imaginary neutral C would be represented by the condenser C_B. The capacitance of the combination of wires A and B would be the capacitance of the series combination of the two condensers C_A and C_B. Now the capacitance of a series combination of two condensers having equal capacitance is one-half the capacitance of each condenser. Therefore the capacitance of the two-wire line of Fig. 215, which is the capacitance of the series combination of C_A and C_B, must be one-half the capacitance of either C_A or C_B.

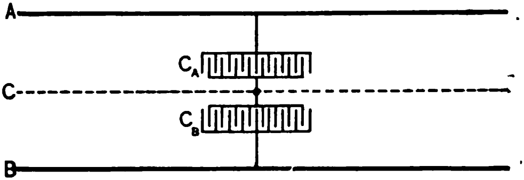

FIG. 215. The capacitance of a two-wire line may be assumed to be composed of the capacitance of a series arrangement of two condensers of capacitance C_A and C_B. C_A represents the capacitance of wire A to the neutral C, and C_B the capacitance of the wire B to neutral.

The capacitance represented by C_A is called the capacitance of wire A with respect to the neutral, and the capacitance represented by C_B is called the capacitance of wire B with respect to the neutral.

Thus the capacitance of a two-wire line is one-half the capacitance of one wire with respect to the neutral; or conversely, the capacitance of one (either) wire with respect to the neutral must be twice the capacitance of a two-wire line.

Accordingly, since the formula for the capacitance per mile of a two-wire line is $\frac{0.0194}{\log \frac{s}{r}}$, then the formula for the capacitance per mile of one wire with respect to the neutral is

$$C_0 = 2 \times \frac{0.0194}{\log_{10} \frac{s}{r}},$$

or

$$C_0 = \frac{0.0388}{\log_{10} \frac{s}{r}},$$

when

C_0 = capacitance in microfarads, with respect to neutral, per mile of conductor.

s = distance between conductor centers, in inches.

r = radius of each conductor, in inches.

This is the formula in general use for computing the capacitance of all types of overhead transmission regardless of the number of wires used.

Example 6. How great a charge will 2000 volts, direct current, force upon a 120-mile circuit consisting of two No. 000 stranded conductors strung 18 inches apart?

Capacitance to neutral per mile of each conductor is found as follows:

Outside diameter No. 000 bare copper cable is 0.470 inch.

$$C_0 = \frac{0.0388}{\log \frac{s}{r}}$$

$$= \frac{0.0388}{\log \left(\frac{18}{0.235}\right)}$$

$$= 0.0206 \text{ mf.}$$

Capacitance of 120 miles of wire would be 120 times as great because it would be equivalent to 120 condensers of this capacitance joined in parallel.

Capacitance of 120 miles = 120 × 0.0206
= 2.472 mf.
= 0.000002472 farad.

Since this is the **capacitance of one wire to neutral** we must use the **voltage from one wire to neutral** in computing the charge.

Voltage between wires = 2000 volts.
Voltage from one wire to neutral = $\frac{2000}{2}$ = 1000 volts.
Volts (to neutral) × capacitance (to neutral) = charge.
1000 × 0.000002472 = 0.00247 amp-sec.

This method produces the same result as though we considered the capacitance of the two real line wires as being $\frac{1}{2}$ of 2.472 = 1.236 mf. To find the charge on the line we would multiply this **capacitance** between the two wires by the **voltage** between the wires.

2000 × 0.000001236 = 0.00247 amp-sec., as before.

Prob. 10-6. What is the capacitance of one wire to neutral, for the transmission line from Big Bend to Oakland described on page 362? The distance between wires is 10 feet and the conductors are not transposed. See page 342.

Prob. 11-6. The transmission line from Great Falls to Greenville, S. C., is 96 miles long and consists of two sets of three No. 00 stranded copper cables. Each set is strung in horizontal plane 8 feet apart and is transposed. Considering each set separately, what is the capacitance from each line conductor to neutral?

Prob. 12-6. What direct-current pressure would be required between any two adjacent wires in the same set of the line in Prob. 11 to put a charge on them of 0.002 amp-sec.?

Prob. 13-6. In Table IV, Appendix B, is given a table of values of the capacitance to neutral for standard solid conductors and standard spacings. Check values for one mile of

(*a*) No. 2 wire with 24-inch spacing.
(*b*) No. 00 wire with 8-foot spacing.

Prob. 14-6. In Table VI, Appendix B, is a table of capacitance to neutral for standard strands. Check value of one of

(*a*) 500,000 cir. mils spaced 30 inches.
(*b*) 750,000 cir. mils spaced 15 feet.

Prob. 15-6. According to Table VI, Appendix B, what should be the capacitance of one conductor to neutral of the transmission line from Big Bend to Oakland? Compare with result calculated by formula, in Prob. 10.

80. Effect of Impressing an Alternating E.M.F. upon a Condenser. Charging Current. We have seen that when an alternating e.m.f. of 90,000 volts was impressed across the terminals of the Big Bend-Oakland line, a current of 48 amperes flowed into the line, even when the line was open or unloaded. We have seen that this current is due to the line acting like a condenser; — the alternating e.m.f. alternately filling the line with electricity and emptying it. Let us further investigate this **charging current**, as it is called.

Capacitance in a circuit acts like air chambers in a pipe line, tending to keep the pressure constant. Thus condensers in an alternating current circuit may be thought of as reservoirs in which electricity is being stored as the pressure rises and charges them. These condensers or reservoirs then use the reacting pressure of the stored charge to maintain the current when the impressed e.m.f. dies out. A current will thus be flowing into the line as long as the voltage continues to rise. When the voltage stops rising, the current stops flowing. The current will be the greatest when the rise in voltage is most rapid. Now an alternating

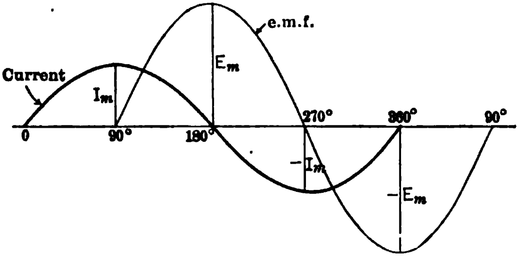

FIG. 216. Note that the wave of charging current is always 90° ahead of the wave of impressed e.m.f. in a circuit containing capacitance only.

e.m.f. is rising most rapidly just as it is passing up through the zero value. The charging current into the line will therefore be flowing at its maximum rate at the instant when the voltage is rising and going through its zero value. Thus the current leads the e.m.f. by 90° while the voltage is growing. Note that in Fig. 216 as the curve of e.m.f. passes up through its zero value, the curve of current is at its maximum value. Just as a reservoir would start to discharge water as soon as the pressure fell below the level of the water in it, so a current begins to flow out of a condenser as soon as the alternating e.m.f. begins to fall, because the impressed e.m.f. then becomes lower than the opposing internal pressure across the condenser. The faster the pressure falls, the greater is the current flowing out of the condenser. Since the pressure falls fastest as it passes down through its zero value, the current must be flowing out fastest at this instant. Thus the current reaches its greatest negative value just as the e.m.f. is at zero and is about to start its negative values. The current is therefore still 90° ahead of the impressed voltage. Note in Fig. 216, how the current curve reaches its maximum value $-I_m$ just as the voltage has dropped to zero. The voltage is thus at its 180° value when the current has reached its 270° value. Similarly it can be shown that at every instant:

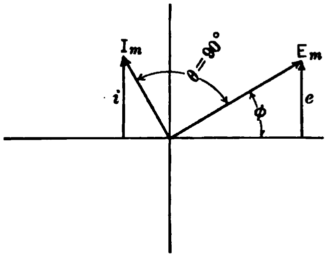

FIG. 217. Vector diagram of charging current and impressed e.m.f. in a circuit containing capacitance only.

In a circuit containing capacitance only, an alternating current leads the impressed alternating e.m.f. by 90°.

It can be shown that this capacity current has a sine wave-form if the impressed e.m.f. has a sine wave-form. We may thus represent relations of both the current and the e.m.f. of such a circuit by vector diagrams. Fig. 217 shows the capacity current vector I_m leading the voltage vector E_m by 90°. The instantaneous value of the capacitance current can be found from the diagram by writing the equation

$$i = I_m \sin(\phi + 90°)$$

in which

i = instantaneous value of capacitance current, in amperes.
I_m = maximum value of capacitance current, in amperes.
ϕ = electrical degrees through which e.m.f. has passed.

81. Charging Current. Capacity Reactance. Since a long transmission line acts as a condenser, an alternating capacitance current, or charging current, flows in it when an alternating e.m.f. is applied to it. The value of this charging current can be computed as follows:

The opposition which the alternating e.m.f. has to overcome in setting up a current in a circuit containing capacitance only is called the **Capacity Reactance** of the circuit, and is measured in **Ohms** just as the Resistance and Inductive Reactance are measured in **Ohms.** And just as the current in a circuit containing resistance only can be found by dividing the e.m.f. by the **resistance,** and the current in a circuit containing inductance only can be found by dividing the e.m.f. by the **inductive reactance,** so the current in a circuit containing capacitance only can be found by dividing the e.m.f. by the **capacity reactance.**

The equation for this relation is

$$I = \frac{E_c}{X_c},$$

in which

I = effective charging current, in amperes.
E_c = effective e.m.f., in volts, required to overcome the capacity reactance.
X_c = capacity reactance, in ohms.

Of course this may be written in the forms

$$E_c = X_c I$$

and

$$X_c = \frac{E_c}{I}.$$

Example 7. An unloaded transmission line is a practical case of a circuit containing capacitance only, since the resistance is negligible when compared with the capacity reactance. The capacity reactance of one wire to neutral of a single-phase 50,000-volt line is 3500 ohms. How large charging current flows in this line?

Capacity reactance for one wire to neutral = 3,500 ohms.
Voltage to neutral = 25,000 volts.

$$I = \frac{E_c}{X_c}$$

$$= \frac{25{,}000}{3{,}500} = 7.14 \text{ amperes.}$$

Thus 7.14 amperes would flow out and in at the sending end of the line, even though the far end were open. This current of 7.14 amperes would lead the e.m.f. by 90°. Fig. 218 shows the vector diagram for this current and voltage relation.

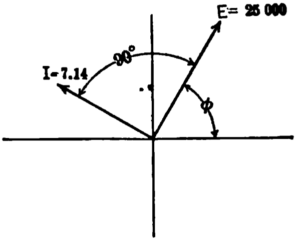

FIG. 218. The vector I representing a charging current of 7.14 amp. leads by 90° the vector E representing the impressed voltage 25,000 volts.

Prob. 16–6. Draw the vector diagram for above example with voltage 15° after it has passed up through its zero value. Find instantaneous values of current and voltage.

Prob. 17–6. Effective alternating voltage of 220 volts (frequency 60) is impressed upon a circuit containing a condenser only. If the current is 2 amperes, what is the capacity reactance of the condenser?

Prob. 18–6. When the instantaneous voltage of Prob. 17 is 120 volts, positive and increasing, what instantaneous value will the current have?

Prob. 19–6. What voltage is necessary to force a maximum current of 20 amperes through a circuit containing 50 ohms of capacity reactance?

Prob. 20–6. (*a*) Draw vector diagram and determine instantaneous value of voltage when instantaneous current in Prob. 19 is 5 amperes, positive and decreasing. (*b*) How many electrical degrees will the voltage have passed through by that time?

82. Capacity Reactance of Long Transmission Lines. In working with the capacitance of long transmission lines, it is convenient in our computations to assume that the capacitance of the line is all contained in two condensers of equal capacitance, one at each end of the line, instead of being distributed among an infinite number of small condensers strung all along the conductor. Similarly, we find it convenient for the sake of simplifying our computation to consider the capacity reactance to neutral of the line to consist of two equal capacity reactances placed one at each end of the line. Thus, in Fig. 219, the capacity reactance X_c of one wire to neutral may be represented by the two equal reactances P and Q. The reactance of each can be represented by ($2\ X_c$), since they are equal and are in parallel with each other. Assume X_c equals 3500 ohms, as in Example 7, and that the voltage to neutral at the sending end equals 25,000 volts and

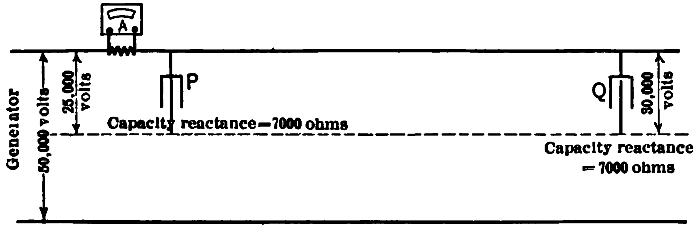
A
Generator
50,000 volts
25,000 volts
P
Capacity reactance = 7000 ohms
Q
30,000 volts
Load
Capacity reactance = 7000 ohms

Fig. 219. The charging current as indicated by the ammeter *A* would equal the sum of the currents through the equal capacity reactances *P* and *Q*.

at the open receiving end 30,000 volts to neutral. The charging current may then be found as follows:

$$I \text{ (of } P) = \frac{\text{voltage across } P}{\text{reactance of } P}$$

$$= \frac{25{,}000}{7000} = 3.57 \text{ amp.}$$

$$I \text{ (of } Q) = \frac{\text{voltage across } Q}{\text{reactance of } Q}$$

$$= \frac{30{,}000}{7000} = 4.28 \text{ amp.}$$

Since these two currents are in parallel and practically in phase with each other, being 90° ahead of the voltage, we have merely to add them arithmetically to get the charging current as read by the ammeter *A*: 3.57 + 4.28 = 7.85 amp. total charging current. The value 7.14 amperes is obtained in Example 7, where we consider the capacitance to be concentrated in one condenser at the middle of the line. The above two-end-condenser method is the simpler and rather more precise and should ordinarily be used in practical computations.

Note by examining Fig. 220, which represents a direct-current circuit, that this two-condenser method is exactly similar to the method we use in considering direct currents, to find the line current when the line is loaded with two equal resistances at *P* and *Q*.

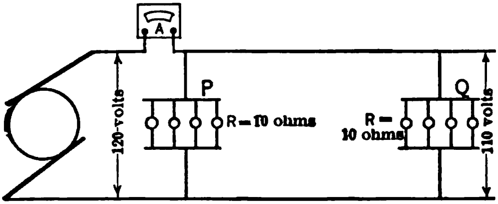

FIG. 220. The line current indicated by the ammeter *A* would equal the sum of the currents through the equal resistances *P* and *Q*.

$$I \text{ (through } P) = \frac{\text{voltage across } P}{\text{resistance of } P}$$

$$= \frac{120}{10} = 12 \text{ amp.}$$

$$I \text{ (through } Q) = \frac{\text{voltage across } Q}{\text{resistance of } Q}$$
$$= \frac{110}{10} = 11 \text{ amp.}$$

The current through the ammeter A, then, equals the sum of these two currents since they are flowing in parallel.

$$11 + 12 = 23 \text{ amp.}$$

Notice that we may set this computation down as follows:

$$\text{Current through ammeter } A = \frac{120}{10} + \frac{110}{10}$$
$$= \frac{120 + 110}{10}$$
$$= 23 \text{ amp.}$$

Similarly we may set down the computation of the charging current in Fig. 219:

$$\text{Charging current} = \frac{25{,}000}{7000} + \frac{30{,}000}{7000}$$
$$= \frac{25{,}000 + 30{,}000}{7000}$$
$$= 7.85 \text{ amp.}$$

Thus, if we distribute our capacities in this manner, we have merely to divide the sum of the voltages across the two assumed condensers by the capacity reactance of one of the condensers or by twice the capacity reactance of one line wire to neutral, and we thereby obtain the value of total charging current for one line wire.

It must be noticed, however, that the capacity reactance of one wire to neutral is not the sum of the reactances of the two condensers, P and Q, but rather it is just one-half the capacity reactance of either. This is clear from the direct current analogy of Fig. 220. The resistance of the load P and Q is not the sum of the resistances of the two, but rather half the resistance of either. Just as the resistance of the combination of P and $Q = \frac{1}{2}$ of 10 or 5 ohms, so the capacity reactance of the parallel combination of P and Q, Fig. 219, equals $\frac{1}{2}$ of 7000 = 3500 ohms. Or, conversely, the capacity reactance of each assumed condenser equals twice the capacity reactance of the line to neutral. The rule, then, for finding the charging current of any transmission line is:

Divide the SUM of the voltages to neutral at each end by TWICE the capacity reactance of one line wire to neutral.

Similarly if we know the charging current and the voltage to neutral at each end, we divide the sum of these two voltages by the charging current. This gives us the capacity reactance of one of the assumed condensers. We have now merely to remember that the line consists of two such condensers in parallel and its capacity reactance is therefore one-half the capacity reactance of one assumed condenser.

Of course, the actual conditions in a circuit are that the line constitutes an infinite number of minute condensers. Thus the charging current entering a line from the generator terminals **gradually** decreases until at the far end it becomes zero. All methods, therefore, which assume the capacitance to be concentrated at one or more points are merely more or less close approximations. The two-end-condenser method usually results in an error of less than 1 per cent and is precise enough for ordinary practical computations, — especially as the assumption that the charging current was a sine wave-form may produce a much greater error, often as great as 40 per cent. See § 84.

Prob. 21–6. Compute from data on page 362 the capacity reactance of one wire to neutral of the Big Bend to Oakland line. In computing voltage from each line wire to neutral, remember that this is a three-phase line, with 89,600 volts between line wires at one end, and 111,000 volts at the other end.

Prob. 22–6. From "Charging Current" table in Appendix B, compute the capacity reactance to neutral of 140 miles of No. 000 solid conductor spaced 3 feet from the other similar line wire, if the system is single-phase 60-cycle.

Prob. 23–6. What is the capacity reactance of one wire to neutral for a three-phase line, strung with 350,000 cir. mils stranded cables, spaced 15 feet apart? Length of line, 80 miles; frequency, 60 cycles. See Appendix B.

Prob. 24–6. What charging current will flow in each wire of Prob. 23, if the open circuit voltage at the sending end is 95,000 and at the receiving end is 114,000? Frequency, 60 cycles.

Prob. 25–6. What would be the charging current of Prob. 24 if the frequency were 25 cycles?

Prob. 26–6. A 350-mile, three-phase, 60,000-volt transmission system, operating at 60 cycles, uses 2800 reactive kv-a. in charging the line.

(*a*) What is the charging current?

(*b*) What is the capacity reactance of the line, one wire to neutral?

(*c*) What is the charging current per mile per 1,000,000 volts to neutral?

83. Computation of Capacity Reactance. When the capacitance of a line-conductor or of a condenser, and the frequency of the impressed voltage are known, the capacity reactance for that frequency may be computed by the following method:

In a circuit containing capacitance, the alternating current charges or discharges the condenser four times each cycle, as seen from Fig. 221 which represents the current curve in such a circuit for one cycle:

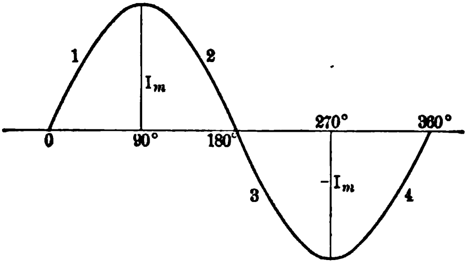

FIG. 221. One cycle of the current sine curve in a current containing capacitance only. Note that the current makes the change between zero and the maximum value four times each cycle.

(1) Condenser is being charged by increasing positive current being forced into it from the line.

(2) Condenser is discharging by delivering a decreasing positive current to the line.

(3) Condenser is being recharged by increasing negative current being forced into it from the line.

(4) Condenser is discharging by delivering a decreasing negative current to the line.

If the current makes f cycles per second, and the current charges or discharges 4 times each cycle, it must charge or discharge 4 f times each second. The time consumed, then, for each charge or discharge must be $\frac{1}{4\,f}$ second.

The quantity of electricity Q sent into or out of the conductor or condenser each time must be equal to the average rate of flow (Av. I) multiplied by the time t during which it is flowing.

Thus

$$Q \text{ (ampere-seconds)} = \text{Av.}\, I \text{ (amperes)} \times t \text{ (seconds)} = \text{Av.}\, I \times t$$

but
$$t = \frac{1}{4f}.$$

Therefore

$$Q = \text{Av.}\, I \times \frac{1}{4f}$$
$$= \frac{\text{Av.}\, I}{4f}.$$

But we have learned that the quantity of electricity sent into or out of a condenser at each charge or discharge also equals the product of the voltage E times the capacitance C (in farads).

$$Q = EC.$$

Then the maximum value of Q is given by the equation

$$Q_m = E_m C.$$

Therefore, since Q_m is equal to both E_mC and $\frac{\text{Av. } I}{4f}$,

$$E_mC = \frac{\text{Av. } I}{4f},$$

or

$$E_m = \frac{\text{Av. } I}{4fC}.$$

But Av. $I = 0.636\ I_m$, because it is assumed that I has a sine wave-form when E is harmonic. Therefore, substituting $0.636\ I_m$ in the place of Av. I, we have

$$E_m = \frac{0.636\ I_m}{4fC} = \frac{I_m}{6.28fC} = \frac{I_m}{2\pi fC}.$$

This is usually written in the form

$$E_m = \frac{1}{2\pi fC} \times I_m,$$

or, since there is the same fixed ratio between effective and maximum values of volts and amperes when they both vary harmonically,

$$E = \frac{1}{2\pi fC} I,$$

where

E = impressed alternating e.m.f., in effective volts.
I = charging current, in effective amperes.

But the impressed e.m.f. must be equal to the capacity reactance times the charging current, as we found in § 81. Thus

$$E = X_cI = \frac{1}{2\pi fC} I.$$

Therefore

$$X_c = \frac{1}{2\pi fC}.$$

The capacity reactance is generally represented by the expression $\left(\frac{1}{2\pi fC}\right)$ just as the inductive reactance is generally represented by the expression $(2\pi fL)$.

Since we are usually more interested in the value of the charging current, we more often use the equation in the form

$$I = \frac{E}{\frac{1}{2\pi fC}};$$

that is, $$I = 2\pi fCE.$$

Example 8. What is the charging current of a line having a capacitance to neutral of 3 mf., if the terminal voltage to neutral is 44,000 volts? Frequency, 60 cycles.

$$\begin{aligned} I &= 2\pi fCE \\ &= 2 \times 3.1416 \times 60 \times 0.000003 \times 44{,}000 \\ &= 49.7 \text{ amp.} \end{aligned}$$

Prob. 27–6. What is the capacity reactance of one wire to neutral of the line in the above example?

Prob. 28–6. What will the capacity reactance of the line of Prob. 27 become if the frequency is reduced to 25 cycles?

Prob. 29–6. What would be the charging current of line in Prob. 28 at the voltage of Example 8?

Prob. 30–6. From the table in Appendix B, find the capacitance to neutral of a 200-mile line composed of No. 00 stranded copper cable hung 4 feet apart. Compute the capacity reactance of this line from one wire to neutral on the basis of a 60-cycle e.m.f.

Prob. 31–6. What would be the charging current of line in Prob. 30, at a voltage to neutral of 52,000 volts?

Prob. 32–6. Electric power is transmitted from Keokuk, Iowa, to St. Louis, Mo., 143.6 miles, at 110,000 volts, three-phase, 25 cycles. The line consists of stranded cable, 300,000 cir. mils area, strung in vertical plane and spaced 10 feet apart without transposition, as shown in Fig. 221a. Compute:

(*a*) Capacitance of line, one wire to neutral.
(*b*) Capacity reactance of line, one wire to neutral.
(*c*) Charging current to each line wire.

84. Effect of Irregular Forms of E.M.F. Wave upon the Charging Current. Owing to the difficulties in properly proportioning the pole tips of a generator and in distributing the armature windings, the wave-form of the e.m.f. produced is rarely a true sine curve. It usually contains more or less well-defined ripples as explained and illustrated in Chapter VIII of the First Course. The greatest cause of these ripples is the fact that the wave-form produced does not consist of a simple wave but is usually made up of several waves. There is not only the e.m.f. wave of a given frequency and given effective value which the machine was designed to produce, but there are also other waves of greater frequencies and usually of much smaller effective values. Each of these waves approximates a true sine wave in form and the resultant wave-form of e.m.f. is merely a combination of them all.

FIG. 221a. A section of the 110,000-volt line from Keokuk, Iowa, to St. Louis, Mo. *General Electric Review.*

The wave-form which the machine was designed to produce is called the **fundamental** or **primary harmonic**; the others are called the **minor harmonics.** The minor harmonics produced in the line by the modern generator usually consist of very small waves which have a frequency of three, five, or seven times the frequency of the fundamental wave. Even higher frequency waves are sometimes produced, and like the lower frequencies, their relation to the fundamental frequency is usually an odd number.

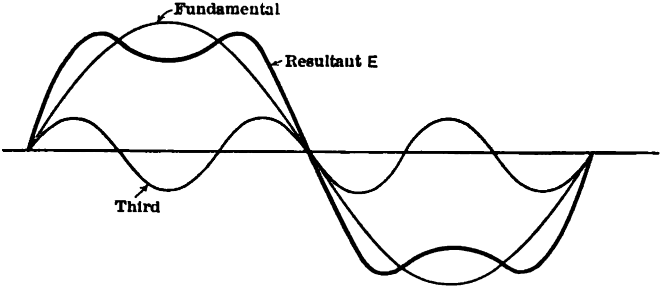

FIG. 222. A wave-form produced by a fundamental sine wave and a third harmonic in phase with the fundamental.

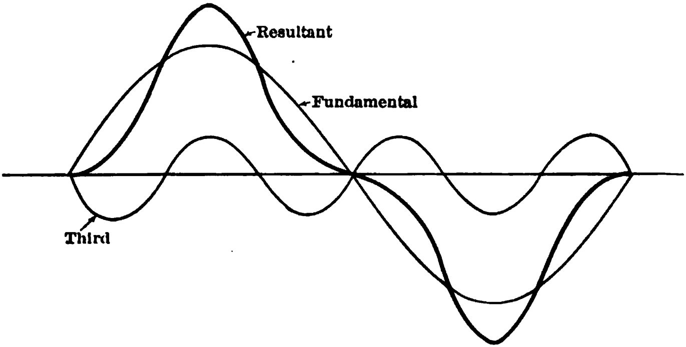

FIG. 223. A wave-form produced by a fundamental and a third harmonic which have a different phase relation than that of Fig. 222.

Fig. 222 shows the fundamental and a third harmonic of much smaller effective value. Each has its own sine wave-form. Note how they combine to produce a wave which has well-defined ripples. The third harmonic is made much larger in proportion to the fundamental than would exist in a properly designed machine. But a third harmonic of a smaller effective value would distort the fundamental in the same way, only to a less degree. Fig. 223 shows the same fundamental and third harmonic at a different phase with each other, and the resulting wave-form which they produce. In Fig. 224, a fifth harmonic combines with the fundamental to produce the resultant shown, and in Fig.

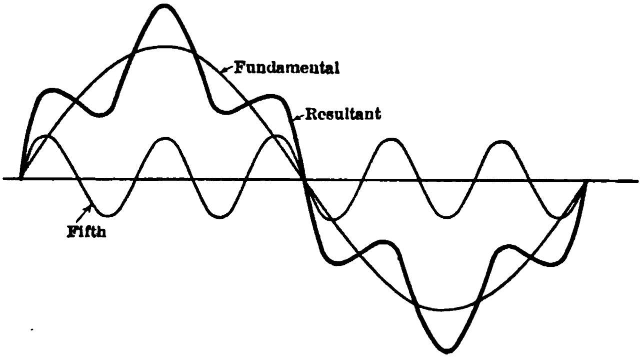

FIG. 224. The wave-form produced by the combination of a fifth harmonic with the fundamental.

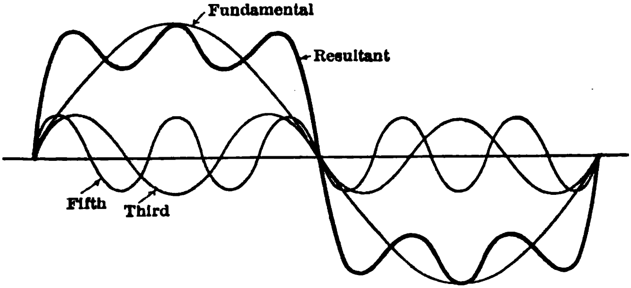

FIG. 225. The wave-form produced by the combination of both a third and a fifth harmonic with the fundamental.

225, both a third and a fifth of the same effective values combine with the fundamental to produce still another wave-form. Fig. 308a, First Course, contains an oscillogram of an e.m.f. wave, marked E, which is the resultant of a fundamental and a fifth harmonic. It also shows the oscillogram of the charging current which an e.m.f. with such a wave-form produces in transmission lines of large capacitance. Note how the small ripples in the e.m.f. curve are magnified in the current curve so that a very irregular wave-form is produced. The reason for this increase in irregularity in the current curve in a circuit possessing large capacitance can be seen from the following example.

Example 8a. The e.m.f. E, Fig. 225, is the resultant of a fundamental sine wave of 100 volts (effective), a third harmonic of 10 volts (effective), and a fifth harmonic of 10 volts (effective).

What are the component parts of the resultant curve of current when this e.m.f. is impressed upon a circuit containing:

(*a*) Resistance only, of 5 ohms?
(*b*) Inductive reactance only, 5 ohms (at 60 cycles)?
(*c*) Capacity reactance only, 5 ohms (at 60 cycles)?

(a) Circuit containing resistance only.

Since the resistance of the circuit would not change to any appreciable extent with the frequency of the impressed voltage, the line would offer practically the same resistance to the currents set up by each component of the e.m.f.

$$\text{Fundamental current} = \frac{E}{R} = \frac{100}{5} = 20 \text{ amperes.}$$

$$\text{Third-harmonic current} = \frac{E_3}{R} = \frac{10}{5} = 2.0 \text{ amperes.}$$

$$\text{Fifth-harmonic current} = \frac{E_5}{R} = \frac{10}{5} = 2.0 \text{ amperes.}$$

Thus the minor current holds the same relation to the fundamental current that the minor e.m.f.'s hold to the fundamental e.m.f. The ripples in the current curve would, therefore, be no more nor less pronounced than those in the e.m.f. curve.

(b) Circuit containing inductive reactance only.

The inductive reactance to the fundamental current is 5 ohms.

Since the inductive reactance of a circuit is proportional to the frequency (being equal to $2\pi fL$), the inductive reactance to the third-harmonic current will be 3 × 5, or 15 ohms, because the frequency is three times as great; and the inductive reactance to the fifth-harmonic current will be 5 × 5, or 25 ohms.

$$\text{Fundamental current} = \frac{100}{5} = 20 \text{ amperes.}$$

$$\text{Third-harmonic current} = \frac{10}{15} = 0.67 \text{ ampere.}$$

$$\text{Fifth-harmonic current} = \frac{10}{25} = 0.40 \text{ ampere.}$$

The minor components of the current are much smaller parts of the fundamental current than the minor components of the e.m.f. are of the fundamental e.m.f. Thus in an inductive circuit, the current curve is smoother than the irregular e.m.f. curve which produces it.

(c) **Circuit containing capacity reactance only.**

Since the capacity reactance of a circuit is inversely proportional to the frequency of the impressed e.m.f. $\left(\text{being equal to } \frac{1}{2\pi fC}\right)$, the capacity reactance offered to the third-harmonic current equals $\frac{5}{3}$ or 1.67 ohms; and the reactance to the fifth-harmonic current equals $\frac{5}{5}$ or 1 ohm.

$$\text{Fundamental current} = \frac{100}{5} = 20 \text{ amperes.}$$

$$\text{Third-harmonic current} = \frac{10}{1.67} = 6 \text{ amperes.}$$

$$\text{Fifth-harmonic current} = \frac{10}{1} = 10 \text{ amperes.}$$

The minor-harmonic currents have thus become much greater in proportion to the fundamental current than the minor harmonic e.m.f.'s are to the fundamental e.m.f. The irregularities of the current wave in a circuit containing capacity reactance only would thus be much greater than the irregularities in the e.m.f. curve producing the current. The following problems bring out the effect of resistance, inductive and capacity reactance on the effective value of the current produced by an e.m.f. of irregular wave-form.

Prob. 33–6. An e.m.f. with an irregular shaped wave-form, produced by a fundamental and a fifth harmonic, is impressed upon a circuit containing resistance only, of 4 ohms. The maximum value of the fundamental harmonic of e.m.f. is 100 volts; the maximum value of the fifth harmonic is 20 volts.

(*a*) Plot to large scale one cycle of the component and resultant e.m.f.'s with the fifth harmonic holding the same phase relation to the fundamental as in Fig. 224.

(*b*) Plot the component and resultant curves of current.

(*c*) Plot the squared values of one-half loop of the resultant current curve and find the effective current (root-mean-square value). If convenient, use a planimeter for finding area under the squared curve.

Prob. 34–6. Assume that the e.m.f. of Prob. 33 is impressed upon a circuit containing inductive reactance only, of 4 ohms at the frequency of the fundamental. Complete (*b*) and (*c*) of Prob. 33, and compare this value for effective current with that of Prob. 33.

Prob. 35–6. Assume that the e.m.f. of Prob. 33 is impressed on a circuit containing capacity reactance only, of 4 ohms at the frequency of the fundamental. Complete (*b*) and (*c*) of Prob. 33, and compare this value of effective current with that of Prob. 33.

85. Why the Voltage is Sometimes Higher at the Receiving End than at the Sending End. In considering this question, let us take as an example the transmission line of the Great Falls Power Company, which transmits 15,000 kw. to a distance of 130 miles from Great Falls to Butte, Montana, at a pressure of 100,000 volts.

Two three-phase lines are run on separate towers. The conductors of each line are No. 0 stranded hard copper cable with hemp centers, outside diameter 0.398 inch, and are strung in a horizontal plane and spaced 10 feet 4 inches, with no transpositions. The charging current at a generator voltage of 100,000 volts and 60 cycles was measured when the receiving end was open and was found to be 39 amperes per wire. This checks closely the computed value, as will be seen in Prob. 36–6.

By reference to Table III in Appendix B and using "equivalent" spacing of 13.0 feet, we find that the inductive reactance per mile of each wire of this line must be 0.8613 ohm, approximately.

The inductive reactance of the whole line equals

$$\begin{aligned} X_L &= 130 \times 0.8613 \\ &= 112 \text{ ohms.} \end{aligned}$$

The resistance of each wire must be

$$\begin{aligned} R &= 130 \times 0.530 \\ &= 68.9 \text{ ohms.} \end{aligned}$$

Let us examine the conditions in the line when it is open at the receiving end at Butte, and with enough voltage at the Great Falls end to supply the charging current of 39 amperes. We will assume that the voltage at Butte is 100,000 volts between conductors. We must then compute what voltage is required at the sending end to produce a charging current of 39 amperes in a line of 70 ohms resistance and 112 ohms inductive reactance, while maintaining 100,000 volts at the receiving end.

We will consider but one wire, and this with respect to neutral. The diagram for this arrangement is as in Fig. 226, which represents one-half of the total capacity of the line as a condenser at the receiving end, and the resistance and inductive reactance of the conductors lumped and joined in series along the wire instead of distributed uniformly as they are in the actual line.

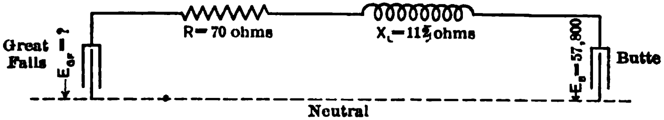

FIG. 226. Diagrammatic representation of the resistance and the inductive and capacity reactance of a single conductor of the Great Falls-Butte line.

The voltage between conductors at the receiving end at Butte is to be 100,000 volts, therefore the voltage to ground at Butte is equal to

$$E_B = \frac{100,000}{\sqrt{3}} = 57,800 \text{ volts.}$$

Draw vector OE_B, Fig. 227, to represent this voltage at the open receiving end.

The line OI_C, drawn at right angles to OE_B, will then represent the average charging current of 19.5 amperes which traverses the full length of the line and which leads the voltage producing it by 90°. But the generator has to maintain not only the voltage OE_B, but also the voltage to overcome the resistance and the inductive reactance of the line.

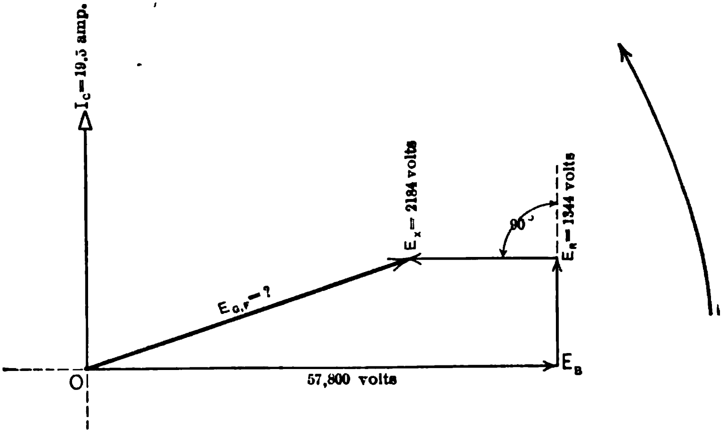

FIG. 227. The vector diagram of the voltage relations in an unloaded long transmission line. The voltage E_{GF} at the sending end is less than the voltage E_B at the receiving end.

The voltage to overcome the resistance is equal to

$$\begin{aligned} E_R &= 68.9 \times 19.5 \\ &= 1344 \text{ volts.} \end{aligned}$$

Since this voltage must be in phase with the current, we draw the vector E_R from the end of E_B and at right angles to E_B (parallel to the current vector I_C).

The voltage to overcome the inductive reactance must equal

$$\begin{aligned} E_x &= 112 \times 19.5 \\ &= 2184 \text{ volts.} \end{aligned}$$

Since this voltage must lead the current 90°, and therefore must lead E_R by 90°, we draw E_x 90° ahead of E_R.

The vector E_{GF}, joining O to the end of E_x, will then represent the voltage at the sending end, because it is the resultant of the series combination of E_B (the voltage at the receiving end), E_R (the voltage to overcome resistance of line) and E_x (the voltage to overcome the inductive reactance of the line).

The numerical value of the voltage at the generator end equals

$$\begin{aligned} E_{GF} &= \sqrt{(57{,}800 - 2184)^2 + 1344^2} \\ &= 55{,}634 \text{ volts.} \end{aligned}$$

The voltage between conductors at the generator end must equal

$$1.732 \times 55{,}634 = 96{,}458 \text{ volts.}$$

This value is distinctly lower than the 100,000 volts which is the pressure between conductors at the receiving end.

Note that this decrease in voltage is due to the presence of both capacity and inductance in the circuit. One tends to neutralize the other; thus the voltage (E_x) used to overcome the inductive reactance is in the opposite direction to the voltage (E_B) used to overcome the capacity reactance of the line. The resistance voltage E_R does not affect, to any appreciable extent, the value of the voltage E_{GF} at the generator end of the line.

Prob. 36–6. Compute from data of size, spacing, etc., in text above, the capacitance of one wire to neutral of the Great Falls-Butte line. From this value of capacitance and the voltages in the above text compute the charging current per wire and check against value taken from tables in Appendix B.

Prob. 37–6. Compute the open-line voltage at the generator end of the line from Great Falls to Greenville, S. C., if the voltage at Greenville is 100,000 volts. Frequency = 25 cycles. For remaining data see Prob. 11–6. Consider that the two sets of conductors are so far apart that they do not affect each other.

Prob. 38-6. What is the voltage between line conductors at the generating end of the line, in Prob. 30, when the voltage to neutral at the receiving end is 52,000 volts?

Prob. 39-6. How many kilovolt-amperes are used in charging the three wires of the Great Falls-Butte line, if the voltage at the open receiving end is 100,000 between conductors?

86. Regulation of Transmission Line Containing Capacitance. The presence of capacitance in a line is often advantageous to the system, especially when the load has a lagging power-factor. To show this, we merely have to note the effect of capacitance upon the regulation of a given line for a given load and power-factor.

The full load of the Great Falls-Butte transmission is 15,000 kw. at 85 per cent power-factor distributed equally between the two lines, or 7500 kw. on each.

To find the current per conductor:

$$P = \sqrt{3}\, IE \cos\theta$$

$$I = \frac{P}{\sqrt{3}\, E \cos\theta}$$

$$= \frac{7{,}500{,}000}{1.73 \times 100{,}000 \times 0.85}$$

$$= 51 \text{ amp. per conductor.}$$

At a power-factor of 0.85 the current lags practically 32° behind the voltage. Draw the vector E_{load}, Fig. 228, to represent the voltage across the receiving end, 32° ahead of the vector I_{load}, which represents the current of 51 amperes taken by the load. The charging current of 19.5 amperes is then represented by the vector I_C, which is 90° ahead of the voltage vector E_{load}, and consequently $90° + 32°$ or 122° ahead of the I_{load}. The current which the line carries must be the combination of the load current and the capacity current. This is represented by the vector I_{line} which is the resultant of the vectors I_{load} and I_C.

The value of I_{line} is found by means of the equation for the diagonal of a parallelogram as given in Appendix A, First Course.

$$I_{\text{line}} = \sqrt{I_{\text{load}}^2 + I_c^2 + 2\,I_c I_{\text{load}} \cos 122^\circ}$$

$$I_{\text{line}} = \sqrt{51^2 + 19.5^2 + 2 \times 19.5 \times 51 \times \cos 122^\circ}$$

$$= 43.9 \text{ amp.}$$

Thus a line current of only 43.9 amperes is able to supply a current of 51 amperes to the load, because of the effect which the leading charging current has when it combines with a lagging load current to produce the total line current.

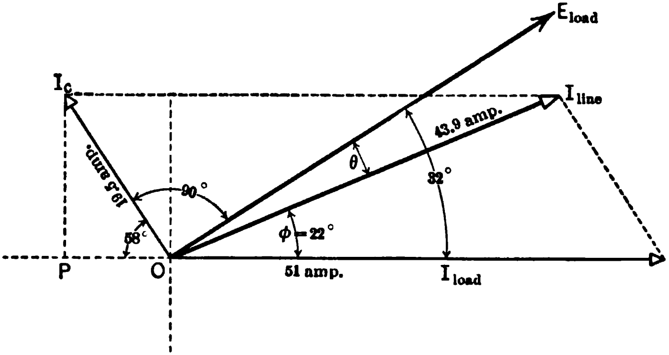

FIG. 228. Current and voltage relations at the load end of a long transmission line carrying an inductive load. The line current I_{line} is less than the load current I_{load} because of the capacity current I_c.

The angle ϕ by which this line current I_{line} leads the load current I_{load} can be found as follows:

Drop a perpendicular from end of vector I_c.

$$I_cP = 19.5 \sin 58^\circ$$

$$= 16.5.$$

A perpendicular dropped from the end of vector I_{line} will have the same length, 16.5.

Thus

$$\sin \phi = \frac{16.5}{43.9}$$

$$= 0.376$$

$$0.376 = \sin 22^\circ.$$

$$\phi = 22^\circ \text{ approx.}$$

The angle θ which is the phase difference between the voltage of the load E_{load} and the line current I_{line} can then be found:

$$\theta = 32^\circ - 22^\circ$$
$$= 10^\circ.$$

To find the voltage regulation of the line with this load and power-factor, construct Fig. 229, repeating Fig. 228 as a basis in order to get the proper phase relations. Draw vector E_{load} at an angle of 32° leading the vector I_{load}, and 10° ahead of line current I_{line}.

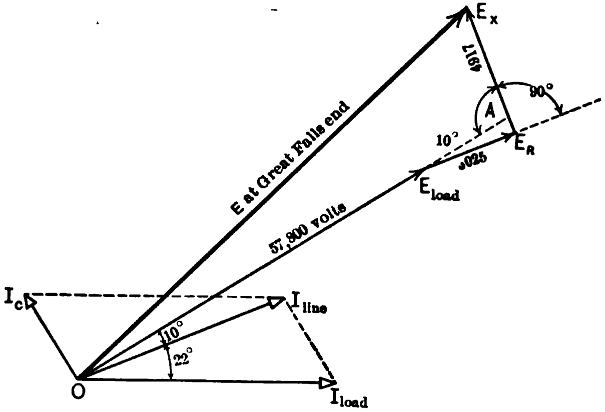

FIG. 229. Vector diagram for finding the voltage E at the sending end of a long transmission line which is carrying an inductive load.

The voltage necessary to send the line current of 43.9 amperes against a resistance of 70 ohms per wire equals

$$E_R = 70 \times 43.9$$
$$= 3025 \text{ volts.}$$

This voltage is in phase with the line current and is represented by the vector E_R drawn lagging 10° behind the load voltage vector E_{load}.

The voltage necessary to send the line current of 43.9 amperes against the reactance of 112 ohms per line wire equals

$$E_x = 112 \times 43.9$$
$$= 4917 \text{ volts.}$$

This voltage must be 90° ahead of the current and thus the vector E_x representing it is drawn 90° ahead of vector E_R.

The vector E, joining O and the end of the vector E_x, must represent the voltage which is necessary to overcome these three components, and thus represents the voltage at the Great Falls end of the line.

To find the value of E, the voltage at the sending end, extend the line E_{load} until it meets E_x at A. The angle at A equals $90° + 10° = 100°$.

$$\begin{aligned}
&\text{The amount cut from } & E_x &= 3025 \tan 10.0° = 533. \\
&\text{The line} & AE_x &= 4917 - 533 \\
&&&= 4384. \\
&\text{The extension of} & E_{\text{load}} &= \frac{3025}{\cos 10.0°} \\
&&&= 3072. \\
&\text{The line} & OA &= 57{,}800 + 3072 \\
&&&= 60{,}870.
\end{aligned}$$

The triangle OAE_x can be solved as follows:

$$\begin{aligned}
E &= \sqrt{OA^2 + AE_x^2 - 2\,OA \times AE_x \cos 100°} \\
&= \sqrt{60{,}870^2 + 4384^2 - 2 \times 60{,}870 \times 4384 \times \cos 100°} \\
&= 61{,}780 \text{ volts.}
\end{aligned}$$

The voltage to neutral at the generator end at full load then equals 61,780 volts.

We have computed that 55,630 volts at the generator end at no-load produces a receiving-end voltage of 57,800 volts. A voltage of 61,780 at the generator end would therefore produce approximately $\frac{61{,}780}{55{,}630} \times 57{,}800$ or 64,200 volts at the receiving end when the line was open.

The voltage regulation at 85 per cent power-factor will therefore be $\frac{64{,}200 - 57{,}800}{57{,}800} = 11.0$ per cent. We thus have a line the receiving end of which has a voltage between conductors of 100,000 at full load, but of 111,000 volts at no-load. (Data, pages 2002–3–4 of Proc. A.I.E.E., 1911.)

In the above examples, the results are only approximations, due to the fact that the impressed voltages do not have a pure sine wave-form. As explained in a previous article and in Chap. VIII, of "First Course," ripples, or harmonics, occur (to a slight extent, to be sure) in the wave-form of all commercial generators. These ripples are greatly magnified by the line capacity and tend to make the charging current, and the voltage values depending upon it, somewhat larger than the usual computed values. The above method, however, gives values which differ so little from tested values, that it can be used with confidence in all commercial computations.

Prob. 40–6. Compute the voltage regulation of the Great Falls-Butte line at unity power-factor. Load, 7500 kw. at 100,000 volts.

Prob. 41–6. (*a*) Compute the voltage regulation of the Great Falls-Butte line at 90 per cent leading power-factor. Load, 7500 kw. at 100,000 volts.

(*b*) What effect does the capacity of the line have upon the regulation when the power-factor is (1) Lagging? (2) Unity? (3) Leading?

Prob. 42–6. The three-phase transmission line from Shoshone to Denver, Colorado, is 153.5 miles long. The conductors are arranged in a horizontal plane, 124 inches apart with no transpositions, and consist of No. 0 six-strand hemp-center copper cables. When 100,000 volts at 60 cycles are impressed on the Shoshone end, what current will flow per wire if the Denver end is open?

Prob. 43–6. What will be the voltage at the Denver end of the line in Prob. 42?

Prob. 44–6. What voltage at the generator end is necessary to deliver 5000 kw. at 100,000 volts at 0.80 power-factor at the Denver end of line in Prob. 42?

Prob. 45–6. What is the voltage regulation of the line under the conditions of Prob. 44?

Prob. 46–6. What would be the voltage regulation of the line in Prob. 44 if the load of 5000 kw. had unity power-factor?

87. Capacitance of Underground Cables. The capacitance of underground cables is very high in comparison with that of overhead cables because the cables are laid with very little space between them. The insulation material, rubber or impregnated paper, also makes the capacitance from two to four times higher on account of a certain **dielectric power** which it possesses to a much greater degree than air. All of these conditions combine to produce a condenser of large capacitance. Even two- and three-tenths of a microfarad per mile are not uncommon values. This, together with the fact that the breakdown strength of the insulation limits the voltage, renders it impracticable to transmit power by alternating current any great distance underground or by submarine cables. In most large cities cables are laid in underground ducts up to distances of 10 miles and at voltages between 11,000 and 23,000 volts. Of course this disadvantage does not exist in the transmission of direct-current power.

Data for the following problems were furnished by the Standard Underground Cable Co.

Prob. 47–6. In a certain three-conductor three-phase cable, each conductor is No. 00 B. & S. and is covered with paper insulation $\frac{9}{32}$ inch thick over each conductor, and over the three insulated conductors is placed a paper belt $\frac{9}{32}$ inch thick. Compute the capacitance of one conductor to neutral for one mile of this cable. The dielectric power of the paper causes the capacitance to be 3.7 times as high as it would be if the space between the cables were air.

Prob. 48–6. What would be the charging current per conductor of a line consisting of 10 miles of the cable of Problem 47–6, if the voltage was 23,000 between conductors? Frequency, 60 cycles.

Prob. 49–6. Compute the reactance to neutral of one conductor of the cable in Prob. 48–6. See page 233, First Course.

Prob. 50–6. The cable in Prob. 48–6 has, under certain (average) conditions, a safe carrying capacity of 7500 kv-a. What is the voltage regulation of the line when carrying its maximum safe load at 90 per cent power-factor and 22,000 volts?

88. Current Surges and Oscillations in Long Lines.

A long line is subjected to current surges from two causes, — (*a*) lightning discharges in the vicinity of the line; (*b*) the necessary switching operations. The more serious of these are likely to be the lightning disturbances. A cloud, generally charged positively as in Fig. 230, comes near a portion of the line, and attracts a large negative charge to this part of the conductor. When the cloud is discharged by a lightning-flash either to earth or to another cloud, this large negative charge on the wire is suddenly released and rushes along the wire, just as a flood of water rushes along a narrow valley when the retaining wall of a reservoir at its head suddenly gives way.

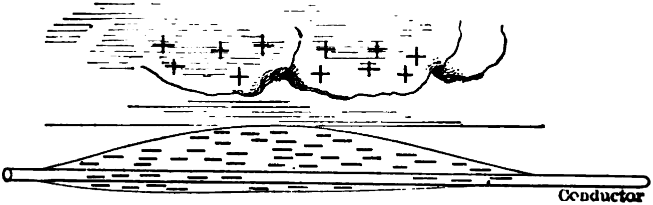

FIG. 230. The positive charge on the cloud draws a negative charge to that part of the conductor nearest the cloud.

If the wave-front of this surge or electric flood hits the windings of a transformer or generator, these windings act as a wall acts to the sudden rush of water. The inductance of the windings opposes any sudden passage of electric charge or growth of the current through them, and the electric charge "piles up" against the transformer. This induces such an excessive pressure between the windings that a charge may be forced through the insulation, and an arc started. While the normal voltage between the turns is never enough to start an arc, once the insulation has been broken down and an arc has been started by a momentary higher voltage, the line voltage is usually sufficient to maintain the arc long enough to severely damage the machine.

In addition to the damage done to the generator or transformer, this arc also sets up very disturbing oscillations in the line, which may damage other machines connected to it.

Similar surges and oscillations may be set up in switching the current on and off the line. The larger the current switched on or off, the greater the disturbance.

As a general rule, in switching on the current it is best to connect the step-down transformers to the receiving end before connecting the step-up transformers to the generator.

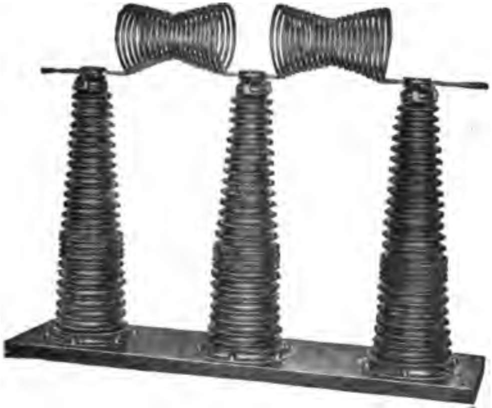

FIG. 231. Hour glass choke coils for 110,000-volt lines. These coils choke back the steep-front and high-frequency waves due to lightning discharges in the vicinity of the line. *The General Electric Co.*

89. Lightning Protection. In order to keep the machines from being damaged by line surges, choke coils are connected between the lines and the various machines. Fig. 231 is an illustration of a common form of a choke coil. This allows the regular current waves to pass with very little impedance but chokes back the surges and waves of high frequency because the impedance of the coil is practically all due to its reactance, which is directly proportional to the frequency of the current waves. The surges dash up against these choke coils just as the waves dash against a breakwater, and, of course, the pressure to ground is raised to a high value at this point. In order that this high pressure may not send the surges rebounding back along the line, an arrangement for conducting the charge to ground is tapped on the line at this point. This consists of a **horn gap,** shown in Fig. 232. One side of the horn is connected to the ground and the other side to one of the conductors. The ordinary line voltage is not enough to cause an arc across the gap, but a dangerously high voltage breaks down the air insulation at the smallest space and forms an arc. The heated air around the arc, and the magnetic effect of the arc cause the arc to travel up the gap. The horns are so

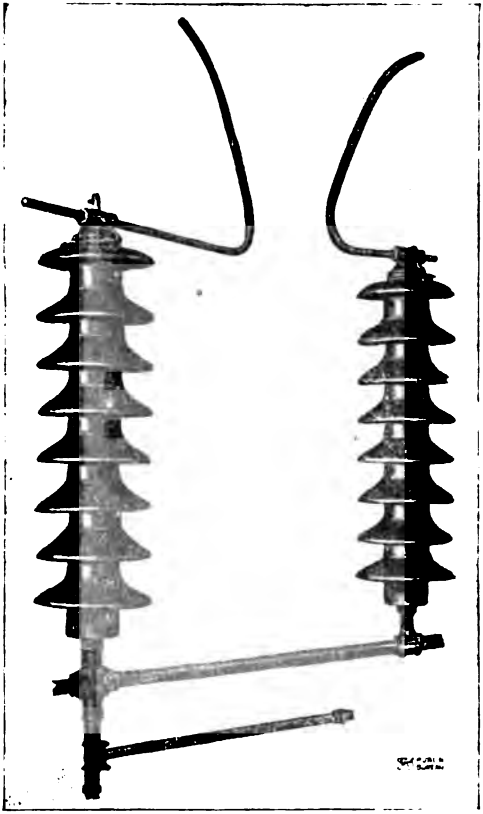

FIG. 232. Horn gap for 110,000-volt aluminum lightning arrester. One horn is connected to the line and the other to the ground, generally through an aluminum cell. *The General Electric Co.*

constructed that the distance between them gradually increases toward the top. Thus as the arc travels up, it soon reaches a place where the distance is too great for the voltage to maintain the arc, and it is thus extinguished. The excess charge on the conductor is thus harmlessly conducted to the ground instead of being sent back over the line.

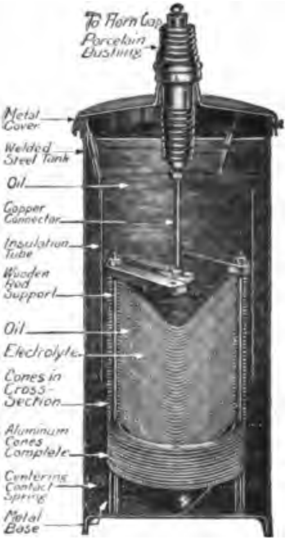

FIG. 233. Cross section of an aluminum lightning arrester. *The General Electric Co.*

Fig. 233 shows the construction of an aluminum cell lightning arrester which is often connected in series with the grounded end of the horn gap. It consists of a number of aluminum plates immersed in an electrolyte. A coating of aluminum hydroxide is formed over the plates, which requires from 320 to 340 volts per pair to break down and set up a current. By placing the proper number of these plates in series, and setting the horn gap very close to the point of arcing at the usual line voltage, the line can be relieved continually of any excess charge which may be concentrated at that point by the choke coil. The aluminum arrester also has the advantage of not allowing any current to flow in the reverse direction through it. It therefore checks all oscillations across the horn gap. It has the disadvantage of deteriorating if a charge is not sent through it every day or two. This necessitates a daily closing of the horn gap enough to allow a charge to pass through. It will not do to leave cells connected directly to the line without a small gap in series, as they allow too much current to leak through them.

90. Corona Loss. When the voltage between two conductors has been raised somewhat above a certain value called the "critical disruptive voltage," the conductors begin to glow and a sort of halo surrounds the wire. The name of **corona** has been given to this glowing of the conductor. There is a certain amount of power dissipated into the air as soon as the critical voltage is reached, even before any glow is apparent. The name of **corona loss** is applied to all the power lost in this way.

The "disruptive critical voltage" may be found by means of the following formula:*

$$E_0 = 105{,}000\, r \log_{10} \frac{s}{r},$$

in which E_0 = **critical voltage to neutral,** effective value.
r = radius of conductor, in inches.
s = spacing between centers of conductors, in inches.

Note that this critical voltage depends upon the radius of the conductors and upon the distance between them. Other things being equal, corona loss will begin at lower voltage when the diameter of the wires or the distance between them is reduced.

The above formula applies to the stranded conductors only and to them under ordinary fair-weather conditions only.

For a round smooth wire the critical voltage is somewhat higher, while fog, sleet and snow lower it to a marked extent. The current which this voltage forces into the surrounding air is in phase with the voltage.

The corona loss under the above conditions can be found from the following approximate formula:

$$P_m = \frac{5.54\, f \sqrt{\frac{r}{s}}\,(E - E_0)^2}{10^6},$$

---

* This and the following formula for corona loss are adapted from formulæ given by Mr. F. W. Peek, Jr., in Proc. A.I.E.E., 1912.

in which P_m = loss per mile per conductor, in **watts.**

f = frequency, in **cycles per second.**

E = voltage of line to neutral.

E_0 = "critical disruptive" voltage to neutral.

Note that the corona loss is proportional:

(*a*) Directly to frequency (for commercial ranges).

(*b*) Directly to the square of the excess of voltage to neutral above critical value.

(*c*) Directly to the square root of the radius of the conductor, and inversely to the square root of the spacing of the conductors.

Example 9. What is the fair-weather corona loss on a 150-mile three-phase line, operating at 100,000 volts, 60 cycles? The cables are No. 0 stranded copper and are spaced 9 feet 2 inches apart in the form of an equilateral triangle. (Outside diameter of No. 0 bare cable is 0.373 ins.)

The critical voltage is

$$\begin{aligned} E_0 &= 105{,}000\, r \log \frac{s}{r} \\ &= 105{,}000 \times 0.187 \log \frac{110''}{0.187''} \\ &= 54{,}400 \text{ volts.} \end{aligned}$$

The corona loss per cable per mile is

$$\begin{aligned} P_m &= \frac{5.54 f \sqrt{\frac{r}{s}}\,(E - E_0)^2}{10^6} \\ &= \frac{5.54 \times 60 \sqrt{\frac{0.187}{110}}\,(57{,}800 - 54{,}400)^2}{10^6} \\ &= 160 \text{ watts.} \end{aligned}$$

For a 150-mile three-wire line the loss equals

$$\begin{aligned} P &= 3 \times 150 \times 160 \\ &= 72{,}000 \text{ watts} \\ &= 72 \text{ kw.}^* \end{aligned}$$

Prob. 51–6. Compute the corona loss in the line of Example 9 at a frequency of 25 cycles.

---

[*] This assumes no voltage drop along the line. Usually the drop is so great that only part of a line at any one time suffers a corona loss.

Prob. 52-6. (*a*) What would be the loss in the line of Example 9 if the pressure between conductors were raised to 110,000 volts? (*b*) Compare ratio of loss with ratio of voltages.

Prob. 53-6. Show that for pressures of 44,000 volts and under, the corona losses are negligible on a line of standard spacing and commercial frequencies.

Prob. 54-6. What would be the corona loss in Example 9 if for the copper line conductors were substituted aluminum conductors of equivalent conductivity?

91. Efficiency of Transmission Lines. By efficiency of transmission lines is meant the efficiency of the conductors only. The transformers or other apparatus are not to be included as part of the line. This efficiency must be measured under standard conditions, with a non-inductive load at the receiving end, with voltage of rated value and rated frequency and of sine wave-form. Since a line rarely operates under standard conditions, it is often desirable to find the efficiency under given conditions. But if no conditions are specified as to power-factor, sine wave-form, etc., standard conditions are understood to be meant. In computing the efficiency of the line, therefore, we have merely to divide the kilowatts **delivered by the line wires** to the apparatus at the receiving end by the kilowatts **received by the line wires** at the generator end under standard conditions.

The values are most easily arrived at by the following means:

$$\left.\begin{array}{l}\text{Power}\\ \text{received by}\\ \text{line wires}\end{array}\right\}\text{must equal}\left\{\begin{array}{l}\text{Power}\\ \text{delivered by}\\ \text{line wires}\end{array}\right\}+\left\{\begin{array}{l}\text{Power lost in}\\ \text{line wires.}\end{array}\right.$$

$$\left.\begin{array}{l}\text{Power}\\ \text{lost in line}\\ \text{wires}\end{array}\right\}=\left\{\begin{array}{l}I^2R\text{ loss}\\ \text{in line}\\ \text{wires}\end{array}\right\}+\left\{\begin{array}{l}\text{Corona loss}\\ \text{in line wires}\end{array}\right\}+\left\{\begin{array}{l}\text{Leakage loss}\\ \text{in line wires.}\end{array}\right.$$

In a well-constructed power transmission line the "leakage loss" is negligible, on account of the relatively small number of points of support where leakage may occur, therefore:

Efficiency of line equals

$$\frac{\text{Power delivered (by line wires)}}{(\text{Power delivered}) + (I^2R \text{ loss}) + (\text{Corona loss})}.$$

Example 10. The following data for a typical three-phase transmission line are adapted from the *Electric Journal*, 1913, page 839.

Length of line	200 miles.
Frequency	60 cycles.
Load delivered to step-down transformers at load end of line	11,250 kw.
Power-factor lagging, at high-tension terminals of step-down transformers	85 per cent.
Voltage between conductors at receiving end of line	108,000 volts.
Conductors, copper cables	250,000 cir. mils.
Mean spacing of conductors	12.6 feet.
Resistance of transformers and protective coils at each end referred to high-tension side	4.1 ohms.
Reactance of transformers and protective coils at each end referred to high-tension side	64.5 ohms.

Find the efficiency of the line under these conditions. Note that the conditions are not quite standard, in that the power-factor is less than unity.

$$\text{Volts to neutral} = \frac{108{,}000}{1.73} = 62{,}400 \text{ volts}.$$

The resistance of each conductor of the line, from table, equals

$$200 \times 0.2165 = 43.3 \text{ ohms}.$$

The reactance of each line conductor equals

$$200 \times 0.804 = 161 \text{ ohms}.$$

From Table 7, Appendix B, we find that the charging current of each conductor equals

$$200 \times 5.41 \times \frac{62{,}400}{1{,}000{,}000} = 67.5 \text{ amp}.$$

Power taken by each step-down transformer equals

$$\frac{11,250}{3} = 3750 \text{ kw.}$$

Current taken by each transformer equals

$$I = \frac{3,750,000}{0.85 \times 62,400} = 70.6 \text{ amp.}$$

Fig. 234 shows the high-tension side of the transformers at each end of the line. The resistance and reactance of the transformers referred to the high-tension side are represented, and also the resistance and inductive reactance of each line wire are represented. Let us consider line wire AB only.

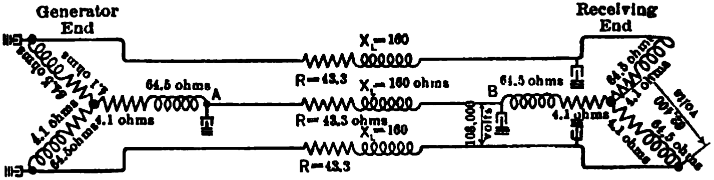

FIG. 234. Diagram of a transmission line showing the values and relative arrangement of the resistance and reactance of line and transformers.

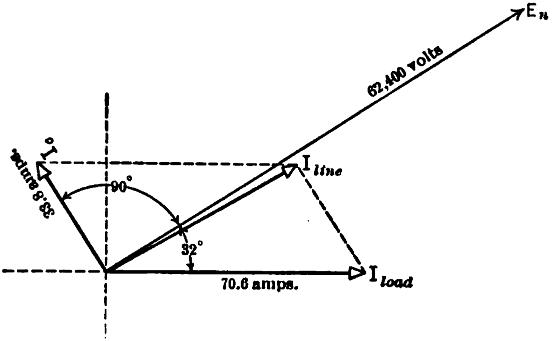

FIG. 235. The line current I_{line} is the resultant of the load current I_{load} and the charging current I_c.

The current in AB is a combination of the charging current, $\frac{67.5}{2}$ or 33.8 amperes, and the transformer current of 70.6 amperes, and may be found by constructing the vector diagram of Fig. 235.

$$I_{\text{line}} = \sqrt{70.6^2 + 33.8^2 + 2 \times 70.6 \times 33.8 \cos 122^\circ}$$
$$= 60.0 \text{ amp.}$$

The I^2R loss in one line wire equals

$$60.0^2 \times 43.3 = 156 \text{ kw.}$$

The I^2R loss in the three conductors equals

$$3 \times 156 = 468 \text{ kw.}$$

The disruptive critical voltage equals

$$E_0 = 105{,}000\, r \log \frac{s}{r}$$
$$= 105{,}000 \times 0.288 \log \frac{151}{0.288}$$
$$= 82{,}300 \text{ volts.}$$

Since the voltage to neutral is only $\frac{108{,}000}{\sqrt{3}}$ or 62,400 volts, it is less than the critical disruptive voltage, and there would be no corona loss in fair weather.

The corona loss = 0.

Total loss in line wires therefore equals the I^2R loss

= 468 kw.

Total input into line is equal to:

Load delivered to step-down transformers = 11,250 kw.
I^2R loss in line = 468 kw.
Total input into line = 11,718 kw.

$$\text{Efficiency of transmission line} = \frac{11{,}250}{11{,}718} = 96.0 \text{ per cent.}$$

Note that this is the line efficiency under the special condition of a load with a lagging power-factor of 85 per cent.

Prob. 55–6. Find the efficiency of the line in Example 10, if the voltage at the receiving end were raised to 150,000 volts. Amount of power delivered, power-factor, and all other conditions the same as in the example.

92. Over-all Efficiency of Transmission. It is often necessary to compute the efficiency of transmission from the generator terminals to the load terminals, and to determine the power-factor of the generator. In this case it is necessary to count the losses in the transformers, feeder regulators, current limiting reactances and choke coils as well as the line losses. The simplest way to arrive at the total amount of power delivered by the generator is to combine all the power quantities taken by the several parts of the system.

First. Resolve the power taken in each part of the system into two components at 90° to each other, namely: the **Effective Power,** and the **Reactive Power.**

Second. Add all the effective power quantities together to obtain the total effective power, and all the reactive power quantities to obtain the total reactive power.

Third. Total apparent power delivered by the generator equals the square root of the sum of the squares of the total effective and the total reactive power.

Fourth. The power-factor of the generator equals the ratio of the effective power to the apparent power, delivered by the generator.

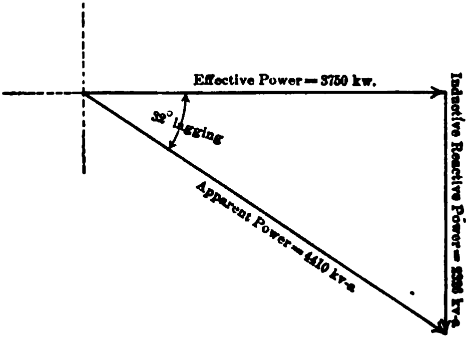
Effective Power = 3750 kw.
32° lagging
Apparent Power = 4410 kv-a
Inductive Reactive Power = 2326 kv-a

FIG. 236. The apparent power 4410 kv-a. is the resultant of the inductive reactive power 2326 kv-a. and the effective power 3750 kw.

Example 11. Find the over-all efficiency of transmission in Example 10.

First. The power delivered to each step-down transformer equals 3750 kw. at 85 per cent power-factor. By constructing Fig. 236, we see that this produces an apparent power of $\frac{3750}{0.85} = 4410$

kv-a., of which $\sqrt{4410^2 - 3750^2}$ (or 4410 sin 32°) equal to 2326 kv-a. is inductive reactive power.

The effective power consumed by each line wire equals $60.0^2 \times 43.3 = 156$ kw.

Construct Fig. 237, adding the 195 kw. in phase with the effective power of 3750 kw. delivered to one of the step-down transformers by the line wire connected to it. The effective power consumed by each step-up transformer with accompanying choke coils and current-limiting reactance coils is composed of the I^2R loss and the core losses. The core losses in a well-designed transformer of this size average about 80 per cent of the I^2R loss. Total effective power loss of one step-up transformer equals

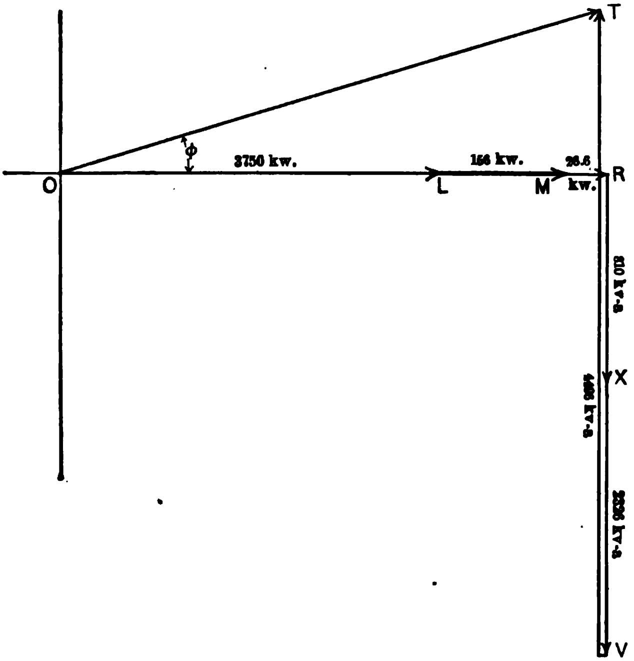

Fig. 237. Total apparent power *OT* delivered to the step-up transformers is the resultant of the total reactive and the total effective power delivered.

$$1.80 \times (60.0^2 \times 4.1) = 26.6 \text{ kw.}$$

Add this in Fig. 237 to the lines representing the previously determined effective power.

The inductive reactive power taken by one line conductor and one step-up transformer equals

$$(64.5 + 161)\, 60.0^2 = 810 \text{ kv-a.}$$

Draw, in Fig. 237, the line *RX* at right angles (lagging) to the line *OR*, to scale representing this 810 kv-a. inductive reactive power. Add to this line the line *XV* representing the inductive reactive power delivered to each step-down transformer, namely 2326 kv-a. By means of a diagram similar to Fig. 229, the equivalent voltage to neutral at the sending end is found to equal approximately 66,600 volts.

The capacity reactive power in each phase equals approximately

$$67.5 \times 66{,}600 = 4496 \text{ kv-a.}$$

Draw the vector *VT* in the opposite direction to the inductive-reactive-power vectors. This completes the effective and reactive power which must be supplied to each step-up transformer by the generator. The resultant vector *OT* will thus represent the power supplied by the generator.

$$OT = \sqrt{RT^2 + OR^2}$$

$$OT = \sqrt{(3750 + 156 + 26.6)^2 + (4496 - 810 - 2326)^2}$$

$$= 4160 \text{ kv-a.}$$

$$\text{Power-factor of generator} = \cos\phi = \frac{OR}{OT} = \frac{3750 + 156 + 26.6}{4160}$$

$$= 94.6 \text{ per cent leading.}$$

Total power delivered to the three step-up transformers

$$4160 \times 3 = 12{,}480 \text{ kv-a., representing}$$

$$3 \times (3750 + 156 + 26.6) = 11{,}800 \text{ kw. at } 94.6\% \text{ power-factor.}$$

We have only to find the power delivered by the step-down transformer to the load in order to determine the over-all efficiency of transmission.

Effective power consumed by each step-down transformer:

$$I^2R \text{ loss} = 70.6^2 \times 4.1 = 20.42 \text{ kw.}$$

$$I^2R + \text{core loss} = 1.8 \times 20.4 = 36.8 \text{ kw.}$$

Inductive reactive power taken by each step-down transformer:

$$70.6^2 \times 64.5 = 321 \text{ kv-a.}$$

Referring to Fig. 236, which represents the total power delivered to the step-down transformer, we see that the effective power delivered to each phase of the load equals

$$3750 - 37 = 3713 \text{ kw.}$$

The inductive reactive power delivered to each phase of the load equals

$$2326 - 321 = 2005 \text{ kv-a.}$$

From Fig. 238, constructed from these values, we find total apparent power delivered to load by each step-down transformer equals

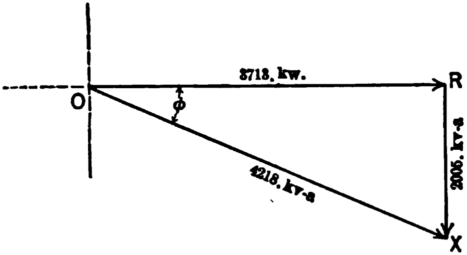

FIG. 238. The total effective power delivered by the step-down transformers to the load is represented by the vector *OR*. Reactive power delivered equals *RX*. Effective power equals *OX*.

$$\sqrt{3713^2 + 2005^2} = 4218 \text{ kv-a.}$$

$$\text{Power-factor of load} = \cos\theta = \frac{3713}{4218} = 88.0 \text{ per cent.}$$

Total effective power delivered to load equals

$$3713 \times 3 = 11{,}139 \text{ kw.}$$

$$\text{Over-all efficiency of transmission} = \frac{11{,}139}{11{,}800} = 94.4 \text{ per cent.}$$

Prob. 56-6. Find the over-all efficiency of transmission of Example 11 under standard conditions when delivering the same power to the load transformers.

Prob. 57-6. What would be the over-all efficiency of transmission in Example 11 if the voltage were raised to 150,000 volts, all other conditions remaining as in Example 11? Assume that the transformer and reactance coils are rewound so that they have the same losses as before, when transforming the same kilovolt-amperes.

SUMMARY OF CHAPTER VI

LONG TRANSMISSION LINES, in addition to resistance and inductive reactance, have a capacity reactance.

CAPACITANCE is a sort of electric elasticity, and a line possessing it may be likened to an elastic pipe-line.

THE AMOUNT OF CAPACITANCE, OR ELASTICITY, that a line possesses, is measured in the number of ampere-seconds of electricity which one volt can force on the line. If one volt could force one ampere-second on the line, the capacitance of the line would be one FARAD.

The equation for the electric charge upon a condenser is therefore

$$Q = EC,$$

where Q = quantity of electricity, in AMPERE-SECONDS.
E = steady pressure across line, in VOLTS.
C = capacitance of line, in FARADS.

A MICROFARAD is the common unit of capacitance, and equals one-millionth of a farad.

A CONDENSER consists of two conductors separated by an insulating material called the DIELECTRIC. The larger the plate area and the thinner the dielectric, the larger the capacity of the condenser. The unlike charges of electricity upon the plates attract and bind each other.

A TRANSMISSION LINE forms a condenser, one wire constituting one conductor, the ground or another wire the other conductor, the air between being the dielectric.

THE CAPACITANCE of one wire to neutral is

$$C_m = \frac{0.0388}{\log_{10} \frac{s}{r}},$$

in which C_m = the capacitance per mile of wire, in MICROFARADS.
s = distance between centers of wires.
r = radius of wire.

s and r MUST BE EXPRESSED IN TERMS OF THE SAME UNITS.

THE CAPACITANCE between two wires is one-half of that of one wire to ground, because the two-wire condenser is a series combination of two condensers each formed by one wire and the ground. This assumes a symmetrical arrangement of wires, as is usual.

THE CAPACITANCE OF CONDENSERS IN PARALLEL equals the sum of their separate capacitances.

THE CAPACITANCE OF CONDENSERS IN SERIES equals the reciprocal of the sum of the reciprocals of their separate capacitances.

AN ALTERNATING E.M.F. ACROSS A CONDENSER causes an alternating current, called the charging current, to flow in the condenser. If the e.m.f. has a sine wave-form, the charging current will have a sine wave-form.

THE CHARGING CURRENT leads the impressed voltage across the condenser by 90 electrical degrees.

THE CAPACITY REACTANCE of a circuit is the ratio of the impressed volts to the charging current at a given frequency.

$$X_c = \frac{E_c}{I_c}.$$

If the frequency of the current and the capacitance of the line are known, the capacity reactance can be found from the equation

$$X_c = \frac{1}{2\pi fC}.$$

THE CHARGING CURRENT OF LINE may be found by dividing the sum of the voltages at sending and receiving end, by twice the capacity reactance of the line. This assumes that the capacitance of the line is concentrated in two condensers of equal capacitance, one situated at each end of the line. One-half the charging current is assumed to be sent the full length of the line.

Tables of capacitances and charging currents are available for lines of standard sizes of conductors and standard spacings.

THE PRESENCE OF MINOR HARMONICS in the wave-form of the e.m.f. impressed upon a circuit of large capacitance causes the charging current to be much more irregular in form than the e.m.f. curve. The higher the frequency of the harmonic the greater the distortion of the current wave.

THE VOLTAGE AT THE RECEIVING END of long unloaded lines is usually higher than the voltage at the sending end. This is due to the fact that the effect of the voltage to overcome the resistance is negligibly small, so that the voltage at the sending end has to overcome only the combination of the inductive reactance and the capacity reactance. Since these two reactances are opposite in effect, the voltage at the sending end equals the difference between the voltage required to overcome the capacity reactance and the voltage to overcome the inductive reactance. This difference is smaller than the voltage to overcome the capacity reactance alone, which is the voltage at the receiving end.

THE CAPACITY CURRENT MAY IMPROVE THE POWER-FACTOR of the line if the load has a lagging power-factor.

UNDERGROUND AND SUBMARINE CABLES HAVE SUCH LARGE CAPACITANCE and the breakdown voltage of the insulation is so low that it is impractical to transmit alternating-current power economically to any great distance by means of them, ten miles being about the greatest distance, and 23,000 volts the highest pressure.

LIGHTNING DISCHARGES NEAR A LINE CAUSE SURGES of current which may raise the e.m.f. high enough to damage the highly inductive machinery, if the surges or oscillations are allowed to enter them.

CHOKE COILS inserted in the line protect the machinery by allowing the surge to raise the voltage at the points where the choke coils are situated. This excess voltage is then made use of to cause the charge to "spill over" a gap to a ground connection, and flow off to the ground.

A HORN GAP is introduced in order to break the power arc which may persist after the momentary high voltage has broken down the air resistance and has established an arc from the line wires to ground.

ALUMINUM CELLS AND A SMALL AIR GAP allow no current to flow through them at normal voltage, but a slight rise in voltage is enough to break down the internal resistance of the cells and allow an excess charge to flow to ground. The cells allow practically no current to flow in the opposite direction and thus stop all oscillations set up by the arc.

CORONA LOSS begins when the DISRUPTIVE CRITICAL VOLTAGE is reached and increases very rapidly as the voltage is raised beyond this point. It is so called from the halo-like glow which appears around the conductors with the pressure a little above the disruptive critical voltage.

The following formula is approximately correct for fair-weather losses on commercial transmission lines using standard cables. Fog, sleet and snow cause greater losses. Smooth round solid wires cause less loss.

$$P_m = \frac{5.54\, f \sqrt{\frac{r}{s}}\, (E - E_0)^2}{10^6},$$

where

P_m = corona loss per conductor per mile, in WATTS.
f = frequency, in CYCLES PER SECOND.
r = radius of cable, in INCHES.
s = spacing of cables, in INCHES.
E = impressed voltage to neutral.
E_0 = disruptive critical voltage to neutral.

THE DISRUPTIVE CRITICAL VOLTAGE for clean stranded cables can be found approximately by the following formula:

$$E_0 = 105{,}000\, r \log_{10} \frac{s}{r}.$$

PROBLEMS ON CHAPTER VI

Prob. 58-6. Power is transmitted from Meppen, Ill., to Alton, Ill., a distance of 28.7 miles, at 66,000 volts, three-phase, 25 cycles. The line consists of No. 2, stranded copper, strung in horizontal plane, 7½ feet apart, with no transpositions, as shown in Fig. 239. Compute the charging current of this line.

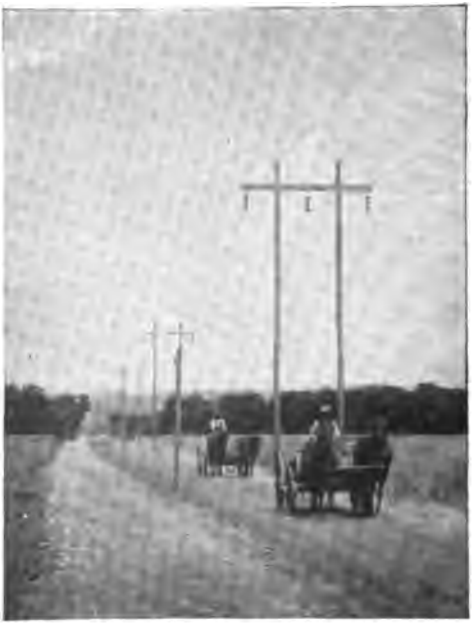

FIG. 239. The three-phase three-wire line from Meppen, Ill., to Alton, Ill. *The General Electric Review.*

Prob. 59-6. What is the voltage in the generator end of line in Prob. 58, when the line is open at the receiving end and the voltage there is 66,000 volts?

Prob. 60-6. The Sierra and San Francisco Power Co. transmit 34,000 kw. from Stanislaus to San Francisco, a distance of 138 miles, by means of two three-phase circuits at a pressure of

104,000 volts. Frequency, 60 cycles. Conductors are No. 00, copper, six-strand, hemp-center, arranged in vertical plane, spaced 96 inches apart. Compute, by formulas, and check from tables in Appendix B:

(*a*) Capacitance of line per conductor to neutral.

(*b*) Inductive reactance of line per conductor to neutral.

Prob. 61–6. What is the charging current of the Stanislaus-San Francisco line?

Prob. 62–6. When the line is open at the San Francisco end, and the pressure there is 104,000 volts, what is the pressure at the Stanislaus end?

Prob. 63–6. Compute the regulation of the Stanislaus-San Francisco line with load of 85 per cent power-factor lagging.

Prob. 64–6. What is the voltage at the Stanislaus end at half-load, 0.95 power-factor lagging? Assume voltage at San Francisco to be maintained constant at 104,000 volts.

Prob. 65–6. If the voltage at the Stanislaus end of the line should become 118,000 volts when San Francisco is taking a load of 10,000 kw. at 0.80 lagging power-factor, what will be the voltage at San Francisco?

Prob. 66–6. Compute the corona loss of the line in Prob. 62.

Prob. 67–6. What is the efficiency of transmission of the Stanislaus-San Francisco line at

(*a*) Full load, unity power-factor?

(*b*) Full load, 85 per cent lagging power-factor?

Prob. 68–6. An electric power company is planning to transmit 22,500 kw. at 110,000 volts, three-phase, 60-cycles, over a distance of 200 miles. Assume energy to cost 8 mills per kw-hr., and estimate interest, depreciation, taxes, etc., at 10 per cent. If there are two lines per tower, operated in parallel, and the line carries full load 16 hours per day and half load 8 hours per day every day of the year,

(*a*) What size aluminum cable would you advise be used at 35 cents per pound in place?

(*b*) What size copper cable at 20 cents per pound in place?

Prob. 69–6. (*a*) What spacing of conductors would you advise be used on the line in Prob. 68?

(*b*) What will be the charging current if copper cables are used? Neglect line drop.

Prob. 70–6. What will be the fair-weather corona loss, neglecting line drop, on the line if installed as in Prob. 69?

Prob. 71–6. Compute the regulation of the line in Prob. 69, at 0.80 lagging power-factor.

Prob. 72–6. What is the efficiency of the transmission line and the power-factor at the generator under the conditions in Prob. 69 and 70?

Prob. 73–6. The "Electrical World," April 25, 1914, gives the following data on the Cheat Haven-Butler, Pa., transmission line: the line is 106 miles long and operates at 125,000 volts, three-phase, 60-cycles, 2 lines per tower. Conductors No. 0, copper, six-strand, spaced in vertical plane, 60 inches apart. Compute the fair-weather corona loss using these data.

Prob. 74–6. What is the charging current on the Cheat Haven-Butler line?

Prob. 75–6. What is the line regulation of line in Prob. 26 when transmitting full load of 32,000 kw. at 80 per cent lagging power-factor?

Prob. 76–6. Compute the efficiency of transmission of the line in Prob. 75, neglecting the transformers.

Prob. 77–6. What is the power-factor at the generators of the Stanislaus-San Francisco line when full load at 85 per cent power-factor and 104,000 volts is being taken from the receiving end?

Prob. 78–6. What regulation will the line have and what power-factor will the generators have in the project of Prob. 68 as you have planned it, using aluminum cables, when the full load has a power-factor of 80 per cent?

CHAPTER VII

ASYNCHRONOUS MOTORS

IT is shown in Art. 28 and in Chapter VIII that an alternating-current generator may operate as a "synchronous motor," taking electrical power into its armature winding, and delivering mechanical power at its shaft. If the frequency of the power supply is maintained constant, the speed of such a motor will be the same at all loads that may be put upon it up to the point where it "pulls out" and stops. This speed is the "synchronous speed" and its value in revolutions per second is equal to the number of cycles per second divided by the number of pairs of poles in the a-c. machine under consideration; it is the speed at which the machine would have to be driven as a generator to produce an e.m.f. of the same frequency as that of the supply line.

We shall also see that no considerable torque can be exerted by an a-c. generator operated in this manner as a motor, until it has been "synchronized," or brought up to synchronous speed. If suitable **polyphase** alternating current is supplied to its armature winding, we shall see that a comparatively weak torque will be produced in the polyphase synchronous motor, if the field is stationary or is rotating at any speed less than synchronous speed. This torque tends to start it or to bring it up to synchronous speed and is due to eddy currents induced in the pole-pieces or pole-faces by the magnetic flux produced by the polyphase currents in the polyphase armature windings.

If we supply **single-phase** alternating current to the armature of a single-phase a-c. generator no starting torque is produced. This difference of action is due primarily to the fact that in the polyphase machine the flux produced by the supply currents is rotating around the shaft of the machine, whereas in the single-phase machine the direction or position of the flux is fixed with relation to the windings, although the amount of flux varies in approximately harmonic relation to time.

The essential difference between the "induction motor" and the "synchronous motor" can best be understood by studying carefully the difference between the manner in which the synchronous motor produces the torque to carry its normal load at synchronous speed, and the manner in which the same motor produces the torque to start it from standstill. The **load torque** is due to magnetic attraction of the poles, produced by the alternating currents in the "armature" windings for the poles produced by the direct current in the "field" winding, this direct current being supplied from some external source. This load torque can only be exerted when the "field," or rotor, is turning at exactly synchronous speed; otherwise, the torque is alternately in one direction and the other, its average value being zero, regardless of the strength of the field poles or the amount of armature current. On the other hand, the **starting torque** is due to magnetic attraction (or repulsion) between the same stator poles produced by the alternating currents taken from the supply line, and the local currents or eddy currents induced in the rotor by the variations of the stator flux. We have here during starting, therefore, a sort of transformer action whereby the activity of the rotor is produced inductively by the stator magnetism rather than by an external source of direct current. While being started from the a-c. supply mains, and until it reaches exact synchronous speed, the synchronous motor really operates as an "induction motor." At synchronous speed, the variations of flux in the rotor, and therefore the e.m.f.'s and currents induced in the rotor, are reduced to zero, and the induction-motor torque disappears, being replaced by the synchronous-motor torque.

The induction-motor torque exerted by a synchronous motor while starting is usually improved by any means which increases the amount of induced currents in the rotor or the amount of the inducing flux produced by the stator windings. Thus, the synchronous motor will have a higher starting torque if its pole-faces are of solid iron than if they are of laminated iron (although the efficiency of the machine is thereby reduced) on account of the increased amount of eddy currents in the pole-faces. Thus, also, the starting torque is much increased by the "squirrel-cage" made of copper bars, which is often inserted in slots prepared for the purpose in the pole-faces, as illustrated in Fig. 240, because the conductivity of the copper paths thus provided for the eddy currents is much greater than the conductivity of the iron paths in the pole-faces which these currents would otherwise be compelled to follow. It will be explained that the original purpose of this copper squirrel-cage on the rotor of synchronous generators and synchronous motors is to reduce the tendency of such machines to "hunt" or oscillate; but it serves also very usefully to increase the induction-motor torque for starting.

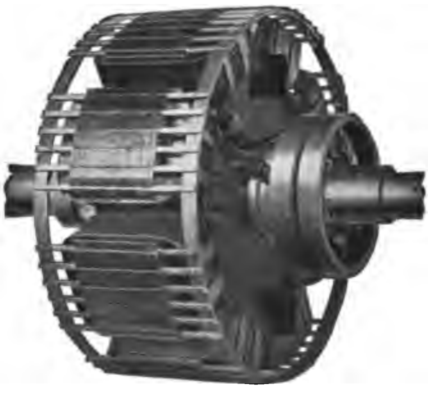

FIG. 240. Field with squirrel-cage winding for 75 kv-a. and larger belted alternators. *Westinghouse Electric and Mfg. Co.*

93. Construction of the Polyphase Induction Motor. The fundamental structural members of the induction motor are the **stator** and the **rotor.** The stator is the stationary frame, made of steel laminations so punched and bolted together as to form a hollow cylinder, the inside of which is accurately formed and is grooved axially with slots into which an insulated winding is laid. This winding does not differ in any essential respect from those employed for alternating-current generators, which are explained in detail in Chapter IX of the "First Course." Quite commonly it is of the two-layer type with diamond-shaped formed coils as in Fig. 241, which is a copy of Fig. 353, First Course. In the case which we are at present considering, it is arranged or connected for two or three phases, and is connected to a polyphase power supply having a similar number of phases and an e.m.f. of such value that an equal counter e.m.f. can be induced in the stator winding with a flux density in the stator core not large enough to cause excessive iron losses and heating. Fig. 241 is really the stationary armature of a revolving-field a-c. generator, but it is exactly like the stator of a majority of induction motors.

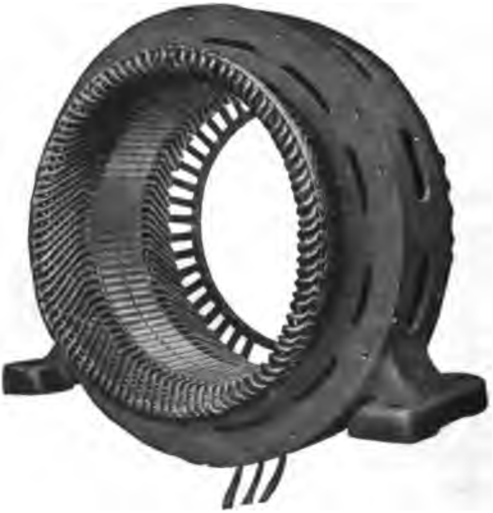

FIG. 241. Frame and armature winding for an alternating-current generator or motor. *Westinghouse Electric and Mfg. Co.*

The rotor is the rotating member of the motor. It also is made up of laminations bolted together, punched in such manner as to form grooves in which the rotor winding is placed. The outside cylindrical surface of the rotor is accurately formed to a diameter that is only enough less than that of the stator in which it revolves to give a safe mechanical clearance. The radial depth of the air gap between rotor and stator is commonly between 0.02 and 0.08 inch; larger values cause the power-factor of the motor to be low, and smaller values make it difficult to adjust and maintain the rotor in proper mechanical and magnetic relation to the stator.

Induction motors are classified as "squirrel-cage" or as "wound-rotor" motors, according to the method of placing in the rotor slots the copper circuits in which the stator magnetism induces the currents that react to produce the torque of the motor. In Fig. 242 we see a fully wound stator resting on its base-plate, and surrounded by an end-shield (or bearing) and three different rotors, *K*, *L*, *M*, any one of which may be used with this stator. *K* is a squirrel-cage winding consisting of lightly-insulated copper bars which have been pushed through the rotor slots and then welded, riveted, or soldered at each end to a ring of copper or bronze which connects them all together electrically. This squirrel-cage rotor does not differ essentially from the rotor shown in Fig. 240, except that the latter has salient or definite poles with coils carrying direct current, which are not necessary in the induction motor. The slots of rotors *L* and *M* in Fig. 242 are filled with a well-insulated winding very similar to that which occupies the stator slots. The total resistance of each rotor circuit of *M* and *L* may be adjusted by inserting suitable rheostats between the terminals of the rotor windings; the effect is (as will be shown) to enable us to control the speed of the motor, to improve the power-factor, and to adjust the starting torque to any value that may be desired, thereby making the motor much more flexible than when a squirrel-cage rotor is used. The adjustable resistances may be mounted within the rotor and controlled by means of a sliding collar on the shaft as in *L*, or they may be removed entirely from the motor and connected to the rotor windings through collecting-rings as in *M* (Fig. 242).

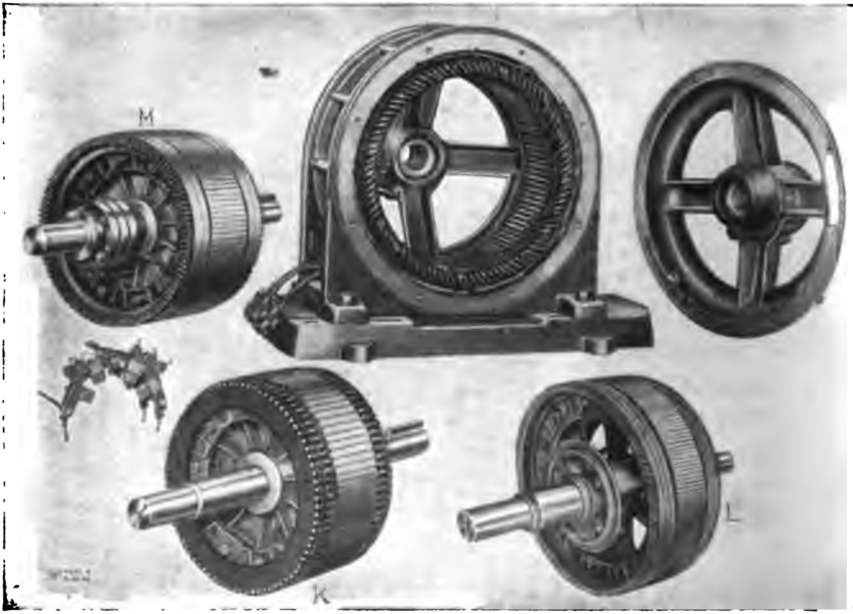

FIG. 242. Rotor *K* is a squirrel-cage type of rotor for an induction motor; *L* is a wound rotor with extra resistances mounted within, and *M* is a wound rotor to be connected with extra resistances through the collecting rings. *General Electric Co.*

94. Physical Theory of the Induction Motor. When polyphase currents are caused to flow properly in the polyphase windings of the stator, magnetic polar regions are produced on the cylindrical surface, which move progressively around the axis — in other words, the flux rotates around the shaft of the rotor. When the rotor is standing still, this moving flux induces e.m.f.'s and currents lengthwise of the rotor from the points where the flux is most dense toward the points where the flux is less dense or is of opposite sense. The rotating flux exerts a mechanical force upon the conductors which carry these currents, tending to turn the rotor. In accordance with Lenz's Law, the direction of induced e.m.f. and current is such as to move the rotor in a direction which will reduce or limit the induced e.m.f. and the current; in other words, the torque produced tends to turn the rotor in the same direction that the flux is moving.

If the resisting torque is not too great, the rotor begins to turn and is gradually accelerated. As its actual speed increases, its relative speed with respect to the revolving flux decreases, hence the induced e.m.f., rotor current and torque also decrease. When the rotor speed has increased to such a value that the induced e.m.f.'s and rotor currents are reduced to a value just sufficient to overcome the resisting torque due to the rotor losses and to whatever load may be upon the motor, there is no longer any excess torque tending to accelerate the motor, and its speed becomes steady. Therefore, the final speed will not be as high for a heavy load as it will be for a light load. If there were no load on the motor and no frictional or magnetic losses in the rotor, the speed would become equal to the speed of the revolving flux or the "synchronous speed" — that is, the number of revolutions around the shaft which any given pole on the stator makes per minute or per second; because then no torque and no rotor current would be necessary, hence no induced rotor e.m.f. and no speed difference between the stator flux and the rotor would be required.

The difference between the actual speed of the rotor and the synchronous speed is called the "slip," and it is usually expressed in percentage of the synchronous speed. Thus, if a motor has a speed of 1140 r.p.m. at full load with a slip of 5 per cent, the stator magnetism of this motor is revolving at a speed of $\frac{1140}{0.95}$, or 1200 rev. per min. At zero load, the rotor of this motor will turn at very nearly 1200 rev. per min., as the I^2R, hysteresis and eddy-current losses in the rotor will be practically zero and the friction losses very small.

Suppose that the e.m.f. applied to the stator of the motor mentioned in the preceding paragraph has a frequency of 60 cycles per second. The stator must therefore be wound for three pairs of poles (6 poles), because the moving stator poles induce the counter e.m.f. in each stator conductor as well as the active e.m.f. in each rotor conductor, and the number of pairs of poles passing any given conductor per second is the same as the frequency of e.m.f. induced in that conductor in cycles per second. Thus, frequency of counter e.m.f. in stator (equal of course to the frequency of impressed e.m.f.) is equal to synchronous speed in revolutions per second times number of pairs of poles; or, $60 = \frac{1200}{60} \times 3$. The synchronous speed of the induction motor is calculated from the number of poles on the stator and the frequency of impressed e.m.f., in the same manner as for a synchronous motor or generator.

The production of torque by means of induced currents in the rotor is illustrated by Fig. 243. We consider here a 4-pole motor, which will have a synchronous speed of

$$\frac{60 \text{ (cycles per second)}}{2 \text{ (cycles per revolution of stator flux)}} = 30 \text{ rev. per sec.}$$
$$= 1800 \text{ rev. per min.}$$

The four polar regions on the stator move progressively clockwise, let us say, at an angular speed of 30 rev. per sec. The rotor turns also clockwise, but at some lower speed, say 1620 r.p.m., or 27 r.p.s. (10 per cent slip). That is, the stator poles move with respect to the rotor conductors at a speed of only 3 rev. per sec. (slip speed), and in a clockwise direction. To analyze the action, we may consider either that the rotor is held stationary while the stator poles turn clockwise at a speed of 3 r.p.s., or that the stator poles remain stationary while the rotor turns counter-clockwise at a speed of 3 r.p.s. In Fig. 243 it is assumed that the stator poles turn clockwise.

At any instant, the rotor conductor situated under the middle of the polar regions of the stator, or at the points where the stator flux is most dense, will have the largest

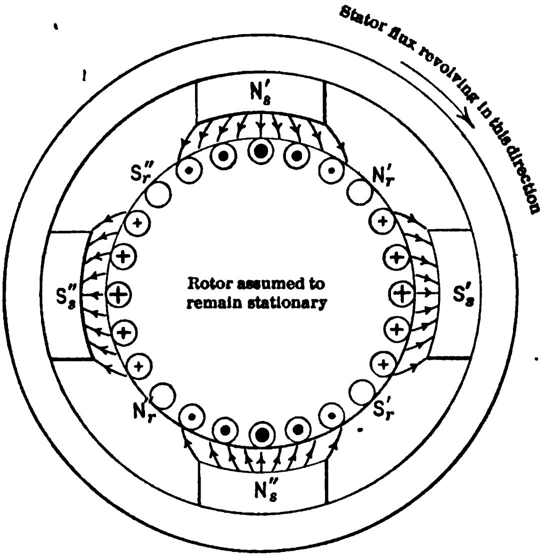
Stator flux revolving in this direction
N_s'
S_r''
N_r'
S_s''
Rotor assumed to remain stationary
S_s'
N_r
S_r'
N_s''

FIG. 243. The rotor conductors are here assumed to have resistance but no inductance. The dot ⊙ represents the induced current as flowing toward the observer, the ⊕ as flowing away from him.

value of e.m.f. induced, and the rotor conductors midway between stator poles will have no e.m.f. induced in them. As usual, an e.m.f. in direction toward the reader is represented by a dot ⊙, and an e.m.f. away from the reader is represented by a cross ⊕. Now, Fig. 243 is drawn on the supposition that the electrical circuits of the rotor are non-inductive, — that they offer resistance but have zero inductive reactance. Then, at any instant, the conductors which have the greatest induced e.m.f. have also the greatest current flowing through them, and this current flows in the direction of the induced e.m.f. The relative magnitudes of these currents in the various rotor conductors is indicated roughly by the relative size of the direction-symbols in the small circles which represent cross-sections of rotor conductors in Fig. 243. It is easily seen that the effect of these rotor currents is to produce a set of poles N_r', S_r', N_r'', S_r'' on the surface of the rotor **midway between the stator poles** N_s', S_s', N_s'', S_s''. As the rotor pole N_r' is pushed by the stator pole N_s' and pulled by the stator pole S_s', it is evident how the torque of the motor is produced. As the stator poles move, the e.m.f.'s and currents which are induced in the rotor also progress, and the rotor poles (" & rotor field") therefore move in exact synchronism with the stator poles and maintain the same position with relation to them, regardless of how the rotor circuits may be wound or connected together.

Fig. 244 is drawn to show what effect would be produced if the rotor conductors were to have zero resistance but considerable inductive reactance. In this case, the maximum current would not be attained in any rotor conductor until ¼ period after the e.m.f. in that conductor had passed through its maximum value in the same direction, or until the pole which would induce an e.m.f. in that direction has reached a point one-half pole-pitch beyond the rotor conductor in question. This results in a distribution of currents such as is shown in Fig. 244 for a particular instant; in effect, there is a sheet of current where the conductors lie on the surface of the rotor, and this sheet travels in time-synchronism with the stator poles and in fixed space-relation to them. The current-density is not uniform at all points in the current-sheet, but the whole effect (when X_r is large compared to R_r) is to form a set of rotor poles (N_r', S_r', N_r'', S_r'') directly beneath the corresponding stator poles N_s', S_s', N_s'', S_s''. It is plain that with such relative position of rotor and stator poles no torque can be produced, all the magnetic forces being either exerted radially or balanced tangentially.

From the foregoing discussion we learn one reason why the torque of an induction motor is often weaker at the moment of starting than it is after the rotor has attained some speed. At standstill, the frequency of the rotor currents has a maximum value, in fact equal to the frequency of stator currents; and when the usual amount of inductance is associated with each rotor conductor or circuit, this represents a relatively large value of reactance in the rotor. Although the induced e.m.f.'s in the rotor are relatively large on account of the large value of slip, the torque may be relatively weak at starting because of the unfavorable position of rotor poles, i.e., of rotor current-sheet with respect to the stator poles. When the rotor turns near synchronous speed (as it ordinarily does at full load or less), the frequency of rotor e.m.f. is small and the inductive reactance of the rotor is small for the same inductance, hence each ampere of rotor

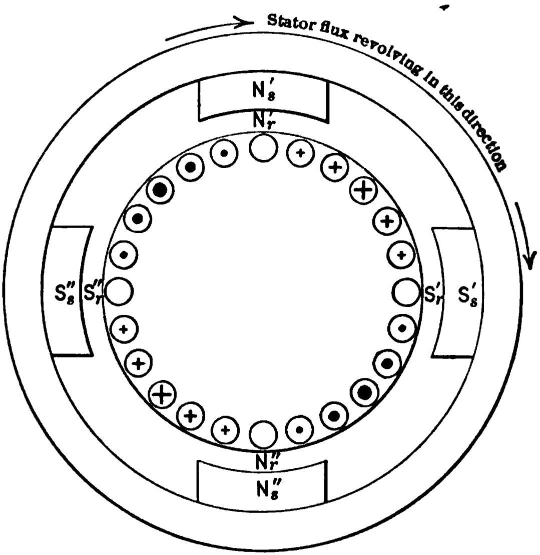

FIG. 244. The rotor conductors are here assumed to have reactance but no resistance. Note that the poles on the rotor are directly opposite similar poles on the stator.

current will contribute much more torque than at standstill, because of the more favorable position of rotor poles relative to stator poles as illustrated by Fig. 243 and 244.

Prob. 1-7. What is the synchronous speed, or angular velocity of the rotating flux in rev. per min., for an induction motor stator wound for 8 poles and connected to (*a*) a 60-cycle circuit, (*b*) a 25-cycle circuit?

Prob. 2-7. If the zero-load speed of an induction motor is 718 r.p.m., when connected to 60-cycle mains, what must be the number of poles for which its stator is wound?

Prob. 3-7. The actual rotor speed at full load for the motor in Prob. 2 is 698 r.p.m. What is the per cent slip at (*a*) zero load, (*b*) full load?

Prob. 4-7. A certain induction motor driven from 60-cycle mains has a full-load speed of 860 r.p.m. and a zero-load speed of 896 r.p.m. Calculate: (*a*) For how many poles the stator must be wound, (*b*) the synchronous speed, (*c*) the per cent slip at full load, (*d*) the per cent speed regulation of motor, (*e*) the per cent slip at zero load.

Prob. 5-7. When the motor of Prob. 1 is at standstill, what is the frequency of e.m.f. induced in each conductor on the rotor?

Prob. 6-7. When the motor of Prob. 4 is running at zero load, what is the frequency of e.m.f induced in each conductor on the rotor?

Prob. 7-7. If the rotor of the motor in Prob. 4 is wound with the same number of coils as the stator, these coils having also the same number of turns and the same relative positions (in other words, if the rotor winding is a duplicate of the stator winding) and if the voltage per phase of the stator winding is 134 while the frequency is 60 cycles per second, calculate the voltage per phase induced in the rotor windings, (*a*) with rotor at standstill, (*b*) with motor running at zero load.

Note. In this problem assume that all flux produced by the stator windings links also with the rotor windings — i.e., there is no magnetic leakage. This is a theoretically perfect condition which cannot be realized practically particularly on account of the air gap between rotor and stator. See Art. 41 and 64 on the transformer.

Prob. 8-7. What would be the answers to the questions of Prob. 7 on the assumption that 10 per cent of the total flux which links with the stator winding leaks through the air gap from one stator pole to another without linking the rotor winding?

Prob. 9–7. If the electrical circuit is opened in each phase of the rotor winding in Prob. 4 when the motor is carrying full-load torque, what will the speed become? (*b*) Approximately what will be the starting torque of the motor under this condition?

Prob. 10–7. When the electrical circuit is complete in each phase of the rotor in Prob. 4, at what angle (mechanical- or space-degrees) from the stator poles would the corresponding rotor poles tend to be formed, under the following conditions:

(*a*) Ohms reactance (x_2) of each rotor circuit equal to the ohms resistance (r_2) of the same circuit;

(*b*) $\frac{r_2}{x_2} = \frac{3}{4}$;

(*c*) $\frac{x_2}{r_2} = \frac{3}{4}$? Illustrate by diagrams similar to Fig. 243 and 244.

Prob. 11–7. What would be the answers to the questions of Prob. 7 if the number of turns per rotor coil is only 25 per cent of the number of turns per stator coil, the two windings being in all other respects exactly alike?

Prob. 12–7. What would be the answers to the questions of Prob. 8 on the additional assumption stated in Prob. 11?

Prob. 13–7. If the circuit of each phase of the rotor winding in the motor of Prob. 4 be opened and the rotor is coupled to an external source of mechanical power which drives it at synchronous speed but in a direction opposite to that of the rotating field, what will be the frequency of the e.m.f.'s induced between terminals of each phase of the rotor winding?

Prob. 14–7. What will be the voltage between terminals of each rotor phase in Prob. 13, under the specifications stated in Prob. 7 for windings and flux?

Prob. 15–7. If the circuit of each phase of the rotor winding in the motor of Prob. 4 be opened and the rotor is coupled to an external source of mechanical power which drives it at a speed of 1200 r.p.m. in the same direction that the stator flux is moving, what will be the frequency of the e.m.f.'s induced between terminals of each phase of the rotor winding?

Prob. 16–7. What will be the voltage between terminals of each rotor phase in Prob. 15, under the specifications stated in Prob. 7 for windings and flux?

Prob. 17–7. If the circuit of each phase of the rotor winding in the motor of Prob. 4 be opened and the rotor is coupled to some external mechanical power, at what speeds may it be driven in order that the frequency of the e.m.f.'s induced between the terminals of each phase of the rotor winding shall be 25 cycles when the frequency of polyphase e.m.f.'s impressed upon the stator windings is 60 cycles?

Prob. 18–7. What will be the voltage between terminals of each rotor phase in Prob. 17, under the specifications stated in Prob. 7 for windings and flux? (Note that the induction motor in Problems 13 to 18 acts as a frequency-changer.)

Prob. 19–7. (*a*) If the rotor of the motor in Prob. 4 is wound exactly like the stator, and turns in the direction of the torque produced by stator flux, and at one-half of synchronous speed, what will be the frequency of e.m.f.'s induced in the rotor?

(*b*) If the terminals of the rotor windings in this motor be connected to the stator terminals of an exactly similar motor, at what speed will the rotor of the second motor tend to rotate?

(*c*) If the motor of part (*b*) be aligned with the motor of part (*a*) in such manner that they tend to rotate in the same direction, and the two be coupled together mechanically in this position, at what speed should the shafts rotate when polyphase e.m.f.'s of 60 cycles frequency are impressed on the stator of No. 1 while the rotor terminals of No. 2 are short-circuited together?

95. The Rotating Magnetic Field. The production of torque in the induction motor depends primarily upon a movement of the flux or polar regions on the stator, around the axis of the rotor. This fact is so important, and so difficult sometimes to follow, that we should pause to study it carefully and in detail. Fig. 245 represents the stator of a two-phase two-pole induction motor. When a current flows positively through phase A (that is, let us assume, from A toward A' through the rear end-connection), while no current flows in phase B, a compass needle R pivoted at the axis of the stator would point its north end upward, as shown by the full arrow a. When current flows positively (from B toward B' through the rear end-connection) in phase B while no current flows in phase A, the north pole of the compass *R* would point to the right, as shown by the dotted arrow *b*.

Now connect coils *A* and *B* to a two-phase source of power, so that a current flows in each of them, these two currents being equal in value (on account of the symmetry of the circuits) and having 90 electrical degrees phase difference. Let us find the relative strength of the resultant magnetic field due to the stator windings at successive instants of time $\frac{1}{8}$ period apart, and its direction as indicated by the

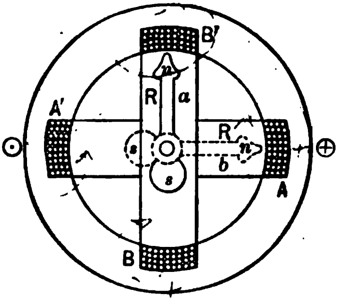

FIG. 245. The stator of a two-phase induction motor. A positive current in phase *A* causes the magnetic needle *n* to point up. A positive current in phase *B* causes a magnetic needle *n* to point to the right.

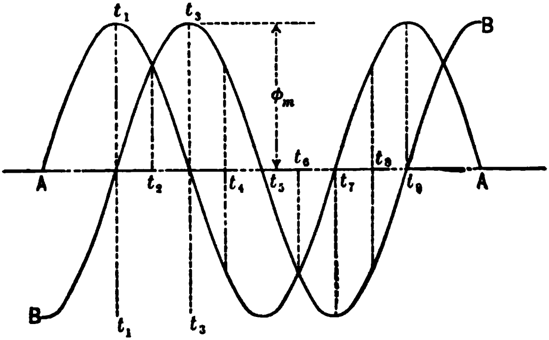

FIG. 246. The curves show the values and directions of the component stator fields of motor in Fig. 245 at the various instants during one and one-half cycles. Note that the field due to phase *A* leads the field due to phase *B* by 90°.

compass needle *R*. The chosen instants are represented by t_1, t_2, t_3, etc., in Fig. 246, and the method of finding the corresponding direction and magnitude of the resultant field *R* is indicated correspondingly at t_1, t_2, t_3, etc., in Fig. 247. At t_1, the current in phase *A* has its maximum positive value and the current in phase *B* is zero, consequently the resultant field *R* is exactly the same as the field due to phase *A* alone. Thus if ϕ_m represent the maximum strength of field, or amount of flux, due to maximum current in either phase alone, *R* is equal to ϕ_m and is directed vertically upward at the instant t_1. At t_2, $\frac{1}{8}$ period later, the field due to phase *A* has reduced to the value $a = \phi_m \cos 45° = 0.707\ \phi_m$, although

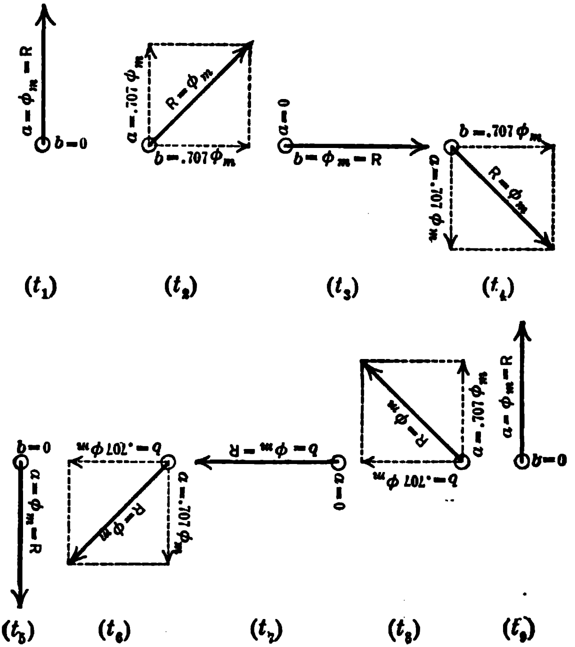

FIG. 247. Vector diagrams showing the amount and direction of the stator field of the motor in Fig. 245, at the instants marked t_1, t_2 . . . in Fig. 246.

it is still in the positive or upward direction; at the same instant (as may be seen from Fig. 246), the field due to phase B has increased from zero to the value $b = \phi_m \sin 45° = 0.707\ \phi_m$, and its direction is positive or toward the right. The resultant of these components taken together is $R = \sqrt{2} \times 0.707\ \phi_m = \phi_m$, as shown at t_2 in Fig. 247.

In similar fashion we follow the resultant field during one complete cycle from t_1 to t_9, and we find that any given magnetic polar region on the stator will travel over the distance occupied by two polar regions of the stator winding. That is, in the case under consideration in Fig. 245, 246 and 247, although there are always two magnetic polar regions on the surface of the stator (diametrically opposite, it being a two-pole winding), both of these poles will sweep around the axis while keeping a fixed relation to each other, making one complete revolution in the stator for each cycle of e.m.f. impressed on the stator windings, which means 60 rev. per sec. if the frequency is 60 cycles. By a similar analysis, it could be shown that if the stator were wound for 4 poles instead of 2, the time of two cycles of impressed e.m.f. would be required for each of the four poles to move completely around the stator, or the synchronous speed of the stator magnetism would be $\frac{60}{2} = 30$ rev. per sec. corresponding to a frequency of 60 cycles per sec. Each magnetic pole on a stator wound for 6 poles would make $\frac{25}{3} = 8\frac{1}{3}$ revolutions of the stator per second or 500 r.p.m., when the motor is connected to a 25-cycle circuit.

When the stator is wound for three phases (and 2 poles) the analysis of magnetic relations is as shown in Fig. 248, 249 and 250. When current flows in phase A only, in positive direction (A toward A' in the rear end-connection), the flux is in direction indicated by Oa in Fig. 248; similarly, positive direction of current in phases B and C produce flux in the direction Ob and Oc respectively. At the instant t_1, we see from Fig. 249 that phases A, B and C produce fluxes respectively as follows (the currents or component fluxes A, B, C of Fig. 249 being 120° apart):

$a = \phi_m$ (in positive direction, as indicated by Oa in Fig. 248).

$b = \phi_m \sin 30° = 0.5\ \phi_m$ (in negative direction, or opposite to Ob in Fig. 248).

$c = \phi_m \sin 30° = 0.5\ \phi_m$ (in negative direction, or opposite to Oc in Fig. 248).

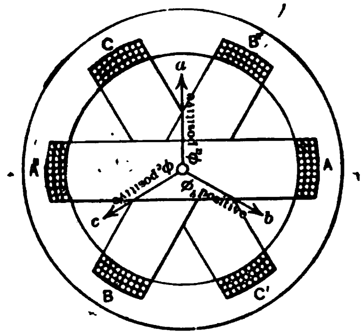

FIG. 248. Diagram of the stator windings of a three-phase two-pole motor. The arrows show the direction of the flux due to positive currents in the phase windings.

In Fig. 250 (t_1), these three component fluxes a, b, c are combined in proper relative values and directions, producing the resultant total flux $R = 1.5\ \phi_m$, where ϕ_m is the flux that would be produced by the maximum instantaneous value of current in any one phase alone.

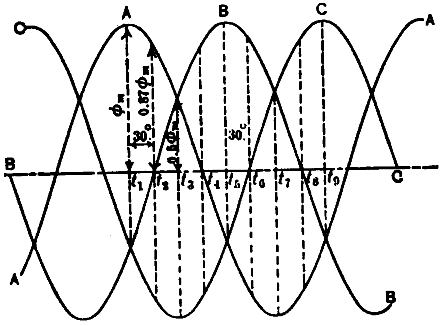

FIG. 249. Sine curves showing the direction and value of the fluxes produced at every instant by the three-phase armature windings of Fig. 248.

At the instant t_2, $\frac{1}{12}$ period or 30 electrical degrees later, the a component has decreased to the value $\phi_m \cos 30°$ or $0.87\ \phi_m$, but is still in its positive direction (along Oa in Fig. 248); the b component has reduced to zero; the c component has increased in value to $\phi_m \sin 60°$ or $0.87\ \phi_m$

and is still negative, or in direction opposite to Oc in Fig. 248. The resultant total magnetic field at this instant is shown as R in Fig. 250 (t_2), and is seen to have moved 30° from its previous position in Fig. 250 (t_1) $\frac{1}{12}$ period earlier, although it has exactly the same numerical strength, namely, 1.5 ϕ_m.

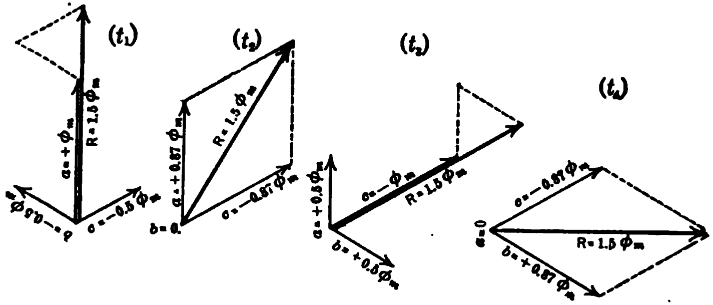

FIG. 250. Vector diagrams showing how the fluxes produced by the three-phase winding of the motor in Fig. 248 combine so as to produce always the same resultant field of 1.5 ϕ. This flux, however, is rotating.

Between the instants t_1 and t_4 the elapsed time is $\frac{1}{4}$ period, and we see from Fig. 250 that meanwhile the position of the resultant flux R has progressed steadily at uniform angular velocity through 90 mechanical degrees, maintaining meanwhile a constant strength represented by 1.5 ϕ_m. Continuing the analysis further along the same lines, we might show that for each cycle of impressed e.m.f. or stator current, every magnetic pole of the stator flux moves progressively over the space occupied by two adjacent polar regions of the stator winding. Thus, just as in the case of the two-phase motor, the synchronous speed of the rotating flux is $\frac{60}{2}$ or 30 rev. per sec. in a 60-cycle motor having 4 poles (2 pairs), and $\frac{25}{3}$ or $8\frac{1}{3}$ r.p.s. in a 25-cycle motor having 6 poles (3 pairs).

Prob. 20-7. The three-phase stator of Fig. 248 is connected in star to a three-phase supply line. The line wire connected to phase B breaks. Explain, by aid of a sketch, what happens to both the position and the value of the resultant flux R.

Prob. 21–7. The stator coils A and B of the two-phase motor of Fig. 245 are connected to two phases (e.m.f.'s of equal value but 120 electrical degrees phase difference) of a three-phase supply line. Draw vectors OR as in Fig. 250 to represent the direction and value of the resultant flux at successive instants of time $\frac{1}{12}$ period apart, beginning when the current in phase A has its maximum positive value.

Prob. 22–7. The stator coils A and B of the three-phase motor of Fig. 248 are connected to a two-phase line (e.m.f.'s of equal value but 90 electrical degrees phase difference). Draw vectors OR as in Fig. 247 to represent the direction and value of the resultant flux at successive instants of time $\frac{1}{8}$ period apart, beginning when the current in phase A has its maximum positive value.

Prob. 23–7. For convenience in mathematical analysis, a magnetic field which has fixed direction but strength varying harmonically with respect to time (that is, a single-phase field) is sometimes considered to be composed of two fields which are of equal and constant strength, and are rotating at uniform and equal angular velocity but in opposite directions. Demonstrate whether this assumption is justified or not.

Prob. 24–7. The stator of a given induction motor is wound for and energized by two-phase e.m.f.'s, while the rotor is wound three-phase and is connected in star. If the windings are similar and the number of slots per phase per pole and of turns per coil are the same for rotor and stator, what will be the ratio of induced stator voltage to induced rotor voltage:

(*a*) Between terminals of each phase;

(*b*) Between terminals of star in rotor? Assume that all flux links completely with both stator and rotor.

Prob. 25–7. What effect would be produced upon the stator magnetism by

(*a*) Reversing the connections of phase A only, in Fig. 245;

(*b*) Interchanging the connections of two line wires to the stator in Fig. 248 (assuming it to be star-connected)?

96. Starting Characteristics of the Polyphase Induction Motor. The facts and relations which interest us particularly about the starting of an induction motor are:

(*a*) The maximum amount of torque which can be developed to start the motor, and the methods for controlling this torque.

(*b*) The current required to start the motor against any given resisting torque.

(*c*) The power-factor of the starting current.

There is the greatest difference between a squirrel-cage motor and a wound-rotor motor in respect to these important factors, consequently we shall discuss first the squirrel-cage motor, and later indicate where the wound-rotor motor differs.

At the moment of starting, a squirrel-cage motor is exactly like a transformer which has a short-circuited secondary and an air-gap in the magnetic circuit between primary (stator) and secondary (rotor). The secondary is stationary, and the rotating flux has the same angular velocity with respect to both rotor and stator. The secondary current per circuit is equal to the secondary induced e.m.f. per circuit divided by the secondary impedance per circuit. This impedance depends upon the values of resistance and reactance. The resistance is a fixed quantity in this type of motor and is made low in order that the efficiency may be high; the reactance depends directly upon the rotor inductance per circuit and upon the frequency of the rotor currents or e.m.f.'s induced in the rotor conductors. The rotor inductance is due to flux (caused by the currents in rotor conductors) which links with rotor but not with stator conductors; its value is approximately constant in a given motor. The rotor frequency is directly proportional to the slip and has a large value, equal to stator frequency, when the motor is at standstill or is being started.

From the foregoing it is evident that at the time of starting a squirrel-cage induction motor, the secondary reactance (x_2) is large in comparison with the secondary resistance (r_2). Therefore, the rotor currents will lag nearly 90 electrical degrees behind the e.m.f. induced in the rotor circuits. The conditions are then as represented in Fig. 244, and the stator flux is revolving at synchronous speed with respect to the rotor.) The rotor poles or magneto-motive forces are almost directly opposed to the stator poles or m.m.f.'s; consequently, a relatively large part of the flux which links with the stator coils and generates the counter e.m.f. in them leaks through the air gap from one part of the stator to another without linking the rotor, and the induced e.m.f.'s in the rotor circuits are less than directly proportional to the slip. This is equivalent to saying that the leakage reactance of the stator, or primary of the transformer, is increased by the presence of the air gap, and that the reacting e.m.f. due to reactance and load component of stator current (taken to balance the ampere-turns of the secondary, and lagging nearly 90° behind the impressed e.m.f.) is almost directly opposite in phase to the e.m.f. impressed on the stator, so that the reduction of induced e.m.f. thereby is relatively large.

We may summarize our analysis as follows, with particular reference to the squirrel-cage motor:

First. The starting current taken from the line by the stator is large because additional primary ampere-turns are required to balance the secondary ampere-turns, thus maintaining the flux and counter e.m.f. in primary as discussed in Chapter III, Art. 35. The secondary (rotor) current tends to be large because the slip is very large, which induces a relatively large e.m.f. in the rotor notwithstanding the fact that the magneto-motive force of the rotor is in nearly direct opposition to the primary m.m.f. and the amount of inducing (mutual) flux is thereby reduced. The rotor current is limited, however, by the relatively large value of secondary frequency due to the 100 per cent slip. The internal torque developed will depend directly upon the starting current, and the acceleration of the motor will increase with the excess of this torque over the resisting torque due to load. To start on full-load torque with an average squirrel-cage motor will require 2.5 to 5 times full-load current from the line. Rather complete data is given in Table I.

Second. The power-factor of the starting current is low. When starting against full-load torque, a squirrel-cage motor will have a power-factor of about 55 to 60 per cent. This is because the ratio of inherent reactance to resistance in the short-circuited secondary (rotor) is large. This in turn is due principally to the large value of slip and of secondary frequency, and to the relatively large values of leakage inductance in both primary and secondary caused by the air gap between them.

Third. The starting torque is low in relation to the starting current, principally because of the unfavorable position of the rotor currents and poles with respect to the stator poles (see Fig. 244). The maximum torque that can be developed in a given squirrel-cage motor to start it from rest is limited by its design (values of r_2 and x_2), and in the average motor is about 1.5 times full-load torque. Table I gives more complete data.

Prob. 26–7. (*a*) Calculate the actual full-load speed in r.p.m. for the 20-h.p. 900-r.p.m. motor of Table I, operating from a 60-cycle 2300-volt circuit.

(*b*) Calculate the current taken from each line wire at full load by this motor, if it is three-phase.

(*c*) Calculate full-load torque in pound-feet at the pulley for this motor.

Prob. 27–7. For the motor of Prob. 26, calculate the following additional items: (*a*) Actual starting current, amperes per line wire if started against rated-load torque at line voltage. (*b*) Torque (pound-feet) per ampere when starting under this condition. (*c*) Torque (pound-feet) per ampere at full load.

Prob. 28–7. Answer the questions of Prob. 26, with relation to the 20-h.p. 1200-r.p.m. motor of Table I.

Prob. 29–7. Answer the questions of Prob. 27, with relation to the motor of Prob. 28.

TABLE I

DATA OF CROCKER-WHEELER POLYPHASE INDUCTION MOTORS

As given by Crocker and Arendt in "Electric Motors," published by Van Nostrand, page 200

H-p.	R.p.m.	Poles	Slip, per cent	Starting current, per cent.	Starting torque, per cent.	Pull-out torque, per cent.	Power-factor, per cent.				Efficiency, per cent.			
							$\frac{1}{2}$	$\frac{3}{4}$	$\frac{4}{4}$	$\frac{5}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{4}{4}$	$\frac{5}{4}$
0.5	1800	4	5.0	350	170	250	50	62	69	74	70	74	75	74
1	1800	4	5.0	400	150	240	62	72.5	78	81	74	78	79	79
2	1800	4	5.0	550	200	320	64	75	83	85	77	81	82	82
3	1800	4	4.4	650	220	350	74	83	88	90	79	83	84	83
5	1800	4	4.4	625	230	350	76	85	89	90	82	84	85	84
7.5	1200	6	5.0	625	250	300	76	84	88	89	84	85	86	85
10	1200	6	5.8	500	200	275	78	86	89	90	84	85	85	83
15	1200	6	5.0	625	200	300	78	86	89	91	85	87	87	86
20	1200	6	5.0	625	250	300	76	85	89	90	85	87	87	86
20	900	8	6.5	550	160	250	72	82	86	88	86	87	87	86
25	1200	6	4.2	650	250	325	79	87	90	92	86	88	88	87
30	900	8	5.5	625	200	300	78	86	90	91	87	89	88	87
40	900	8	4.4	600	200	300	79	86	89	90	86	88	89	88
50	900	8	3.9	650	225	325	74	84	89	90	87	89	90	90
75	720	10	4.1	600	200	300	78	86	89	90	88	90	90	89
100	720	10	4.1	650	210	325	76	85	89	90	88	90	90	90
150	720	10	3.5	725	200	360	81	88	91	92	88	90	91	90

Note. Starting torque and pull-out torque are given in per cent of torque at rated full load. Starting current refers to starting at full rated voltage, and is expressed as percentage of current at rated load.

TABLE II

Efficiencies and power-factors of 60-cycle, 2-, and 3-phase squirrel-cage Induction Motors made by the Richmond Electric Co.

850 R.p.m.

H.p.	Per cent efficiency.					Per cent power-factor.				
	Load.					Load.				
	¼	½	¾	1	1¼	¼	½	¾	1	1¼
1	58	74	77	78	77	45	62	71	75	77
2	61	75	78	80	78	47	63	72	77	78
3	63	77	80	81	80	47	63	73	78	80
5	65	79	81	82	81	50	66	76	80	82
7.5	66	81	82	83	83	52	70	76	81	83
10	66	81	83	84	84	54	72	78	82	83
15	66	83	84	85	84	56	73	79	84	86
20	67	86	88	88	88	56	73	80	86	88
25	67	87	88	88	87	57	74	81	86	87
30	67	86	87	88	88	58	75	82	87	89
35	67	85	86	88	87	57	75	83	87	88
40	67	85	87	88	87	57	76	84	88	89
45	68	86	88	89	88	57	76	84	88	90
50	68	86	88	89	88	58	75	83	88	90
60	68	86	88	89	88	58	75	83	89	91
75	68	87	88	89	89	58	75	84	90	92

1140 R.p.m.

H.p.	Per cent efficiency.					Per cent power-factor.				
	Load.					Load.				
	¼	½	¾	1	1¼	¼	½	¾	1	1¼
0.25	50	62	64	65	63	40	52	57	62	63
0.5	53	66	70	72	71	43	50	66	70	72
1	53	73	76	77	77	45	59	68	75	77
2	60	78	80	81	80	51	66	75	80	81
3	61	79	81	83	81	54	70	78	83	84
5	62	79	82	83	82	54	70	79	85	87
7.5	64	82	84	84	82	56	72	82	86	87
10	65	83	84	85	84	56	73	82	87	89
15	65	83	85	86	85	57	75	84	88	88
20	67	85	86	86	85	57	75	84	89	91
25	66	84	86	86	85	57	75	84	89	91
30	65	84	85	86	86	56	74	84	89	91
35	67	85	86	86	85	55	75	85	89	90
40	68	85	86	87	86	55	75	84	89	91
45	68	85	86	87	86	56	74	84	89	91
50	68	86	87	88	87	58	76	85	90	92
60	69	87	88	88	88	58	77	86	90	92
75	68	87	88	89	88	57	77	85	90	91
85	69	87	88	89	89	58	78	86	90	91
100	70	88	89	89	89	57	77	86	90	92

97. Starting Torque of Polyphase Induction Motors. If we alter the design of the motor so as to change the ratio of x_2 (reactance per secondary circuit at standstill) to r_2 (resistance per secondary circuit), while keeping the total secondary impedance $(\sqrt{r_2^2 + x_2^2})$ constant, we find that the actual maximum starting torque will be attained when $r_2 = x_2$. It may be shown, either mathematically or by test, that the **actual value of this maximum torque at standstill** depends upon factors of design and operation, as follows:*

(*a*) It is directly proportional to the square of the number of turns per secondary circuit.

(*b*) It is directly proportional to the square of the voltage induced per turn of the secondary circuit at standstill.

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* These relations may be explained as follows:

(*a*) Doubling the number of turns per secondary circuit, other things being equal, would double the e.m.f. induced in each secondary circuit. This would cause the current per circuit to be doubled if r_2 and x_2 remain unchanged. With twice as many rotor conductors, each carrying twice as many amperes, all acted upon by the same stator flux as formerly, the turning effort will evidently be doubled twice, or quadrupled.

(*b*) Doubling the voltage induced per turn of secondary circuit, other things being equal, would double the e.m.f. induced in each secondary circuit. A doubled e.m.f. per turn indicates that the amount of inducing flux has been doubled. This doubled flux acts upon the doubled current in each secondary conductor, to produce a quadrupled torque.

(*c*) A doubling of the angular velocity of the rotating flux would be due to a doubling of the frequency of e.m.f.'s applied to the stator, or to halving the number of poles. If we are keeping the same induced volts per turn of secondary circuit while doubling the speed of the rotating flux, we must at the same time halve the amount of the flux. The secondary current per conductor remains constant if the induced volts per turn is constant, and this same current is acted upon by the halved flux to produce a halved turning effort.

(*d*) To produce the maximum starting torque with other conditions as given, r_2 has been made equal to x_2. If now we double the value of both r_2 and x_2, the impedance per secondary circuit is doubled and the current per circuit is halved for the same induced e.m.f. per circuit. This halved current acts upon the unchanged flux to produce a halved torque.

(*c*) It is inversely proportional to the angular velocity of the rotating field.

(*d*) It is inversely proportional to the value of secondary resistance per circuit, which is assumed to be made equal to the reactance per circuit.

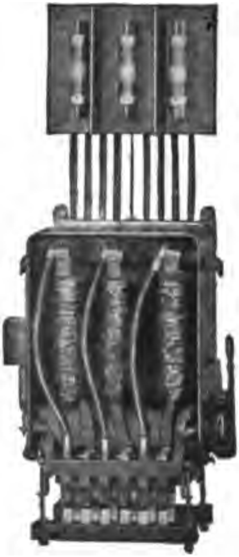

FIG. 251. Three-phase starting compensator for induction motors, equipped with no-voltage release (left side) and fuses to protect against overload. The cylinder switch at the bottom is for making the starting and the running connection.

To **increase the starting torque,** therefore, we have to make one or more of the following changes:

(*a*) **Increase the number of conductors on the rotor.** This can be done when designing a new motor, but is not practicable for a motor already constructed. We must bear in mind also, that the number of secondary turns affects the value of secondary inductance.

(*b*) **Increase the voltage applied to the stator.** This will increase the counter e.m.f. in stator, the flux, and therefore also the voltage induced per turn of the secondary. Large motors would draw too much current from the line if started even at full-load voltage. Thus it is necessary to cut down the line voltage for starting. The stator voltage may be controlled by introducing an adjustable resistance between each stator phase and the corresponding line wires, keeping these resistances always equal to one another so that the phases will remain balanced. This method involves a large I^2R loss in such resistances (as they carry the entire load current) and a correspondingly low efficiency for the motor.

The method usually employed to lower the voltage applied to the stator for starting and raise it again when the rotor has attained its full-load speed is by means of autotransformers. Thus, Fig. 251 represents what is known as a "starting compensator," such as is used for starting three-phase squirrel-cage induction motors of 5 horse power and larger. It consists essentially of three autotransformers connected together in star. The handle on the right operates a drum-switch below, enabling us to connect the motor terminals to taps on the autotransformers for starting the motor and to change over directly to the line after the motor has reached its full speed. The motor is protected against overload by fuses on the board just above the compensator; if desired, these fuses may be replaced by circuit-breakers or by overload relays which give warning when they stop the motor.

Fig. 252 shows the complete connections for this same compensator. When the switch-cylinder is turned so as to connect to the front finger-block, the outer ends of the autotransformers are connected to the line and the intermediate taps are connected to the motor terminals. The motor thereby receives something less than line voltage, depending upon the location of the taps. After the motor has reached full speed the cylinder-switch is thrown over to the running side, and the motor is thereby connected through the back finger-block directly to the line (through the fuses) receiving full rated voltage.

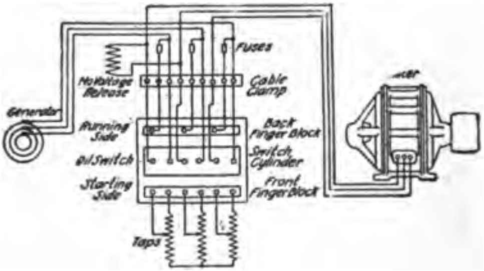

Fig. 252. Connections of the three-phase starting compensator with no-voltage release shown in Fig. 251. The fuses protect the motor when running, but not when starting. *General Electric Co.*

Notice that the fuses in Fig. 252 are not in circuit during starting, as the starting current is usually considerably larger than full-load current, and fuses which would protect the motor in normal operation would be continually blowing when the motor is starting. The cylinder-switch is often arranged so that it will not stay in the starting position unless held there by hand. When it is thrown to the running side, the compensators become disconnected from the line, which avoids the continual core losses in them that would otherwise occur.

Fig. 253 is a simplified diagram showing such arrangements as may be used in starting motors of large size or high voltage. First throw in the switch S_1, energizing the autotransformers. Then throw switch S_2 downward, connecting the motor to intermediate taps on the compensator C. When motor reaches full speed throw S_2 upward, connecting motor directly to line through trip coils, which latter operate the circuit-breaker in case of overload. Finally, open switch S_1, disconnecting compensator from the line.

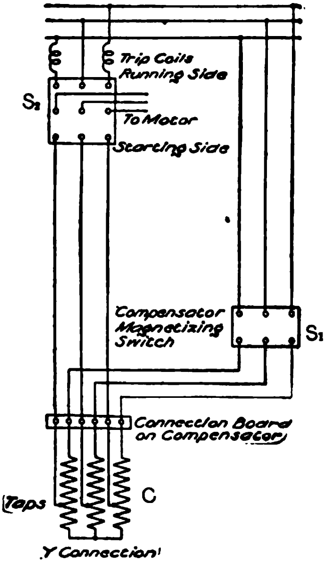

FIG. 253. Starting compensator with separate switches, for starting three-phase induction motors of large size and high voltage.

Motors smaller than 5 horse power are usually started either by means of a star-delta switch (see First Course, Fig. 130 and Prob. 139–3, page 152), or by connecting the stator directly to the line without any starting device.

(*c*) It is not practicable to **change the speed of the rotating field for changing the starting torque of the motor,** because the line frequency is not under the control of the operator, and the winding and switching required to change the number of poles on the stator are too complicated and expensive.

(*d*) The **starting torque would be raised by increasing the rotor resistance** r_2 up to the point where r_2 becomes equal to x_2 (at standstill). If r_2 is increased further than this, the starting torque decreases again, because the gain due to change in the position of the rotor poles is not as great as the loss due to increase of rotor impedance (which cuts down the rotor current). If copper circuits are used in the rotor, it is ordinarily impracticable to hold x_2 (at standstill) down to the same value as r_2. But r_2 may be increased so as to be more nearly equal to x_2, by using alloyed metals for the rotor bars and connections (end-rings), which have higher resistance than copper. Although this does increase the starting torque, it also lowers the efficiency of the motor and causes it to have a poor speed regulation, as we shall see later.

If we use a wound-rotor in our induction motor, it is easy to bring out the terminals of the rotor circuits through collecting-rings to external adjustable resistances as already mentioned in relation to Fig. 242. Adjusting r_2 thus by external means, we can develop in the wound-rotor motor at starting, a torque as large as the motor can develop when running at full voltage, which it is not practicable to do in a squirrel-cage motor. Moreover, the external resistance may be short-circuited or cut out after the motor is up to full speed, thus avoiding the low efficiency and poor speed regulation mentioned above as pertaining to high values of r_2.

Prob. 30-7. The starting torque of a certain squirrel-cage motor is 2.5 times rated-load torque, when full voltage is applied to stator at starting. For about what per cent of rated or line voltage should the taps on the starting compensator be adjusted, in order to start the motor against rated-load torque with as little current as possible?

Prob. 31-7. If the 10-h.p. 1200-r.p.m. motor of Table I gives a starting torque equal to 200 per cent of rated-load torque when full rated voltage is applied to the stator, what per cent of rated voltage is the least that will start the motor against rated-load torque?

Prob. 32–7. From the answer to Prob. 31 and the data given in Table I for starting current at rated voltage, calculate what per cent of rated-load current should be required to start the motor against rated-load torque.

Prob. 33–7. What per cent of rated-load current should be taken by the motor of Prob. 31 and 32 when the applied voltage is just sufficient to start it against 150 per cent of rated-load torque?

Prob. 34–7. Assuming the inductance of the secondary or rotor circuits to be constant, calculate the ratio of the reactance per rotor circuit at rated load (3.9 per cent slip) to the reactance at standstill (starting) for the 50-h.p. 900-r.p.m. motor of Table I.

Prob. 35–7. The starting compensator for the 25-h.p. 1200-r.p.m. induction motor of Table 1 has taps for 40 per cent, 60 per cent and 80 per cent of the line voltage, which is equal to the rated voltage of the motor. What percentages, respectively, of rated-load torque will be obtained when starting on these various taps?

98. Current and Power-Factor when Starting the Induction Motor. Pursuing further the analogy between the induction motor and the transformer (see Art. 96), it is usual to consider the total current which the stator takes from the line, as being made up of an exciting component and a load component. The exciting component is itself considered to consist of a magnetizing component (in phase with the stator flux which links with each phase) and a core-loss component (which supplies the core-loss power and is in phase with the e.m.f.). The load component of stator current is taken to balance the counter magneto-motive force in the magnetic circuit, which is generated by the current in the secondary turns.

For a given voltage applied to the primary or stator, **the exciting current is approximately constant** regardless of what value the secondary or rotor current may have, because the counter e.m.f. in the stator is reduced only very slightly by the I_1r_1 and I_1x_1 reactions therein, consequently the stator flux remains practically unchanged (or proportional to stator voltage). The **load component of stator current,** however, must be **proportional to the secondary current** (the ratio between these two currents being approximately the same as the inverse ratio of turns, as in an ordinary transformer). The power-factor of the stator load component must be equal to the power-factor of the rotor current; this is equal to the cosine of the angle of phase difference between current and induced e.m.f. in the rotor, or (at standstill) $\frac{r_2}{\sqrt{r_2^2 + x_2^2}}$.

To produce a reasonable acceleration of the rotor even at zero load requires a load component which is large compared to the exciting current (being equal to or more than full-load current). Consequently, the power-factor of the entire stator current at standstill is practically equal to that of the rotor current, namely $\frac{r_2}{\sqrt{r_2^2 + x_2^2}}$. If r_2 is adjusted to be equal to x_2 so as to produce the maximum starting torque for a given current and rotor impedance, then the value of this power-factor at standstill becomes equal to $\frac{r_2}{\sqrt{r_2^2 + r_2^2}}$ or $\frac{1}{\sqrt{2}}$ or 0.71. Obviously the power-factor at starting may be made to have almost any desirable value by simply adjusting the ratio of r_2 to x_2, which is usually accomplished by adjusting r_2.

The amount of total starting current taken by the stator will depend almost entirely upon the amount of rotor current, at standstill. This will depend directly upon the e.m.f. induced per rotor circuit, and therefore upon the e.m.f. impressed on the stator; it will also be inversely proportional to the impedance per rotor circuit, which is $\sqrt{r_2^2 + x_2^2}$ at standstill. Theoretically it would be best to make r_2 equal to x_2 so as to develop maximum torque; and then make both r_2 and x_2 of such actual value that the rotor current and the torque developed in the rotor would be not much more than sufficient to overcome the torque against which the motor must start, and to develop a proper acceleration so that it will reach full speed in a reasonable length of time (say one minute). Practically, however, in any given motor x_2 is not adjustable, so that if we do not vary the voltage we must adjust the motor to develop the required starting torque by means of r_2 only. Therefore the starting current and the starting power-factor will each depend upon and vary with the other, and we shall be able to control the starting current independent of the starting power-factor only by adjusting the voltage applied to the motor.

By proper adjustment of the external resistances of a wound-rotor induction motor the power-factor while starting against full-load torque may be made as high as at normal full load (that is, probably 85 per cent to 90 per cent), while the starting current may be very little in excess of rated full-load current. Compare these with corresponding values for the squirrel-cage motor as given in Art. 96.

The effect of increasing the resistance of the secondary (rotor) circuits, upon the current taken by the motor at standstill and upon the starting torque, are shown in Fig. 254. Curve *B* shows that the largest starting torque that can be developed in this particular (typical) motor is 250 per cent of rated full-load torque (or of the torque which will develop rated horse-power output at rated speed), and that this torque is attained when the resistance external to the rotor is equal to about three times the internal resistance of the rotor. The starting torque is less than this for either greater or less values of resistance in the rotor circuit. Curve *A* shows the current taken from the supply line by the stator (expressed in per cent of current taken when running at normal full load), when the resistance of the rotor circuit has various values. Full line voltage is impressed upon the stator in Fig. 254. Both curves *A* and *B* would descend to the zero line at the point where rotor resistance becomes infinitely large (open circuit in the rotor), because then no current could flow in the rotor and therefore no forces could be produced on the rotor. Of course the values for these curves cannot be observed for resistances lower than that of the rotor winding (short-circuit across rotor terminals); but the dotted portions show that the torque would be zero and the current maximum if the entire resistance of the rotor circuit (r_2) should be reduced to zero, leaving only x_2 to limit the current. We have seen from Fig. 244 why the torque is zero under this condition.

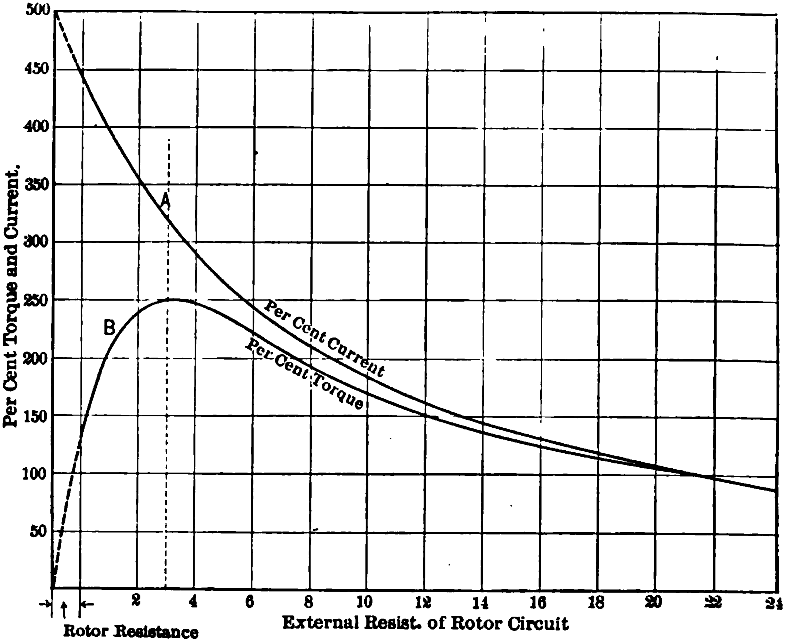

FIG. 254. Curve *A* shows the relation of starting current to rotor resistance, voltage constant. Curve *B* shows the relation of torque to rotor resistance, voltage constant. *Wagner Elect. Mfg. Co.*

Fig. 255 shows the effects of impressing reduced voltage upon the stator at standstill, as is done in starting a squirrel-cage motor by means of a compensator or autotransformers. Curve *A* shows current (in per cent of current taken at normal full load of motor) delivered from compensator to stator, curve *C* shows current (on same basis) taken by compensator from supply line, and curve *B* shows torque (in per cent of torque at normal full load), when the voltage impressed upon the stator has various values expressed in percentage of rated voltage. The current supplied to the motor is directly proportional to the pressure applied to the stator

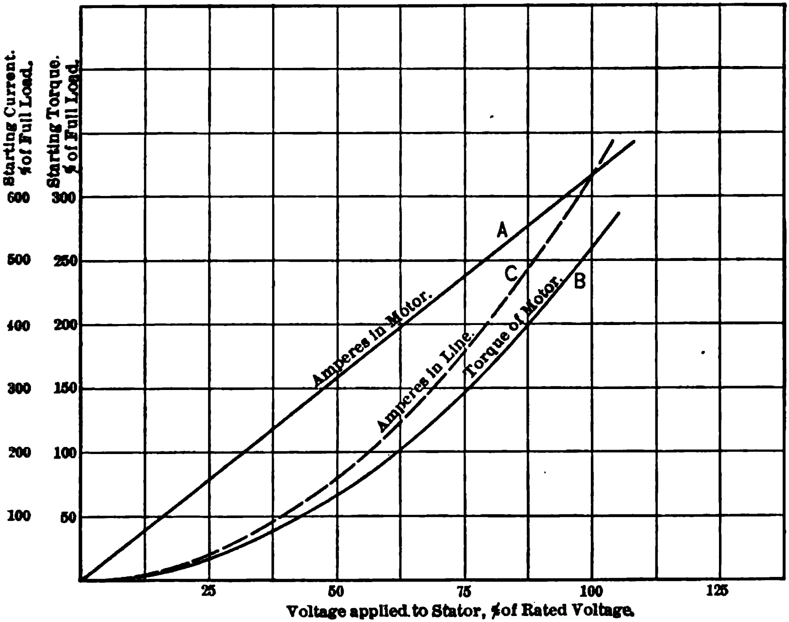

FIG. 255. Curve *A* shows the relation of the starting current taken by a Wagner induction motor to the voltage applied. Curve *C* shows the relation of the line current taken by the compensator to the stator voltage. Curve *B* shows the relation of the starting torque to the voltage. *Wagner Electric Mfg. Co.*

(or induced in the rotor), because the resistance, reactance and impedance of the rotor circuit are all constant at standstill. As already demonstrated, the torque at standstill (in fact, at any fixed value of percentage slip) is proportional to the square of voltage applied to stator or the voltage induced in rotor.

Prob. 36–7. If the power-factor of the starting current for the 15-h.p. 1200-r.p.m. motor of Table I is 60 per cent, what will it be when starting under the following altered conditions: (*a*) Stator voltage reduced 50 per cent, frequency unaltered; (*b*) stator voltage unaltered, frequency reduced 50 per cent; (*c*) stator voltage and frequency both reduced 50 per cent? Neglect the exciting current.

Prob. 37–7. Each rotor circuit of the motor in Prob. 36 has a resistance of 1 ohm, let us say. Calculate the reactance of each circuit, in ohms, (*a*) at starting, normal frequency; (*b*) at rated load, with 5 per cent slip. Assume inductance constant.

Prob. 38–7. The impedance of each rotor circuit in the 2-h.p. 1800-r.p.m. motor of Table I is, let us say, 5 ohms when the motor is being started from standstill. What will be the corresponding value at rated full load with 5 per cent slip and 83 per cent power-factor?

Prob. 39–7. Assuming each motor in Fig. 254 and 255 to be adjusted so as to develop rated-load torque at starting, the total I^2R losses in the motor will be what per cent of the I^2R losses at rated full load, (*a*) for the wound-rotor motor of Fig. 254; (*b*) for the squirrel-cage motor of Fig. 255.

Prob. 40–7. Assuming adjustments to have been made on each motor of Fig. 254 and 255 limiting the starting current on the supply line to 150 per cent of rated-load current, calculate the percentage of rated-load torque developed at starting by (*a*) the wound-rotor motor of Fig. 254; (*b*) the squirrel-cage motor of Fig. 255.

Prob. 41–7. Explain the reason for the difference between curves *A* and *C* in Fig. 255, and show that they correspond exactly.

Prob. 42–7. From the data in Fig. 254 calculate the total reactance of the motor at standstill, as percentage. Consider that, as in a transformer or a-c. generator, the percentage reactance (volts) means the percentage of rated voltage consumed in overcoming reactance when rated-load current flows.

99. Action of Induction Motor when Carrying Load.

"Loading" a motor means offering more opposition to the turning of the motor. The first effect of this, in any motor, is to cause a reduction of speed because, for an instant at least, the power output is greater than the power input, and the excess must come out of the store of kinetic energy in the moving parts of the machine by reduction of their "momentum" and velocity. In all types of motor which we have already studied we have seen that this initial reduction of speed produces changes in the counter e.m.f. and the current taken from the line, which increases the electrical power intake of the motor; the speed reduction will proceed until the input becomes equal to the output plus the losses, at which point the speed becomes steady again.

So it is also with the polyphase induction motor. When we increase the load, the speed falls and the slip increases. The e.m.f. induced in each rotor circuit increases with the slip; the frequency and impedance of the rotor circuits is nearly constant at ordinary loads, so that the rotor current is increased approximately in proportion to the rotor e.m.f. and the slip. The rotating stator flux exerts increased force upon the increased rotor currents, producing a correspondingly greater driving torque. The speed will fall until this driving torque becomes equal to the resisting torque, and will then remain steady as long as the load is steady. A higher resistance of the rotor circuits necessitates a greater rotor e.m.f. and slip in order to produce the amount of rotor current required to carry the same load (torque). Therefore the speed corresponding to a given load will be lower on account of the increase of slip due to the greater rotor resistance.

To simplify our reasoning, let us first consider a nearly ideal motor. If the primary (stator) had no resistance ($r_1 = 0$) and no leakage reactance ($x_1 = 0$), then the primary counter e.m.f. and the flux would be constant for all loads (the e.m.f. impressed upon the stator windings being constant), and therefore the slip would be directly proportional to the e.m.f. that must be induced in the rotor. If also the secondary has zero reactance and constant resistance, this rotor e.m.f. necessary to produce the rotor current is directly proportional to the current and therefore to the torque that must be developed (the other factor of torque, namely, flux, being constant). Under these ideal conditions the slip increases uniformly with torque from a value of zero at the theoretical zero load (no opposition whatever, either external or internal, to the movement of the rotor), to a value of 100 per cent when the motor is "stalled" or meets such opposition that it comes to a standstill. Thus, as shown by curve *A* in Fig. 256, the speed drops correspondingly and uniformly with increase of (internal or driving) torque, from synchronous speed to zero.

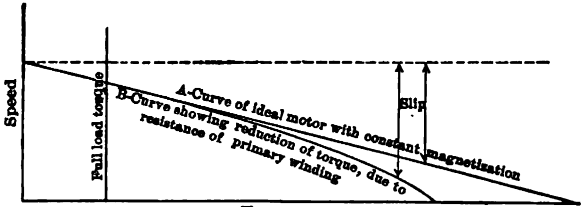

FIG. 256. Curve *A* shows the relation of the speed to the torque of an induction motor with no stator resistance, an ideal case. Curve *B* shows the relation of speed to the torque of an induction motor the stator of which has resistance; this resistance causes the speed to fall at an increasing rate. *Westinghouse Electric and Mfg. Co.*

The **actual speed-torque curve** departs from this ideal straight-line relation, slightly at light loads but very greatly at heavy overloads on the motor. There are several causes for this behavior. Considering them in order as we meet them in following the input through the motor, we come first to the effect of **resistance of the primary windings.** This primary resistance causes the counter e.m.f. to be less than the (constant) impressed e.m.f. by an increasing amount (equal to the Ir_1 drop, I increasing with load). Consequently the flux decreases and there must be a twofold increase of slip, for we have not merely to induce, by means of the decreased flux, the same current as formerly for a given torque, but we must actually produce a larger current to act on the weaker flux with the same force as formerly. Thus, considering only the effect of stator resistance, the speed-torque curve would become like *B* in Fig. 256, departing from curve *A* at an ever-increasing rate as the torque grows, and thus becoming curved downward.

The effect of **magnetic leakage in the motor** is far more important than that of primary resistance, however. On account of the air-gap in the path of the useful (mutual) flux, the reluctance of this path is of the same order of magnitude as the reluctance of the leakage paths of the local fluxes, which link with primary but not with secondary, or vice versa. On account of the primary leakage, the mutual flux which induces the total e.m.f. in the rotor is reduced further than indicated by curve *B* of Fig. 256. On account of secondary leakage (the effect of which is like inductance introduced into the secondary circuit, as explained in Art. 41 and 42 of this book) a considerable and increasing part of whatever e.m.f. may be induced in the rotor is used up in overcoming the secondary leakage reactance so that the rotor current corresponding to a given slip is reduced. Thus, the slip required to produce a given rotor current must be larger than if the secondary circuits had resistance only as in the ideal case of Fig. 256.

Other considerations are also involved with the magnetic leakage, all tending to the same effect of reducing the torque that corresponds to a given slip. The reactance of the secondary depends directly upon frequency of secondary current or induced e.m.f. as well as upon leakage inductance of rotor. The reactance of the rotor is therefore directly proportional to the slip; the secondary impedance increases, and the rotor current corresponding to a given per cent slip decreases, on this account. Moreover, the increase of secondary reactance with load and slip lowers the secondary power-factor and brings the rotor currents or poles into a position relative to the stator poles which is less favorable to the production of torque (see Fig. 243 and 244). Finally, as seen from Fig. 244, the increasing lag of secondary currents (with increasing load, slip and secondary reactance) brings the rotor poles into more nearly direct opposition to the stator poles, which tends to increase the magnetic leakage still further. This effect is not unlike the armature reaction in a direct-current machine.

The combined effect of all these influences is to change the speed-torque relation from that shown by curve *B* in Fig. 256 and 257 to that shown by curve *C* in Fig. 257. The most significant feature about this curve, which is typical of a squirrel-cage motor, is that the resisting torque or load can be increased up to a certain well-defined maximum value, but if it be made greater than this the speed and torque both decrease simultaneously and rapidly and the motor comes to a standstill. This point of maximum load is called the **pull-out torque** or the **breakdown-point**. At standstill with the same (full rated) voltage applied, the torque is

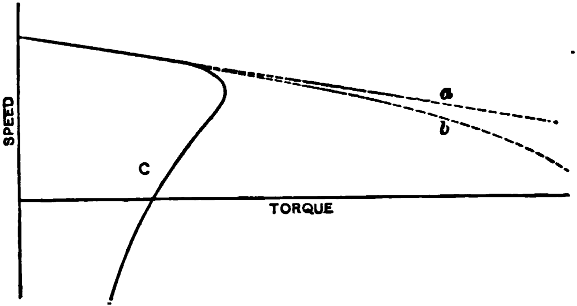

FIG. 257. Speed-torque curves for a polyphase induction motor, showing the effect of magnetic leakage. Curves *a* and *b* same as in Fig. 256. Curve *C* shows the combined effect of resistance, leakage and secondary reactance. *The Westinghouse Electric and Mfg. Co.*

usually considerably less than this maximum. Thus, in Table I we observe that the starting torque is usually about two-thirds as large as the pull-out torque, both being measured with rated voltage applied to the stator.

100. Performance Curves of Induction Motor. When we load a motor, the "independent variable" which we control and upon which all other factors depend, is the **torque**; that is, we increase the resisting torque. It is most logical, therefore, to consider all other variable factors with relation to the torque that the motor develops. Some writers prefer to use horse-power output as abscissas for their performance curves, but such curves are more difficult to reason about because the horse power depends upon speed as well as torque, and speed varies with torque. In any case, the persons who must buy the motor, use its service, or supply it with electrical power, are interested in the power-factor, the speed, the efficiency of the motor, and the manner in which these quantities vary with the load whether it be measured by torque or power output.

Thus, Fig. 258 shows the relation of speed (curve *A*), power-factor (curve *C*) and efficiency (curve *B*) to the useful torque delivered at the pulley, for a three-phase, 25-horsepower, 60-cycle, 675-r.p.m. induction motor manufactured by the Wagner Elect. Mfg. Co. The motor has a wound rotor, but curve *A* shows the speed variations when the rotor is completely short-circuited or operated with zero external resistance. Curves 1, 2, 3, . . . , 8 represent the speed-torque relations for this motor when various amounts of resistance are inserted in series with the rotor windings, curve 1 corresponding to a much larger amount of resistance than curve 8. More will be said of these curves later.

At zero load or zero torque the only current flowing to the stator from the line is the exciting current, there being practically no rotor current and therefore no load component of stator current. The zero-load power-factor of the motor depends therefore upon the relation of power (core-loss)

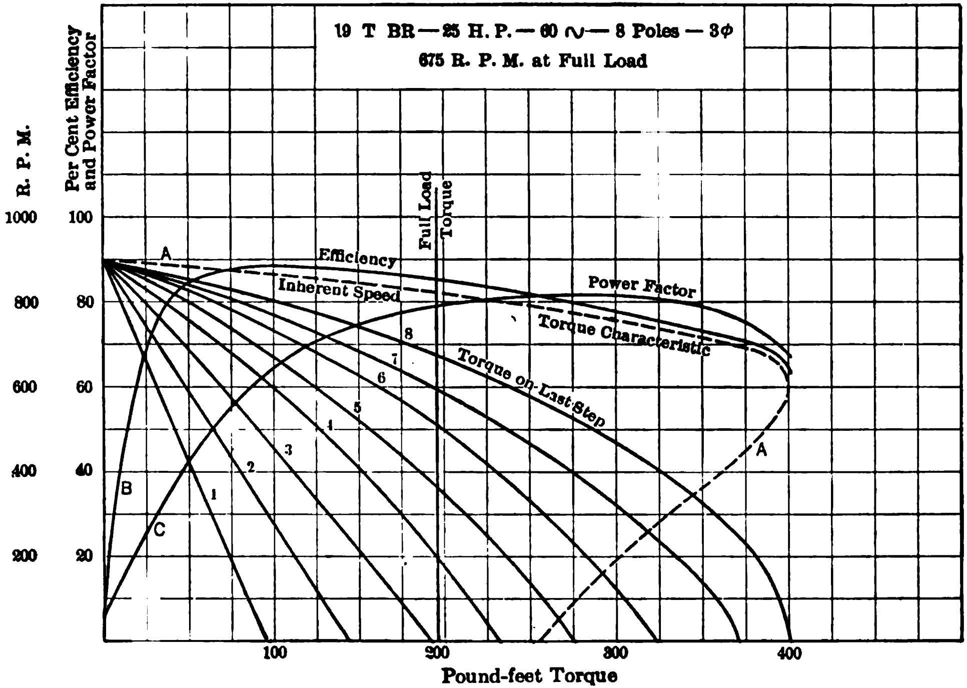

FIG. 258. Characteristics of wound-rotor type of polyphase induction motor, designed for crane service. Curve *A* is the speed-torque curve with all the external resistance cut out. Curves 1, 2, 3, 4, 5, 6, 7, 8 are the speed-torque curves with various amounts of resistance inserted in the rotor circuits, curve 1 corresponding to the largest amount of resistance. Note that the efficiency and power-factor correspond to curve *A*.

component to magnetizing component of this exciting current. Now, on account of the air gap in the magnetic circuit of the motor, the magnetizing component of the exciting current is relatively very much larger than in the ordinary transformer which has an all-iron magnetic circuit. Consequently, the **running** current of the motor at **zero load** has usually a low power-factor; this is to be clearly distinguished, however, from the power-factor of the **starting** current which depends principally upon the design of the rotor as already explained.

When the motor is running at or about normal full load, the current input consists mainly of load-component, and the (constant) exciting component has become relatively insignificant. The power-factor of the motor will now be nearly equal to the power-factor of the rotor current, which depends upon the ratio of rotor reactance to rotor resistance. The rotor reactance is relatively small because the value of slip and rotor frequency are low. When the motor becomes heavily overloaded, however, the corresponding large value of slip and rotor frequency make the rotor reactance large in comparison with rotor resistance; the result is that the power-factor of secondary current becomes lower and therefore the power-factor of stator current also becomes smaller. This is as shown by the curve C in Fig. 258.

When the motor overcomes no resisting torque, the power output is zero regardless of speed; and as there is an input equal to the total losses, the efficiency of the motor is zero. As the load increases the current increases, and the I^2R losses in rotor and in stator both increase in proportion to the square of the current. The friction and core losses are approximately constant (variations due to change of speed and to flux density in iron being slight as long as the impressed voltage and frequency are maintained constant, as usual). The efficiency will rise until it reaches a maximum value when the total variable losses (I^2R in rotor and stator) become equal to the total constant losses (due to friction, hysteresis and eddy currents). At greater loads than this, the efficiency becomes lower again on account of the very rapid increase of I^2R losses at the large values of I.

By properly proportioning the parts of the motor, the designer usually makes the maximum efficiency occur at the load which the motor must carry most of the time. If the load is fairly steady, this would naturally be the "full load" for which the motor is rated and selected. But if the load is fluctuating, and particularly if the requirement of heavy starting torque compels the selection of a motor so large that the average load is considerably less than the rated "full load" (which it can carry without overheating), then the maximum efficiency may be caused to come at less than full load (as in Fig. 258) in order to produce a higher all-day efficiency.*

Table II presents in condensed form the relations of efficiency and power-factor to power output for a complete line of medium-sized motors of the squirrel-cage type, at two different standard speeds. The motors are all designed for a frequency of 60 cycles per second, and are supposed to be operated at this frequency and at the voltage for which they are designed. It makes practically no difference in the operating characteristics whether the windings are arranged for two-phase or for three-phase, and practically no difference whether the voltage is 220 or 2300.

Notice from Table II the following general relations:

First. The efficiency curves are very nearly flat from half-load up to or beyond 25 per cent overload.

Second. The power-factor curves are far from flat. Even at $\frac{3}{4}$ load the power-factors are decidedly less than at full load and the power-factor in general becomes very poor below half-load.

Third. For a given size (horse power) of motor, the efficiency curve is practically the same for the high-speed motor as for the low-speed motor. However, the power-factor is decidedly higher in the high-speed motor than in the low-speed motor.

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* See Art. 4 and 38 of this volume.

Fourth. For a given speed, the larger sizes of motor have the higher efficiencies and also the higher power-factors.

Such conclusions from Table II may be turned to practical advantage. Thus, it appears that we should be very careful not to "over-motor" the loads — that is, not to select a motor larger than is actually required to carry the load. For, if the motor is too large it operates at only a fraction of its full load, and the power-factor and efficiency are much lower than might be if the motor were more closely adapted to its task. This is perhaps the most common fault in applications of induction motors. The tendency is very natural to "play safe" when we do not know exactly the amount of power necessary to drive a given machine, and to install a motor which is much too large. A large amount of information is now available in all electrical handbooks concerning the amount of power necessary to drive various machinery. If this is not sufficient, the manufacturer should be appealed to, or the power required should be measured by a trial motor before making a permanent selection of motor.

In many cases where installations have been remodeled so as to group a number of machines on one motor no larger than was previously used to drive each of the machines separately, or where large motors have been replaced by smaller ones on individual drives, the result has been to produce higher power-factor, better voltage regulation of the distributing lines, reduction of generating capacity used or more generating capacity released for other purposes, and higher efficiencies for all parts of the system including motors, lines and generators.

Furthermore, it is evident from Table II that we improve the power-factor by selecting a higher speed motor of the same size to carry the same load, while we alter the efficiency neither appreciably nor necessarily. Taken in conjunction with the fact that a higher speed motor is smaller, lighter and less expensive for the same rated horse power, this signifies that a considerable improvement both in economy and in quality of service may be had by choosing the higher speed motors, particularly in locations where the machines to be driven require high speed, which would have to be obtained by gearing or belting if low-speed motors were selected.

Prob. 43–7. The motor of Fig. 258 being rated 25-h.p., 675-r.p.m., 60-cycles, three-phase, and being wound with 8 poles for 250 volts, **calculate** the following:

(*a*) Pound-feet useful torque at rated load and speed.

(*b*) Synchronous speed, in r.p.m.

(*c*) Slip, in per cent, at rated load torque (curve 8).

(*d*) Slip, in per cent, at rated load torque, without controller (curve *A*).

Prob. 44–7. From data of the curves *A*, *B*, *C*, for the motor of Fig. 258, and Prob. 43, calculate:

(*a*) Watts input at rated load torque.

(*b*) Volt-amperes input at rated load.

(*c*) Amperes input per line wire at rated load.

(*d*) Equivalent single-phase current at rated load.

(*e*) Horse-power output at rated torque without controller (curve *A*).

Prob. 45–7. From the curves of efficiency, power-factor and speed (curve *A*) for the motor of Fig. 258, calculate the current per line wire at ¼, ½, ¾, 1, 1¼, 1½ and 2 times full-load torque, and draw the current-torque curve to scale. Discuss the form of this curve.

Prob. 46–7. Replot the curves of efficiency, power-factor and speed (curve *A*) of Fig. 258, on a basis of horse-power output as abscissas. Compare these with the corresponding curves of Fig. 258.

Prob. 47–7. If the maximum torque at standstill is obtained when secondary resistance equals secondary reactance, and if secondary reactance varies in direct proportion to the slip, calculate from the data of curve 8, Fig. 258, what should be the power-factor of the rotor currents (or of the load component of primary or stator currents) at rated load with speed of 675 r.p.m. Compare this value with that observed from the power-factor curve of Fig. 258.

Prob. 48–7. From the data in curves of Fig. 258, calculate the total watts lost in the motor at ¼, ½, ¾, 1, 1½ and 2 times rated-load torque. Draw a curve with torque as abscissas and total losses as ordinates. Extend this curve until it cuts the vertical axis and from the intercept thereon measure the power input at zero torque. Should this be equal to stator core losses and friction losses of the motor, approximately?

Prob. 49–7. By means of data from curves *A*, *B*, of Fig. 258 corresponding to the torque at which the efficiency is maximum, calculate the value, in watts, of

(*a*) The constant losses, namely core losses and friction.

(*b*) The variable losses, namely copper losses in stator and rotor. Note whether the value (*a*) equals the answer found in Prob. 48.

Prob. 50–7. From the current-torque curve of Prob. 45 determine the current at 325 pound-feet torque as a percentage of rated-load current. Assuming the variable losses to be proportional to the square of the stator current, calculate from this percentage and from the results of Prob. 49 what the variable losses should be at 325 pound-feet of torque.

Prob. 51–7. From the results of Prob. 49 and 50, calculate the efficiency corresponding to 325 pound-feet of torque, and compare with the value given on the efficiency-torque curve of Fig. 258.

Prob. 52–7. Solve Prob. 50 and 51 with respect to a torque of 200 pound-feet.

Prob. 53–7. How many kilowatts and kv-a. of generator capacity would be saved or lost by grouping together upon a single 5-h.p. 1140-r.p.m. motor (Table II), five machines each of which was formerly driven by a 1-h.p. 1140-r.p.m. motor delivering full rated load? Assume that the losses in the shafting introduced by the group drive amount to 25 per cent of the useful power output of the shaft, and that the load on all machines is perfectly steady.

Prob. 54–7. Assume that the work performed by the five machines of Prob. 53 is such that only two of them are working (at full load, requiring 1 h.p. for each machine) at any time. What size motor would be needed for group drive, and how much more or less of kilowatt and kilovolt-ampere generator capacity would be required for group drive than for individual drive, assuming horsepower loss in shafting same as in Prob. 53?

101. Effect of Rotor Resistance on Performance of Induction Motor. We have already seen (Art. 97 and 98) what effects are produced upon the current, power-factor and torque of the polyphase motor at starting, by increasing the resistance of the rotor circuit, or by increasing the ratio of resistance to reactance of the rotor in designing the motor. Such changes or adjustments also affect the operating characteristics of the motor — particularly the speed-torque and the efficiency-torque curves, and furnish the most convenient means for controlling the speed of an induction motor.

Thus, in Fig. 258 we see that the insertion of a certain amount of resistance in circuit with a given rotor changes the speed-torque relation from curve *A* to curve 8, giving the maximum torque at standstill or at starting instead of at 67 per cent of synchronous speed (or, giving a starting torque of 400 pound-feet instead of 256 pound-feet, with full rated voltage applied to the stator in both cases). The insertion of still more resistance in the rotor circuit changes the speed-torque relation as shown by curves 8, 7, . . . , 2, 1 successively.

From these curves of Fig. 258 we read that:

First. If the rotor resistance is increased so as to give maximum torque (of 400 pound-feet) at standstill or at starting (curve 8), then the speed regulation obtained, when the torque is reduced from normal full-load value to zero, is $\frac{900 - 675}{675}$ or $33\frac{1}{3}$ per cent.

Second. If the rotor resistance is still further increased, so as to give slightly more than full-load torque (232 pound-feet) at starting (curve 4), then the speed regulation from full-load torque (194 pound-feet) to zero is $\frac{900 - 200}{200}$ or 350 per cent.

Third. With the motor working against full-load torque, we can obtain reduced speeds of 0 r.p.m. (curves 3, 2 or 1), 200 r.p.m. (curve 4), 360 r.p.m. (curve 5), 510 r.p.m. (curve 6), 600 r.p.m. (curve 7), or 675 r.p.m. (curve 8), with the steps of resistance provided in this particular controller. At half-load torque we could obtain two additional speeds by use of steps No. 2 and No. 3 on the controller.

Fourth. If the motor has to start against a torque equal to that at normal full load, the rotor would not begin to move until the controller had been turned to point No. 4 after passing points 1, 2, 3. On point 4 there would be torque of 232 − 194 = 38 pound-feet available for accelerating the motor. The speed will increase until it reaches about 200 r.p.m., at which point the torque is reduced to the value against which the motor is turning and the motor turns steadily because there is no torque available for acceleration. If now we turn the controller to point No. 5, the torque at 200 r.p.m. immediately increases to about 235 pound-feet (on curve 5); we now have 235 − 194 = 41 pound-feet of torque available for acceleration, and the speed increases as far as 360 r.p.m., where the motor will continue to turn steadily unless we proceed to point 6 on the controller.

The greater the opposing torque against which the motor must start, the further must we turn the controller before the rotor will move. The more rapidly we desire the motor to pick up speed, the faster must we move the controller in the direction of lower resistance. We must realize, however, that in either of these cases the current taken from the line during acceleration is correspondingly increased.

The effect of secondary resistance upon speed is further illustrated by Fig. 259. For curve *b* the total resistance per rotor circuit is twice as large as for curve *a*. Having doubled r_2, let us adjust the torque until the slip has also been doubled. Then, the induced e.m.f. in the rotor is also doubled (assuming, for the moment, that the mutual flux remains unchanged). Also, the reactance of the rotor, which is directly proportional to secondary frequency, or slip, is also doubled. In consequence, the ratio of secondary reactance to secondary resistance is not altered, and the power-factor remains unchanged; but the secondary impedance has been doubled because both resistance and reactance of rotor have been doubled. With doubled impedance opposing the doubled rotor induced e.m.f., the rotor amperes will obviously have the same value

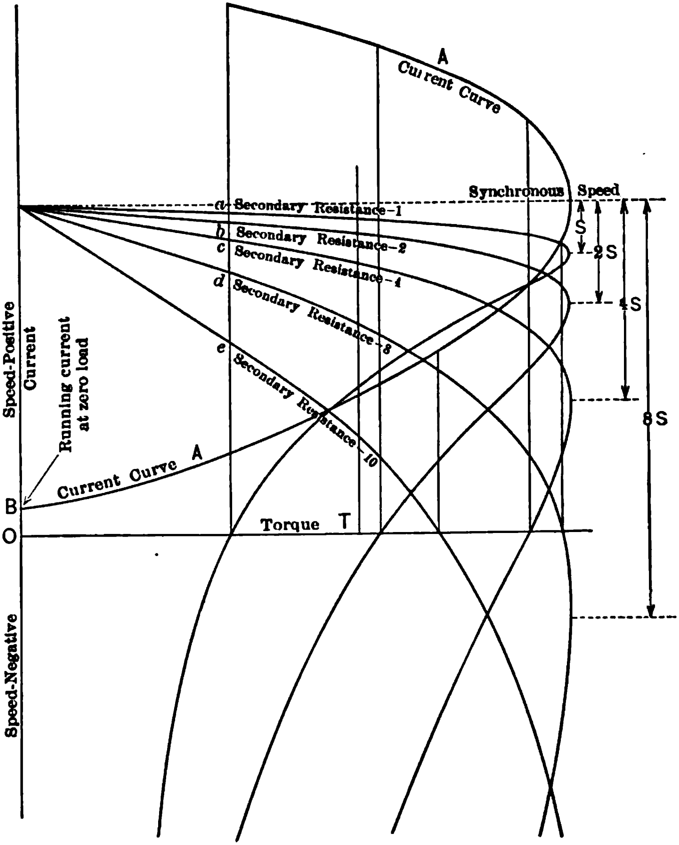

FIG. 259. Speed-torque and current-torque curve for a polyphase induction motor with various values of secondary resistance. *The Westinghouse Electric and Mfg. Co.*

as formerly. If the rotor poles have the same strength (same rotor current) and the same position relative to the stator poles (same rotor power-factor), we see that we were justified in assuming the mutual flux to remain unchanged. With unchanged stator flux and unchanged rotor poles, we see also that the torque must be the same as before.

Speaking in general terms, therefore, we may say that if the total resistance of each rotor circuit is increased *n*-fold, then the same torque will be produced with the same current supplied to the motor but at an *n*-fold increased slip. The power input is unchanged if the motor takes the same current at the same voltage and power-factor. The power output is reduced in exact proportion to the slip or to the reduction of rotor speed, because the torque (the other factor of mechanical power developed in rotor) is unchanged. Therefore, **the percentage efficiency corresponding to the same torque is reduced in direct proportion to the speed, by any increase of resistance in the rotor circuits.** We see, further, that if the resistance of each rotor circuit is increased *n*-fold while the current remains the same (with *n*-fold increase of slip) then the total I^2R loss in the rotor winding must be increased *n*-fold.

Referring again to Fig. 259, we see that for any given value of torque, say *T* pound-feet, curve *b* shows the motor to be twice as far below synchronous speed as curve *a*; that is, the slip corresponding to any given value of torque is doubled when we double the rotor resistance. Similarly, comparing curve *c* with curve *a*, we see that the slip corresponding to a given torque is quadrupled if we make the rotor resistance four times as large; curves *d* and *a* show that the slip is increased 8-fold when we increase the rotor resistance 8-fold, and so forth.

The current curve *A* (Fig. 259) is the same for all speed-torque curves (*a*, *b*, *c*, etc.), as long as the voltage and frequency impressed on the motor are maintained constant. This accords with our previous finding, that the current and power-factor remain unchanged if the torque is constant, although the speed may have any value between zero and synchronous speed depending upon the amount of resistance in the rotor circuits. The current *OB* which the stator takes when no torque is being developed is the exciting current. After the breakdown point or point of maximum torque is passed, we see that the current continues to increase although the torque is decreasing. The intercepts of the current curve upon the vertical straight lines of Fig. 259 measure the "stalling current" which the motor takes at standstill with full voltage applied and various amounts of resistance in the rotor circuit.

We see also from Fig. 259 that increase of rotor resistance does not reduce or alter the pull-out point or the maximum value of torque which the motor can produce for a given voltage applied. The vertical lines have been drawn in Fig. 259 to point out an important fact; namely, that the lowest values of rotor resistance give a relatively small torque at standstill but nevertheless cause the largest starting currents to be taken at given (constant) voltage; and that the starting torque may be increased while the starting current is reduced, by increasing the rotor resistance up to a certain point (corresponding to curve *d*), but that if larger rotor resistance than this be employed, the starting torque is reduced as also the starting current (shown by curve *e*). The extension of the speed-torque curves below the horizontal axis *OT* indicates what values of torque would be encountered by an external source of mechanical power when used to drive the rotor in a direction opposite to that of the rotating flux (that is, produce negative speeds), while the stator remains connected to the alternating-current supply.

To assist the student in reading the ordinary literature of the induction motor, it were well for us to explain that the torque is often expressed in terms of "synchronous watts" instead of "pound-feet." To explain the relation between these terms, let us take for example a machine which develops a torque of 100 pound-feet at 1000 r.p.m.

and whose synchronous speed is 1200 r.p.m. This torque is equal to that produced by a pull of 100 pounds along a belt wrapped around a pulley of 1-ft. radius or 2-ft. diameter. The work done by such a force in turning the pulley through 1 revolution is $2\pi rF = 6.28 \times 1$ ft. $\times$ 100 lbs. = 628 foot-pounds of work or energy. If the motor were to exert this same torque while turning at synchronous speed (1200 r.p.m.), the mechanical power generated would be $628 \times 1200 = 753{,}600$ foot-pounds per minute, or $\frac{753{,}600}{33{,}000}$ = 22.8 horse power, or $22.8 \times 746 = 17{,}020$ watts. This is the so-called "synchronous watts," meaning the mechanical power (expressed in watts) which the machine would develop if it were to turn at synchronous speed while exerting the same torque.

As power is directly proportional to both torque and speed, "synchronous watts" is equal to the product of synchronous speed (r.p.m.) and torque (pound-feet) multiplied by a constant (K) whose value is determined as follows:

$$\begin{aligned} \text{Synchronous watts} &= \text{r.p.m.} \times \text{pound-feet} \times K, \\ 17{,}020 &= 1200 \times 100 \times K, \\ K &= \frac{17{,}020}{1200 \times 100} \\ &= 0.142. \end{aligned}$$

That is, we multiply the torque in pound-feet by the synchronous speed in r.p.m. and by the constant 0.142 to get the torque in synchronous watts. This quantity is useful in analyzing the losses in the induction motor and, like the method of expressing resistance and impedance of transformers and generators in terms of per cent, it enables us more readily to compare the performance of machines of various sizes and speeds.

Prob. 55-7. Calculate the percentage speed regulation of the motor (rated-load torque to zero) corresponding to each of the speed-torque curves given in Fig. 258, and draw from these results a curve having as abscissas the starting torque (as per cent of rated-load torque) and as ordinates the per cent speed regulation corresponding to the rotor resistances which give these values of starting torque.

Prob. 56–7. What values of torque, expressed as percentages of rated-load torque, may be obtained from the motor of Fig. 258 operating at a speed of 500 r.p.m., by means of the particular controller to which these speed-torque curves refer?

Prob. 57–7. What values of speed, expressed as per cent of synchronous speed or zero-load speed, may be obtained from the motor of Fig. 258 operating at half of rated-load torque, by means of the particular controller to which these speed-torque curves refer?

Prob. 58–7. Using data from Fig. 259, draw a starting characteristic for wound-rotor motor at constant voltage and frequency having as abscissas starting torque and as ordinates the corresponding currents taken by the stator at starting (zero speed). Then, from data of Fig. 254, obtain a similar curve showing relation of current to torque, when starting at full voltage but with various values of rotor resistance. Compare the forms of these two curves.

Prob. 59–7. In Fig. 259, assume that the controller, by means of which the amount of resistance in rotor circuits is adjusted, has only five running points corresponding to the speed-torque curves *a*, *b*, *c*, *d*, *e*. The first point on the controller corresponds to curve *e*, this being the maximum resistance available after closing the rotor circuit. If the motor starts against a torque equal to rated-load torque represented by the vertical line *T*, draw a curve having speed as abscissas and current as ordinates. The motor is allowed to reach steady speed on each controller point before being advanced to the next point.

Prob. 60–7. From the data of Fig. 258, calculate what must be the ratio between the total rotor circuit resistances corresponding to speed-torque curves 8, 7, 6, 5, 4, 3, 2, 1, respectively.

Prob. 61–7. If the curve *A* in Fig. 258 represents the speed-torque relation with the rotor terminals short-circuited (that is, only the inherent resistance of rotor windings being present in rotor circuit), calculate the ratio between resistances external and internal to the rotor, corresponding to speed-torque curves 8, 7, 6, 5, 4, 3, 2, 1, respectively, in Fig. 258.

Prob. 62–7. From the data of Fig. 258 calculate the starting torque in terms of synchronous watts, corresponding to each of the eight controller points, and also to the "inherent speed-torque characteristic" with rotor short-circuited.

102. Effect of Air Gap on Operating Characteristics. Changing the length of air-gap in an induction motor, other things being equal, affects all the operating characteristics of the motor, but principally the relation of current, power-factor and efficiency to the torque or the horse-power output. With constant voltage and frequency impressed upon the stator, the counter e.m.f. and the amount of flux must remain approximately constant; therefore, the core losses (watts) and the core-loss component of the exciting current must both be nearly constant for variations of air-gap length as well as for variations of load.

However, if we double the length of air gap we very nearly double the total reluctance of each magnetic circuit in the motor, because most of the reluctance in the magnetic circuits is due to the air gap. To produce the same flux against a doubled reluctance requires that the magnetizing ampere-turns be doubled; that is, the magnetizing component of the exciting current must be doubled. Suppose that originally the exciting current was 3 amperes at 20 per cent power-factor. Its core-loss component was therefore $0.2 \times 3 = 0.6$ ampere, and its magnetizing component was $\sqrt{3^2 - 0.6^2} = 2.94$ amperes. If now we double the air gap, the core-loss component remains approximately 0.6 ampere while the magnetizing component increases to approximately $2 \times 2.94 = 5.88$ amperes, consequently the exciting current is now $\sqrt{5.88^2 + 0.6^2} = 5.91$ amperes. The zero-load current is therefore increased by $\frac{5.91 - 3}{3}$ or 97 per cent, and the power-factor at zero load is reduced from 0.20 to $\frac{0.6}{5.91}$ or 0.102.

The magnitude of the effect of changes in length of air gap upon the current, power-factor and efficiency at loads greater than zero will depend upon the design of the motor — for instance, upon the relation of exciting current and power-factor at zero load to total current and power-factor at rated full load, with normal air gap. In general, the current input corresponding to any given percentage of rated load will be increased, the power-factor will be reduced, the efficiency will be lowered; the speed and the pull-out torque will be affected to the least extent although both will be reduced. For illustration, Fig. 260 exhibits the relation of power-factor to load, corresponding to several different lengths of air gap for the same motor; the air gaps represented here are in the ratio 1 : 2 : 3 : 4 = $A : B : C : D$.

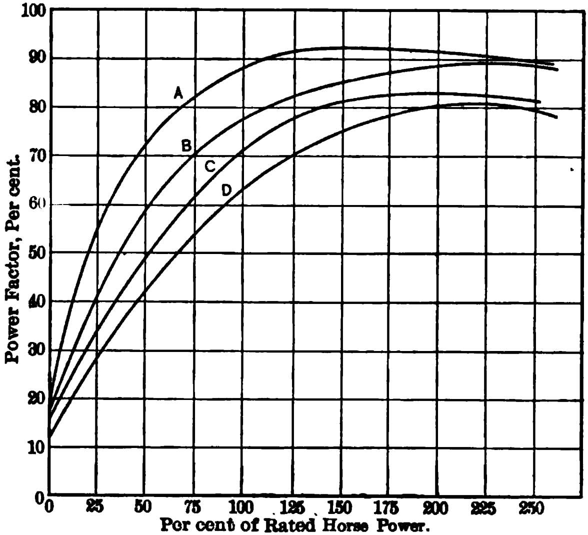

FIG. 260. The effect of changing the air gap of a polyphase induction motor.

Curve *A*, — standard gap.
Curve *B*, — double standard gap.
Curve *C*, — three times standard gap.
Curve *D*, — four times standard gap.

Fitzgerald in Proc. N.E.L.A.

There are practical aspects of great importance in this matter of air gap. If in an effort to raise the power-factor and efficiency, or lower the current, the air gap be decreased too far, then a comparatively slight amount of wear in the bearings will cause the rotor to rub, and therefore adjustments and repairs become necessary more frequently. Typical values of air gap are from 0.02 to 0.05 inch in motors of about 5 to 30 horse power, and 750 to 1200 r.p.m. Though small, these are sufficient for all ordinary uses; for very rough use, in steel mills for instance, gaps as high as 0.10 inch may be employed if, as usual in such cases, the purchaser considers continuity of service to be very much more important than power-factor or efficiency. In any case, it is important that trouble from rubbing of the rotor be remedied by adjusting or changing the bearings and never by turning down the rotor to a smaller diameter, because the latter method brings on all the bad effects of increased air gap.

103. Effect of Wrong Voltage or Frequency on Operating Characteristics. If the voltage impressed on the stator be reduced for any cause, while the frequency remains unchanged, the torque corresponding to any given percentage slip will be reduced in proportion to the square of the voltage. The reason for this has already been explained in relation to the starting of the induction motor. (See Art. 97 and 98, and Fig. 255.) Thus, a motor which gives full-load torque at 5 per cent slip with rated voltage applied to the stator, will give $(\frac{1}{2})^2 = \frac{1}{4}$ times full-load torque at the same slip or speed, if the voltage be reduced one-half.

It follows that, if the resisting torque remains constant while the impressed voltage is reduced to half, the slip must be increased and the speed correspondingly reduced. In fact, in this case the slip would be increased four-fold when the voltage is halved; the four-fold slip with halved flux would induce double e.m.f. in rotor, which (neglecting rotor reactance) would produce doubled rotor current, and this doubled rotor current with halved stator flux would produce the same torque as formerly. The effect of doubled rotor reactance upon rotor current and upon the position of rotor poles relative to stator poles will be such as to cause an even greater increase of slip than that deduced above.

Any reduction of frequency affects the synchronous speed of a given motor in direct and exact proportion. If the voltage meanwhile is maintained constant as by a voltage regulator, the amount of flux is increased in inverse proportion to the frequency. (See Art. 37.) This would not only increase the core losses and the operating temperature of the motor corresponding to a given load, as well as lower the efficiency, but would reduce the speed while also reducing the per cent slip. Supposing that we halve the frequency with constant voltage, we may reason as follows:

First. Half frequency at same voltage means double flux.

Second. Double flux with same torque means half rotor currents (neglecting change of relative position of rotor and stator poles due to change of frequency and power-factor in rotor).

Third. Half rotor currents mean half rotor induced e.m.f. (neglecting change of rotor frequency and reactance due to slip).

Fourth. Half rotor e.m.f., with double flux, means that the slip in terms of actual rev. per min. must be only one-quarter as large as originally.

Fifth. One-quarter as many r.p.m. of slip, with one-half as many r.p.m. of stator flux, means that the per cent slip is only one-half as great as originally.

As the frequency of rotor e.m.f.'s is decreased, we may reason that the effect of rotor reactance will be to cause the reduction in per cent slip to be slightly greater than that deduced above. Since the slip is usually relatively small at rated load and frequency, we may say that the actual rotor speed is reduced to a value slightly more than half its former value when the frequency is reduced to half. Such a large change of frequency is not likely to occur in any motor unless, for instance, we happen to attach a 60-cycle motor to a 25-cycle circuit having the same voltage for which the motor is rated; in that case the excessive core losses and exciting current would soon burn out the motor, so that this is not to be considered as an operating condition.

By reasoning similar to the example explained above, we may arrive at the conclusion that if the voltage is raised or lowered in proportion as the frequency varies, the per cent slip corresponding to a given torque remains unchanged; that is, with constant torque we get exactly half speed corresponding to half frequency, or we get 0.9 of the original speed if we reduce both the frequency and voltage to 0.9 of their original values. There is not much difference between the results of the two sets of assumptions; roughly, the speed of a given induction motor is directly proportional to the frequency.

It may also be shown that if an induction motor must operate on frequency other than standard, the performance will be better if the voltage is changed in proportion to the square root of the frequency. Thus, a 440-volt 60-cycle motor operating on 50 cycles will have very nearly its normal operating characteristics if the voltage is reduced to $440 \times \sqrt{50/60}$ or 402 volts.

Prob. 63–7. From the "inherent speed-torque characteristic" of Fig. 258, draw accurately to scale a curve having as abscissas torque (pound-feet) and as ordinates slip (r.p.m.). From this, calculate another slip-torque curve corresponding to an impressed voltage 60 per cent of rated voltage. Finally, draw the inherent speed-torque curve for 60 per cent of rated voltage.

Prob. 64–7. Repeat the work of Prob. 63 on the basis of the curve 8 of Fig. 258, thus showing the speed-torque relation for the same motor when operated on last step of controller with 60 per cent of rated voltage impressed upon the stator.

Prob. 65–7. (*a*) A 440-volt 60-cycle induction motor is to be operated from 25-cycle mains. What voltage should be impressed upon its stator? (*b*) What percentage of normal flux density will be obtained under the altered conditions?

104. Speed Control of Polyphase Induction Motor. Any desired speed between zero and synchronous speed may be obtained by adjusting the amount of resistance in the secondary or rotor circuits. This method of speed control is fully covered in Art. 101. The method is objectionable in the following respects:

First. After the speed has been adjusted to a given value at a given load, any change of load will be accompanied by a considerable variation of speed. That is, the speed regulation of the motor will be poor; the further from synchronism the speed is adjusted by means of rotor resistance, the worse will be the speed regulation, naturally.

Second. The efficiency is lowered when the rotor resistance is increased to control the speed. (This relation is also covered in Art. 101.)

This method of control is therefore not suitable for service requiring several constant speeds with varying torque, such as the driving of machine tools. It is, however, very useful where constant speeds are not essential, as in operating cranes, hoists, elevators and dredges, and also for service where the torque remains constant at each speed, as in driving fans, blowers and centrifugal pumps.

Various speeds may also be obtained with any given torque, by changing the voltage applied to the stator terminals (by inserting resistance between the supply lines and the stator terminals, which has the same disadvantages as the method of secondary resistances and is not as effective; or by changing from one tap to another on an autotransformer). In Art. 103 it is explained how variations of voltage at constant frequency cause variations of speed. This method is not in commercial use, however, on account of low power-factor, low efficiency, and poor speed regulation obtained when the speed is controlled over an appreciable range in this manner.

Sometimes the stator winding is arranged so that when the connections are changed by means of a special switch, the number of poles is changed. Thus, if the number of poles on a 60-cycle stator is changed from 6 to 4, the synchronous speed is changed correspondingly from 1200 r.p.m. to 1800 r.p.m. Although the operating characteristics are maintained excellently at the changed speed, it is impracticable to have many changes of speed in the same motor, and the scheme does not meet favor on account of its complications.

Next to the method of varying secondary resistance, the most practicable and useful method is the connection of induction motors "in cascade," or in "concatenation." The theory of this method has already been hinted at in Prob. 19 of this chapter. The system requires two motors, and they must be rigidly coupled together by gearing or belting or by being mounted on the same shaft. Furthermore, at least one of the motors must have a wound rotor with slip-rings for connecting it to an external circuit. The stator of motor No. 1 is connected to the line, and the terminals of its wound rotor are connected to the stator of motor No. 2. Motor No. 2 may have either a squirrel-cage rotor or a wound rotor; in the latter case rotor resistances may be used to obtain intermediate speeds between the otherwise abrupt steps obtainable simply by the cascade connection.

With concatenated motors as described above, the speed at which the shaft will turn for a given frequency and voltage impressed upon the stator of motor No. 1 may have various values depending upon:

(*a*) The number of poles in each motor.
(*b*) The polarity of connections between rotor of motor No. 1 and stator of motor No. 2.
(*c*) The amount of resistance in the rotor circuit of motor No. 2, if it is of the wound-rotor type.

As the internal action of concatenated motors is especially instructive, it is explained in some detail in a separate article following.

It is apparent from the above discussion of methods for controlling the speed of polyphase induction motors, that they are all rather unsatisfactory in one way or another. In fact, it appears that the induction motor is inherently what may be called a "constant-speed motor" — not in an absolute sense like the synchronous motor, but in comparison with such flexible adjustable-speed types as may be had among the direct-current motors.

105. Induction Motors in Cascade. The connection variously known as the cascade, or chain, or tandem connection, or concatenation of polyphase induction motors, is described briefly in the preceding article as one of the methods available for controlling the speed of such motors. We shall examine now the internal actions by which motors so connected adjust themselves.

Consider first two motors exactly alike, with the rotor of the second one short-circuited or replaced by a squirrel-cage rotor. Being directly coupled together, they must turn at the same speed. If the stator of No. 2 is connected to the rotor terminals of No. 1 in such sequence that both motors tend to rotate in the same direction, then the synchronous speed of the couple will be just one-half of the speed at which each motor would operate if both stators were connected directly to the supply line in parallel while both rotors were short-circuited. This is in fact the manner in which we get two speeds from the couple, the tandem connection giving us one-half the speed which we get from the parallel connection.

Analyzing the general case, let:

p_1 = number of pairs of poles on motor No. 1.
p_2 = number of pairs of poles on motor No. 2.
f_0 = number of cycles per second, frequency of supply mains.
n = number of rev. per sec., speed of shaft.

Then

$\frac{f_0}{p_1}$ = synchronous speed of motor No. 1, rev. per sec.

$\left(\frac{f_0}{p_1} - n\right)$ = slip of motor No. 1, in rev. per sec.

$\left(\frac{f_0}{p_1} - n\right) p_1$ = frequency of rotor e.m.f.'s in motor No. 1 = frequency of stator e.m.f.'s impressed on motor No. 2.

$\dfrac{\left(\frac{f_0}{p_1} - n\right) \times p_1}{p_2}$ = synchronous speed of motor No. 2, in rev. per sec.

Now, if motor No. 2 is connected to motor No. 1 in such manner that both tend to start in the same direction (direct concatenation), we have

$\dfrac{\left(\frac{f_0}{p_1} - n\right) \times p_1}{p_2}$ = synchronous speed of motor No. 2 = n rev. per sec.

This is because the motors are rigidly coupled together; motor No. 2 with its rotor short-circuited must operate at or near its synchronous speed, and this must be the same as the speed of motor No. 1. Simplifying this equation, we have

$$(1) \quad f_0 - np_1 = np_2, \text{ or } n(p_1 + p_2) = f_0, \text{ or } n = \frac{f_0}{(p_1 + p_2)}.$$

But suppose that motor No. 2 is connected to motor No. 1 in such manner that they tend to start in opposite directions (differential concatenation); we have then

$\dfrac{\left(\frac{f_0}{p_1} - n\right) \times p_1}{p_2}$ = synchronous speed of motor No. 2 = $-n$ rev. per sec.

Simplifying this equation, we have

$$(2) \quad f_0 - np_1 = -np_2, \text{ or } n\,(p_1 - p_2) = f_0, \text{ or } n = \frac{f_0}{(p_1 - p_2)}.$$

By using two motors with different numbers of poles, we can get four different speeds. Thus, suppose that motor No. 1 has 6 poles and motor No. 2 has 4 poles. Then $p_1 = 3$ and $p_2 = 2$. When supplied from a 60-cycle circuit:

(*a*) With motor No. 1 running singly, stator directly to line and rotor short-circuited, speed $= \frac{f_0}{p_1} = \frac{60}{3} = 20$ rev. per sec.

1200

(*b*) With motor No. 2 running singly, speed $= \frac{f_0}{p_2} = \frac{60}{2} =$ 30 rev. per sec. 1800

(*c*) With motors No. 1 and No. 2 in direct concatenation,

$$\text{speed} = \frac{f_0}{p_1 + p_2} = \frac{60}{3 + 2} = 12 \text{ rev. per sec.}$$ 720

(*d*) With motors No. 1 and No. 2 in differential concatenation,

$$\text{speed} = \frac{f_0}{p_1 - p_2} = \frac{60}{3 - 2} = 60 \text{ rev. per sec.}$$

We are thus able to get speeds of 12, 20, 30 and 60 rev. per sec., which is two speeds more than we could get from these motors without the aid of cascade connection.

Prob. 66–7. Two induction motors which are being operated in direct concatenation are exactly alike in all respects except that the rotor of No. 2 is short-circuited. The rated voltage and frequency of each motor is such that the two may be operated in parallel directly from the supply line. If the flux density and the rotor and stator currents may not be permitted to exceed the values obtained under this condition, what is the maximum total power that may be obtained from the pair when operated in direct concatenation, expressed as percentage of the total power when operated in parallel?

Prob. 67–7. What speeds are obtainable by concatenated arrangements of two 60-cycle motors, one having 8 poles and the other 4 poles?

Prob. 68–7. What speed would be obtained by direct concatenation of three 60-cycle motors, having 8, 6 and 4 poles, respectively? The rotor of the last motor is short-circuited.

Prob. 69-7. What would be the result if the 4-pole motor of Prob. 68 were connected in differential concatenation to the first two motors, which are connected in direct concatenation?

106. Induction Motor as Asynchronous Generator. The Induction Generator.

When opposition is offered to the rotation of an induction motor, it takes electrical power into its stator from the line wires, it generates mechanical power in its rotor, and it turns at a speed somewhat less than the synchronous speed. As we decrease the torque resisting rotation and thereby the mechanical power generated, the slip decreases and the speed approaches synchronous speed, while the current and power taken by the stator decrease. If we remove entirely the resisting torque and permit the motor to turn freely, it takes from the line only an exciting current (of relatively small value and low power-factor) and a small amount of power to overcome fixed losses in iron and friction.

Now suppose we couple this machine to an engine which drives it at exactly synchronous speed. The stator continues to take from the line an exciting current and just enough power to overcome the iron losses, while the engine supplies the friction losses. If we speed the engine up slightly above synchronism (negative or reversed slip), the direction of rotor e.m.f. and current is reversed, which causes the stator to carry a load component of current, in direction opposite to that which it takes when the machine turns as motor at less than synchronous speed. That is, when the machine is forced to turn at more than synchronous speed, it delivers electrical power (or carries a component of current 180° out of phase with the e.m.f.) into the line, and absorbs an equivalent additional amount of mechanical power from the engine. While it continues to take from the line an exciting current, or more exactly a magnetizing current to maintain the flux, the power-factor of the total line current is distinctly raised by the comparatively large power component representing electrical power delivered or mechanical power transformed.

When an induction motor is driven thus by a prime mover above synchronous speed and thereby delivers electrical power from its stator into the line, it is called an "induction generator," or an "asynchronous generator." The amount of electrical power delivered is approximately proportional to the slip (which is negative, the speed being above synchronism), just as in the case of induction motor action where the slip is approximately directly proportional to the mechanical power developed. In fact, the variation of line current, line kilowatts, power-factor and efficiency of an induction generator with respect to (negative) slip or (negative) torque are almost exactly like the corresponding curves for the same machine with respect to (positive) slip or (positive) torque when it acts as a motor.

The induction generator cannot operate unless connected to a circuit which is capable of supplying a suitable amount of reactive volt-amperes to excite the iron cores, or produce the flux which assists to generate the e.m.f. and the power; in other words, the machine is not self-exciting. If it drives a synchronous motor, the latter may furnish the necessary reactive volt-amperes for excitation of the induction generator, while the latter gives out the kilowatts of effective power to drive the synchronous motor. Likewise an induction generator may drive a synchronous converter. If the prime mover which drives the induction generator has a governor to keep its speed constant, the negative slip that is necessary to enable it to deliver electrical power can be obtained only by a decrease in the frequency of the line which supplies the exciting volt-amperes and absorbs the electrical power generated.

If the excitation is furnished by synchronous motors or synchronous converters, the reduction of their speed, which naturally occurs when they are loaded, may cause the necessary amount of negative slip. But if the governed induction generator is driven in parallel with synchronous generators the prime movers of which are also governed, then the latter must be designed for a speed regulation somewhat larger or poorer than that of the prime mover driving the induction generator, in order that the induction generator may take its share of the load.

The most common use of induction generators is as an auxiliary to synchronous generators in large power houses. Thus, the synchronous a-c. generator is driven by a reciprocating steam engine; the exhaust steam from this engine, at or about atmospheric pressure, is fed to a steam turbine which drives an induction generator. The stator of the latter is connected directly to the terminals of the synchronous generator. There is no governor on the exhaust-steam turbine which receives all the steam that the reciprocating engine delivers; notwithstanding this, the turbine cannot race because it must deliver electrical power in proportion to any excess of its speed above the synchronous speed (of the main generator, which is governed), and this would lighten the load on the reciprocating engine, whereupon the governor of the latter comes into action.

The advantage of an induction generator in the central station is that when short-circuited its voltage drops and it stops generating, consequently cannot injure itself as can a synchronous generator (see Art. 17). It would not be advisable to use induction generators for more than about half the generating capacity of a station, however, because the relatively large reactive component of exciting current required would seriously reduce the power-factor and consequently the load capacity of the synchronous generators.

The ability of the polyphase induction motor to return power to the polyphase supply line when driven above synchronous speed is very useful in what is known as "regenerative braking." Thus, if a trolley car driven by such motors is allowed to roll freely down hill, its speed increases only until the motors are turning slightly above synchronism. Then the motors become induction generators, returning the stored "energy of position" of the car to the line as electrical power. The motors cannot run much above synchronous speed before they develop a magnetic resisting torque equal to the force tending to speed the car down hill; hence the car moves safely at about its top speed, but does not much exceed top speed. Induction motors are employed similarly to drive hoists; the load while being lowered remains connected to the motor, driving it as induction generator and returning electrically to the line a large part of the energy previously required to lift the load. However, if the power supply fails the braking action is lost.

107. Induction Motor as Frequency Changer. There are decided advantages in using a low frequency (as 25 cycles per sec.) for transmission of power, rather than a higher frequency (as 60 cycles per sec.). Thus, the line drop due to inductance of the transmission line, and the charging current due to capacitance of the line, are both reduced by lowering the frequency. While the operating characteristics of a-c. motors are improved by lower frequency, the size and cost of the machines (including transformers) per horse power or per kv-a. is increased. Everything considered, it has seemed advisable to design and operate several large power systems at 25 cycles per second. For operation of lights, however, this frequency is distinctly unsatisfactory because it produces a flicker; consequently, on such systems the lighting circuits are connected to the transmission lines through a "**frequency changer.**"

Such a frequency changer might consist of a synchronous motor coupled to an a-c. generator having a different number of poles. Prob. 5–6 in "First Course" has made it apparent that there are distinct limitations to the design of such couples. Furthermore, Prob. 13, 14, 15, 16, 17 and 18 in this chapter have served to indicate that the induction motor may be used as a frequency changer. For this purpose, the stator is connected to the supply line (low frequency) and the terminals of the wound rotor are connected to the distributing line (higher frequency) while the rotor is coupled to a synchronous motor which turns it against the rotating stator flux. The synchronous motor takes power from the supply mains at the lower frequency. Thus, to deliver at 60 cycles from the rotor when power is supplied at 25 cycles to the stator requires that the rotor be driven at a speed of $\frac{60 - 25}{25}$ or 1.4 times the synchronous speed of the rotor, and in a direction contrary to that in which the rotor would turn if operated as an induction motor with the same connections of stator to line.

It is instructive to examine the distribution of power in such a frequency changer. To drive the rotor against the stator flux requires no torque (except that necessary to overcome friction and magnetic losses in the rotor) as long as the rotor carries no current. But as soon as the rotor delivers current to a receiving circuit, the (synchronous) driving motor must exert against the rotating flux a torque T (pound-feet) represented by a mechanical force of $\left(\frac{T}{r}\right)$ pounds tangential to the rotor at a radius of r feet. If we neglect the frictional and magnetic losses in the rotor, this force performs work at the rate of $2\,\pi r\left(\frac{T}{r}\right)$ foot-pounds per revolution, or $(2\,\pi T n_r)$ foot-pounds per sec., where n_r represents the actual speed of the rotor, in rev. per sec. But the power required to make the stator flux turn against the resisting torque due to rotor currents, which is equal to the electrical power taken by the stator from the supply lines less the magnetic and copper losses in the stator, is also equal to the product of mechanical force exerted by stator flux upon the rotor times the speed of stator flux. That is, the (mechanical) power transferred from stator to rotor is equal to $2\,\pi r\left(\frac{T}{r}\right)n_s$, where n_s is the synchronous speed (of stator flux) in rev. per sec.

Thus, the rotor receives $(2\,\pi T n_r)$ foot-pounds per ~~minute~~ from the driving motor which turns it against the flux, and it also receives $(2\pi Tn_s)$ foot-pounds per minute from the stator by way of the rotating flux. The total mechanical power received by the rotor and transformed into electrical power is therefore equal to $(2\pi Tn_r + 2\pi Tn_s)$ foot-pounds per minute; and if we neglect the copper losses in the rotor, this is also the amount of electrical power output from the rotor to the higher-frequency circuit. Therefore, if we neglect all losses in the frequency changer, we have the following relations:

(*a*) **Electrical power taken in by stator** from low-frequency line equals

$$\frac{2\pi Tn_s}{2\pi Tn_r + 2\pi Tn_s} \quad \text{or} \quad \frac{n_s}{n_r + n_s}$$

times the total electrical power generated in the rotor.

(*b*) **Mechanical power taken in by rotor** from synchronous motor connected to low-frequency supply line equals

$$\frac{2\pi Tn_r}{2\pi Tn_r + 2\pi Tn_s} \quad \text{or} \quad \frac{n_r}{n_r + n_s}$$

times the total electrical power generated in the rotor.

If we take account of the losses in the frequency changer, the electrical power taken by the stator will be larger than (*a*) by the amount of the iron losses and copper losses in the stator, while the electrical power output from the rotor will be less than the mechanical power input to rotor by the amount of the iron, friction and copper losses in the rotor.

Prob. 70–7. If the rotor and stator of a frequency changer are both wound for 8 poles, how must the rotor be driven in order to transform from 25 cycles to 60 cycles per second?

Prob. 71–7. Calculate what relation must exist between the number of poles for which the frequency changer is wound, and the number of poles on a synchronous motor intended to drive the frequency changer, in order to change from 25 to 60 cycles per second.

Prob. 72–7. A frequency changer of the induction type is used to obtain power at 60 cycles per second from a 50-cycle system. What per cent of the electrical power output demanded at 60 cycles must be put into the frequency changer as mechanical power, and what per cent of it must be supplied to the stator as electrical power from the 50-cycle line? Neglect all losses in the machine. Choose any convenient number of poles if necessary.

Prob. 73–7. Calculate the driving speeds (rev. per min.) and the number of poles required on a synchronous driving motor supplied from the 50-cycle line, for various numbers of poles on the frequency changer of Prob. 72.

108. Circle Diagram for Induction Motor. Having placed various loads upon any given polyphase induction motor, and having measured the value (I_1) of current input to the stator and of power-factor ($\cos\theta_1$) at each load, let us by means of the power-factor resolve the stator current into two components, namely the power component ($I_1 \cos\theta_1$) and the reactive component ($I_1 \sin\theta_1$). From these data we may now lay out a vector representing each measured value of total primary current I_1, all with respect to a vector *OV* (Fig. 261) chosen to represent the e.m.f. of the line.* Thus, in Fig. 261, vector *OE* represents the exciting current taken by stator from line at zero load of motor, as *Om* represents the magnetizing component (in quadrature with line voltage), *Oc* represents the core-loss component (in phase with line voltage) and $\left(\frac{Oc}{OE}\right)$ represents the zero-load power-factor. Similarly, at some other load corresponding to *Od* ($=I_1$) amperes input to the stator, the power component of I_1 is *Op* amperes and the reactive component is *Or* amperes, while the power-factor is $\left(\frac{Op}{Od}\right)$.

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* When the diagram refers to a polyphase motor, *OV* is the voltage between line wires, and the length of each current vector represents "equivalent single-phase amperes" (see First Course, page 204), i.e., in a three-phase system, $\sqrt{3}$ times current per line wire, and in a two-phase system, twice the current per wire.

When we have plotted a sufficient number of such current-vectors, we are enabled to discover that a curve drawn through the ends of the vectors approximates quite closely to the arc of a circle, the center of which lies on the line *cE* drawn through the end of the exciting-current vector and perpendicular to the line-voltage vector. Having located this circle or arc from observations of current and power-factor at a limited number of different loads, we may use it to predict the value and power-factor of stator current corresponding to any number of other loads. This circular "locus" or path of the ends of the current vectors is not

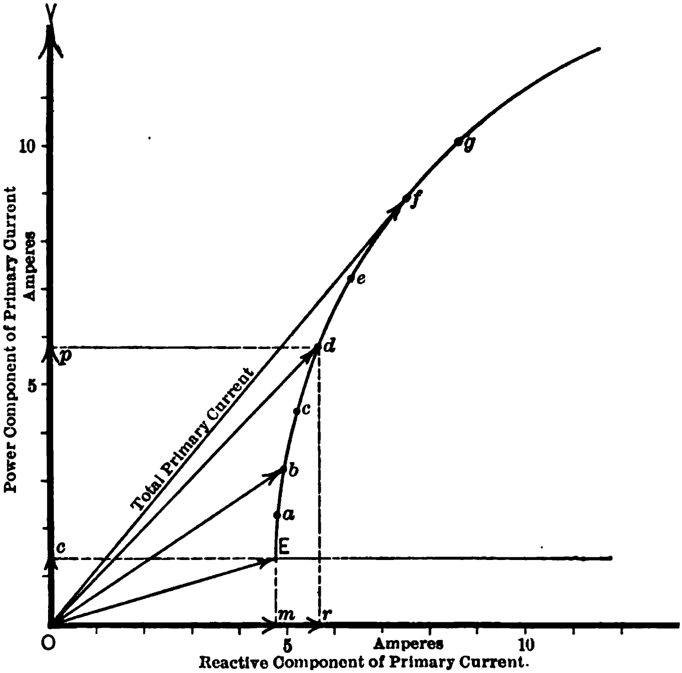

FIG. 261. The vectors represent the primary currents of an induction motor at various loads. A curve drawn through the ends of the vectors proves to be very nearly the arc of a circle.

peculiar to the induction motor but pertains also to the transformer, and in fact to any circuit having constant inductive reactance but variable resistance, upon which constant (harmonic) voltage is impressed.

As the circle of Fig. 261 passes through the ends of all vectors representing the current taken by the motor when turning against various loads, with constant voltage applied, so also it passes through the end of the vector which represents the current taken by the motor when the load becomes great enough to bring it to a standstill (with same voltage applied). Thus, in Fig. 262, the semicircle *EFPTLZ* passes not only through the end of the vector *OE* representing the exciting current taken by the motor at zero load (running freely) but also through the end of the vector *OL* representing the current which the motor takes when its rotor is held stationary, both of these currents being measured while full rated voltage is applied to the stator and both vectors being drawn to the same scale. This fact, together with the knowledge that the center (*O*) of the circle lies on a line *EZ* drawn through *E* perpendicular to the vector (*OV*) representing line voltage, gives us a very simple method for drawing the circle, and thereby of **determining the complete performance of the motor, from only two simple measurements.**

To obtain data for constructing the circle diagram, we connect the motor to a power supply suited to the rating of the motor (as to number of phases, frequency and voltage). Proper instruments are included in the connections, for measuring pressure, current and power (watts). We take one set of readings (V, I_0, W_0) with motor running unloaded, and another set of readings (V, I_L, W_L) with rotor locked and stationary, the pressure V being full rated voltage of motor in both cases. By means of the power-factor $\left(\frac{W_0}{VI_0}\right)$ at zero load we resolve the exciting current I_0 into its two components $\frac{W_0}{V}$, or $I_0 \times \frac{W_0}{VI_0}$, in phase with OV; and

$\sqrt{I_0^2 - \left(\frac{W_0}{V}\right)^2}$ in quadrature with OV. From these components we draw the vector OE (Fig. 262) representing the

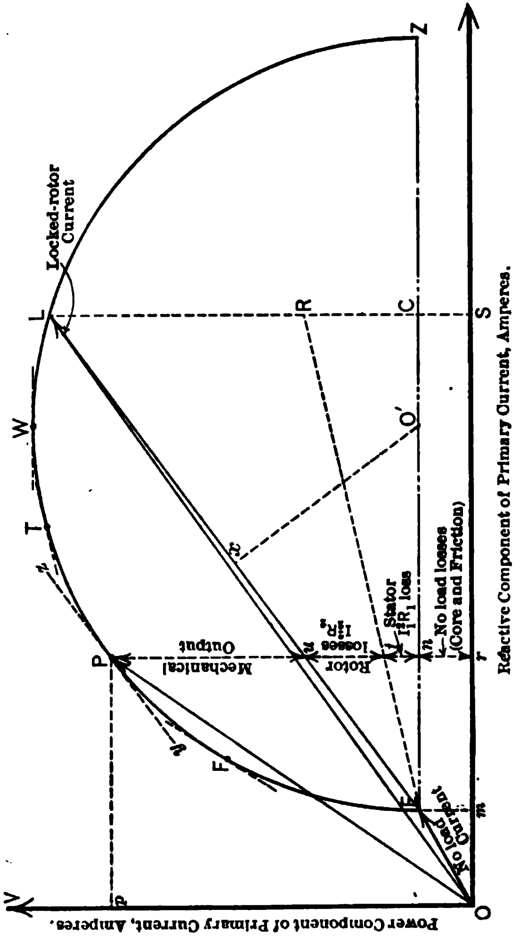
Z
Locked-rotor Current
L
R
C
S
W
O'
T
t
x
P
Output
Mechanical
$I_2^2R_2$ losses
Rotor
Stator $I_1^2R_1$ loss
No load losses (Core and Friction)
n
r
F
f
E
m
No load Current
V
p
O
Power Component of Primary Current, Amperes.
Reactive Component of Primary Current, Amperes.

FIG. 262. Circle diagram for induction motor. The vector OV represents the voltage between line wires. All current vectors are in terms of equivalent single-phase currents.

exciting current. Similarly, from the locked-rotor readings, we draw the vector OL, representing I_L as to both value and phase relation to OV. Through E we draw a line ECZ

perpendicular to OV. Then we draw a straight line from E to L, and perpendicularly from its middle a line xO' intersecting ECZ at O'. This point O' is the center of the semicircle, which we draw as $EFPTLZ$.

As the motor takes an excessive current which would soon damage the windings when full voltage is applied with locked rotor, it is usual to reduce the voltage while taking the locked-rotor readings, to such value that the motor takes say $\frac{5}{4}$ of rated-load current. This requires, let us say, $\frac{1}{4}$ of rated voltage. Now, with locked rotor the motor acts like a (very leaky) transformer whose secondary is short-circuited, and its resistance, reactance and impedance are constant quantities. Consequently, if full rated voltage had been applied, we reason that the value of I_L would have been $\frac{4}{1}$ times the observed value, or ($\frac{4}{1} \times \frac{5}{4} = 5$) times the rated-load current, and the power-factor of this current would have been equal to the value determined from the readings at lower voltage. That is, we locate the position of vector OL from the reduced-voltage readings, and then extend it straight outward to a length equal to

$$\left(\frac{\text{rated voltage of motor}}{V_L} \times I_L\right),$$

where V_L and I_L are respectively the voltage and the current measured with locked rotor.*

So far the "circle-diagram" gives us only the amount of effective power (watts), of reactive volt-amperes, and of power-factor corresponding to any given value of amperes input. But by drawing a few additional lines we may obtain much more than this. To do this, we must measure carefully the resistance of the primary (stator); this should be measured at or near the normal operating temperature, or the measured value should be corrected to operating temperature by calculation.*

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* On account of the iron in the magnetic circuit, the locked-rotor impedance of the motor is not constant, neither is the ratio of reactance to resistance, or the power-factor, constant for all values of locked-rotor current. It is better, therefore, to measure the locked-rotor current for a number of voltages, draw a curve, extend it fairly, and read from the extended part of the curve the probable value of locked-rotor current corresponding to full rated voltage.

We now compute the total copper loss (I_1R_1) in the primary or stator of the motor corresponding to some particular value of current input. It may be shown (see Prob. 74–7) that the total copper loss in any three-phase winding is equal to ($\frac{3}{2} I^2R$), where I is the current per line wire and R is the (average) resistance between any two line wires, regardless of whether the winding is connected in Y or in Δ. It is perhaps most convenient to base this computation upon the locked-rotor current OL (Fig. 262) which was obtained directly by or from measurement. Dividing this computed value of stator copper loss (at full rated voltage, with locked rotor) by the rated voltage V, we obtain that part of the power component of locked-rotor current which supplies stator copper losses. To the same scale employed for all currents in the circle diagram, we lay off this component as CR in Fig. 262. Then we draw a straight line through E and R.

As it now stands, Fig. 262 enables us (without further testing and without loading the motor or demanding power or consuming energy) to determine all operating characteristics of practical importance. Thus, mE or nr or SC is the working or power component of exciting current in amperes; if we multiply it by rated voltage we obtain the no-load losses (practically, total fixed losses, or stator core losses plus friction losses) in watts. Multiplying CR by rated volts gives us stator copper loss with locked rotor. Multiplying RL by rated volts gives us rotor copper loss with locked

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* If the resistance of a conductor is R_1 ohms when measured at temperature t_1 (degrees Centigrade), then the resistance of the same conductor at temperature t_2 (deg. Cent.) will be

$$R_2 = R_1 \times \left(\frac{1 + 0.00427\,t_2}{1 + 0.00427\,t_1}\right) \text{ ohms.}$$

rotor. The total watts input with locked rotor is equal to $(SC + CR + RL)$ times rated volts, because there can be no output when the motor is standing still regardless of the amount of torque produced.

Now let us choose any other point, as P, of the circle diagram (Fig. 262). A perpendicular dropped from P to OS makes intersections at u, t, n and r with various construction lines, as shown. All distances on Pr represent portions of the working component of current input which, when measured on the chosen scale of amperes and multiplied by the rated volts, give corresponding parts of the total wattage input to the motor. Thus:

Pr (amperes) × rated volts (OV) = total watts input to motor;
rn (amperes) × rated volts = no-load losses or constant losses;
nt (amperes) × rated volts = watts lost in stator copper;
tu (amperes) × rated volts = watts lost in rotor;
ru (amperes) × rated volts = total watts lost in motor;
uP (amperes) × rated volts = total watts input − total watts lost;
= mechanical output of power expressed in watts.

Also:

$$\text{Efficiency of motor} = \frac{\text{watts output}}{\text{watts input}} = \frac{uP \times OV}{Pr \times OV} = \frac{uP}{Pr}.$$

$$\text{Power-factor of motor} = \frac{Pr}{OP} = \frac{Op}{OP}$$

$$\text{Slip} = \frac{\text{secondary copper loss}}{\text{secondary input}} = \frac{tu \times OV}{(Pu \times OV) + (tu \times OV)} = \frac{tu}{Pt}.$$

$$\text{Per cent slip} = \frac{tu}{Pt} \times 100 \text{ per cent.}$$

$$\text{Torque} = \frac{\text{mechanical power output}}{\text{rotor speed}}$$
$$= \frac{\text{power transferred from stator}}{\text{synchronous speed.}}.$$

From the definition given in Art. 101 for "synchronous watts," we can see that the torque of the induction motor, expressed in synchronous watts, is equal to the watts output from primary (stator) or input to secondary (rotor), and is equal to Pt (amperes) times OV (line volts). That is:

$$Pt \text{ (amperes)} \times OV \text{ (volts)} =$$
$$= \text{torque of motor, in synchronous watts.}$$

$$\text{Torque (in pound-feet)} = \frac{\text{synchronous watts}}{\text{synchronous speed (r.p.m.)} \times 0.142}$$
$$= \frac{7.04 \times Pt \text{ (amperes)} \times OV \text{ (volts)}}{\text{synchronous speed (r.p.m.)}}.$$

Thus, we may select any number of points like P on the circle of Fig. 262, and determine from each point a set of simultaneous values of amperes and watts input, power-factor, watts or horse-power output, slip and speed, torque and efficiency. From these derived data we may draw characteristic curves in any form that may be desired. Furthermore, it is quite convenient to determine directly from Fig. 262 the maximum values of power-factor, power input, power output or torque, without recourse to the characteristic curves. Thus, if we draw a line through O and tangent to the semicircle, the point of tangency F indicates the load at which the **power-factor** of the motor **reaches its highest value.** Similarly, the **maximum power output** is obtained at P, where a line (yz) parallel to the output line (EL) is also tangent to the semicircle. The **maximum torque** is obtained at T, the point where a line drawn parallel to ER becomes also tangent to the semicircle. The **maximum power input** is obtained at W, the point where a line drawn parallel to OS or EO' becomes also tangent to the semicircle.

Prob. 74–7. Prove that the total copper loss in a three-phase winding is $\frac{3}{2} I^2R$, where I is the current per line wire and R is the resistance between line wires, regardless of whether the phases are connected in Δ or in Y.

Prob. 75–7. The following measurements were made on a 50-horse power, three-phase, 440-volt, 25-cycle, 6-pole induction motor:

(*a*) Running free at no-load: Volts between terminals, 440; amperes per terminal, 16; total kilowatts input, 1.14.

(*b*) With locked rotor (estimated from measurements at reduced voltage): Volts, 440; amperes per terminal, 436; total kilowatts input, 173.

(*c*) Resistance between any two terminals of stator, at 60° C., is 0.1118 ohms.

Draw the circle diagram (like Fig. 262) for this motor, using equivalent single-phase amperes for all current vectors, and as large a scale as possible.

Prob. 76–7. From the diagram of Prob. 75, calculate the maximum values of:

(*a*) Per cent power-factor.
(*b*) Horse-power output.
(*c*) Pound-feet torque.
(*d*) Watts input.

Prob. 77–7. From the diagram of Prob. 75, make measurements and calculations from which to draw a set of characteristic curves for this induction motor, having as abscissa the torque (pound-feet) and as ordinates the following:

(*a*) Amperes per line wire.
(*b*) Kilowatts input.
(*c*) Horse-power output.
(*d*) Per cent slip.
(*e*) Revolutions per minute, rotor speed.
(*f*) Per cent power-factor.
(*g*) Per cent efficiency.

Prob. 78–7. From the diagram of Prob. 75, find the point at which the sum of variable losses is equal to the sum of constant losses, and determine therefrom:

(*a*) The horse-power output at which the efficiency is maximum.
(*b*) The torque (in pound-feet) corresponding.
(*c*) The percentage value of this maximum efficiency.

Compare these results with corresponding values from curves of Prob. 77.

109. Single-phase Induction Motor. After a polyphase induction motor has been started and run up to full speed, it will continue to run even after we have disconnected all phases but one (for instance, after we have opened up one of three line wires supplying a three-phase motor). Under these conditions the motor usually makes a loud humming noise, operates at considerably lower power-factor and efficiency than when running polyphase, and has poorer speed regulation and less pull-out torque. The motor exerts no starting torque when supplied with single-phase power only, but if started turning in either direction with sufficient impulse from some auxiliary device (or even by hand, in case of a small motor) it will speed up to synchronism if there is no load upon the motor.

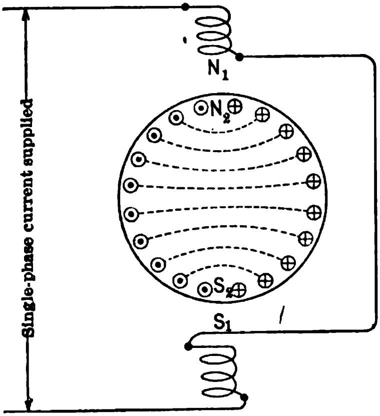

FIG. 263. Conventional diagram of a single-phase induction motor. The poles N_2 and S_2 on the stationary rotor are produced by the rotor currents induced therein by the transformer action of the flux due to the stator poles N_1 and S_1.

It is instructive and useful to inquire briefly into the causes for these differences between the polyphase and the single-phase induction motor. Fig. 263 is a conventional diagram of a two-pole single-phase induction motor. N_1 and S_1 represent the poles of the stator, the flux from N_1 across to S_1 varying harmonically with time. The small circles around the rotor (large circle) represent sections of the rotor conductors (bars, in a squirrel-cage rotor). As indicated by the dotted lines, corresponding rotor conductors, connected through the end-rings, may be considered to form coils. The alternating stator flux from N_1 to S_1, linking with these coils as indicated in Fig. 263, induces in each coil or inductor an e.m.f. as in the secondary of a transformer. These e.m.f.'s are such as to cause rotor currents which produce poles on the rotor at N_2S_2, in direct line with the stator poles N_1S_1. There can be no torque between N_2S_2 and N_1S_1, regardless of how strong may be the stator flux or the rotor currents, because of their disadvantageous relative positions.

Now, suppose we start turning the rotor in say a clockwise direction, as indicated in Fig. 264. An alternating e.m.f. will

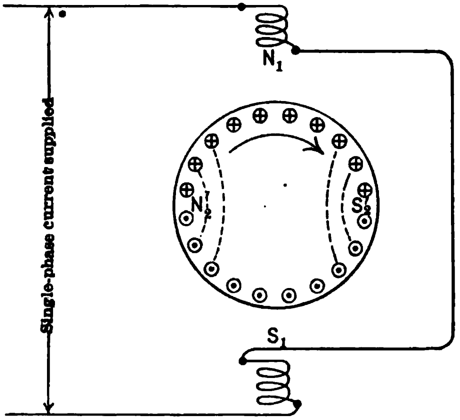

FIG. 264. Diagram of a single-phase induction motor showing poles N_2' and S_2' on the moving rotor produced by the rotor currents induced therein by the movement of the rotor through the flux of the stator poles N_1 and S_1.

be produced in each conductor, which is moving through flux, the amount of this e.m.f. at any instant being directly proportional to the speed of rotation and to the amount of flux from N_1 to S_1 at that instant. According to Lenz' law, the direction of these induced e.m.f.'s and currents is such as to produce poles on the rotor, as shown at N_2' and S_2' midway between stator poles, opposing the motion which produces them. As the stator core completely surrounds the rotor, these rotor poles $N_2'S_2'$ induce corresponding opposite poles on the surface of the stator, at right angles to the main poles N_1S_1.

The "speed e.m.f." which produces the poles $N_2'S_2'$ (Fig. 264) reaches its maximum value at the same instant as the main field N_1S_1. The "speed currents" in the rotor conductors lag nearly one-quarter period behind the corresponding speed e.m.f.'s, because the rotor circuits are highly inductive. Thus the speed flux ($N_2'S_2'$), which is in phase with the speed currents in the rotor, lags one-quarter period behind the main flux (N_1S_1). As these two fields are also at right angles to each other in space, the total or resultant flux will rotate in space (see Art. 95 and Fig. 245–247).

When the rotor is turning slowly, the speed e.m.f.'s, speed currents and speed field are all relatively weak, so that the resultant field has much greater strength when it passes through the region N_1S_1 than when it passes through the region $N_2'S_2'$. It is then said to be an "elliptical field" because a polar curve representing its strength in different directions, or at different instants of time, would have approximately the form of an ellipse. When the rotor turns at synchronous speed, the speed field becomes nearly as strong as the main field (N_1S_1, sometimes called the "transformer field"); the resultant or total field is then of nearly constant strength, and is called a "circular field." As the rotor slows down the circle representing the resultant field becomes an ellipse, which becomes flatter until at standstill it is reduced to a straight line — that is, the resultant field is stationary but alternating.

The rotation of the resultant flux, due to motion of the rotor, produces torque and drags the rotor in the manner of a polyphase motor. As the rotor approaches synchronous speed, the single-phase motor behaves more nearly like a polyphase motor. At any given value of slip the rotating elliptical field will have a minimum strength which is less than the constant strength of the circular field in a corresponding polyphase motor with the same slip. Therefore we see that the torque corresponding to any given slip, and the pull-out torque, are less for the single-phase motor than for the polyphase motor. However, the operating characteristics show a general similarity to those of a polyphase motor, as may be seen from Fig. 265. From "Standard Handbook for Electrical Engineers" we quote as follows:

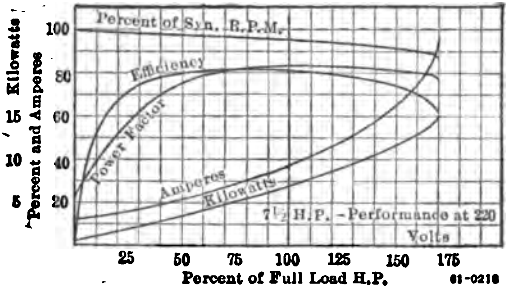

FIG. 265. Speed, efficiency and power-factor characteristics of a Wagner 7½ H.P. single-phase motor. *Wagner Electric Mfg. Co.*

> "When a three-phase motor is operated single-phase with the same voltage between lines, its maximum output will be approximately 40 per cent of the three-phase maximum output. For best conditions, such as best distribution of losses and ratio of rated to maximum output, it is customary in using a three-phase motor single-phase to reduce the rated output of the motor by 25 to 33 per cent and to increase the rated terminal voltage by about 30 per cent."

In general it may be said that a motor of given size, weight and cost can carry less load when operating single-phase; or, conversely, that single-phase motors weigh more, occupy more space, and cost more, per horse power of rated capacity, than polyphase motors. A good general understanding of this may be had by considering the nature of the total power supplied to the motor. The power in a single-phase circuit varies from instant to instant (see Art. 35, 36, 37 in "First Course"). When the power-factor is less than unity, as is usual in a motor, the power has actually negative values during some parts of each period.

This means that a single-phase motor, working against a uniform resisting torque or load, must be able to store the excess energy when the instantaneous power is greater than the average, in order to sustain the load when the instantaneous power is less than the average. When the power-factor is less than unity, its storage capacity must be still greater, because it must return energy to the supply circuit, as well as carry the load, while the instantaneous power is negative. To furnish this storage capacity more iron and other materials must go into the construction of the motor, than would be necessary if (as in any balanced polyphase circuit) the total power is unvarying, or does not change from instant to instant.

110. Method of Starting Single-Phase Induction Motors. Notwithstanding the advantage of the polyphase motor over the single-phase motor in point of cost, size, overload capacity, efficiency, power-factor and starting torque, it is frequently necessary to install small motors (fractional horse power and up to 10 h.p.) so far from the polyphase power supply that it does not pay to run three wires to the motor when two wires will suffice. Thus, a-c. induction motors for driving such loads as fans, sewing machines and washing machines must usually be supplied from lighting circuits, which are invariably single-phase. To be useful even for such purposes, the single-phase motor must be made self-starting, and a number of auxiliary devices are used for this purpose.

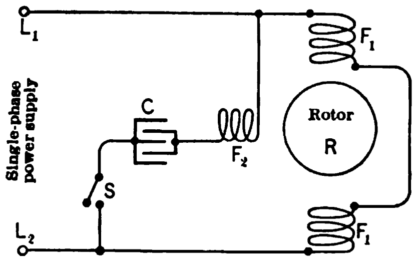

FIG. 266. Single-phase induction motor with split-phase starting coil fed through a condenser.

First: **Starting by hand.** It is shown in Art. 109 that a small single-phase motor may be given a sufficient impulse by hand in either direction to make it come up to full speed.

Second: **Split-phase starting.** As indicated in Fig. 266 and 267, auxiliary coils F_2 are wound on the stator between the main field coils F_1. The main coils are connected directly across the line, while the auxiliary coils are in series with a condenser C (Fig. 266) or a nearly pure inductance I (Fig. 267) through a switch S which is closed only while starting. The effect of C or I is to throw the current and flux in the starting coil (F_2) out of phase with the main field, thereby causing the resultant field to rotate and a torque to be produced. Fig. 268 shows a phase-splitting arrangement for starting and running a three-phase motor from a single-phase circuit; the switch S is to be opened after the motor gets up to full speed.

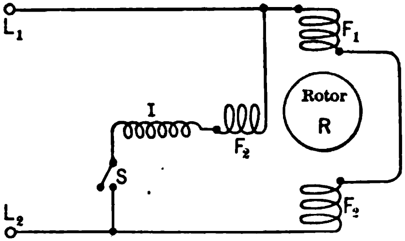

FIG. 267. Single-phase induction motor with a split-phase starting coil fed through an inductance.

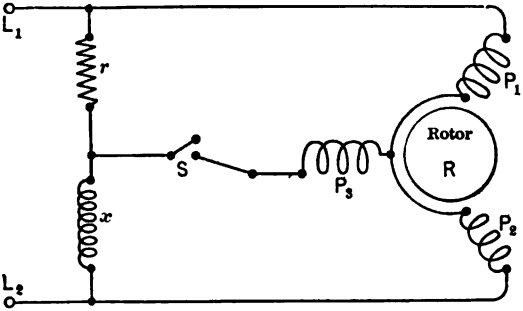

FIG. 268. Single-phase induction motor with a split-phase starting arrangement.

Third: **Shading-coil for starting.** In Fig. 269, a short-circuited coil of copper wire, or a solid ring of copper (c), has been placed around one-half of each main pole (N_1, S_1, N_1', S_1'). When the flux is changing, there is induced in

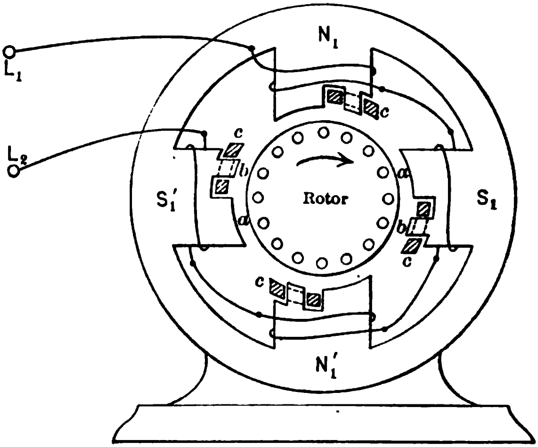

FIG. 269. Single-phase induction motor with shading-coils in the poles in order to produce a starting torque.

each of these coils (c) a current which opposes the change. Consequently, the variations of flux in the leading tip of each pole will lead the variations of flux in the trailing pole-tip. The effect of this is to cause a flux wave to sweep across each pole face, after the manner of flux in a polyphase motor, which produces a starting torque on the rotor.

Fourth: **Starting as Repulsion Motor.** By addition of commutator and brushes to the rotor, the induction motor may produce starting torque by "repulsion motor" action, to be explained in the following article. When the motor has reached synchronous speed, the centrifugal force upon a set of weights linked to and rotating with the shaft brings a short-circuiting ring into contact with all the commutator bars, making the armature practically equivalent to a squirrel-cage rotor, and usually at the same time lifts the brushes away from the commutator.

Prob. 79–7. Show conclusively that the total power in a balanced polyphase circuit does not vary from instant to instant, but has a steady value equal to $\sqrt{3}\,EI \cos\theta$ for a three-phase circuit or $2\,EI \cos\theta$ for a two-phase circuit, where E and I refer to the volts between line wires and the amperes in each line wire, respectively, and $\cos\theta$ is the power-factor of each phase of the total connected load.

Prob. 80–7. Two coils, AA' and BB', as in Fig. 245, are in such relative position that individually they produce fluxes at right angles to each other. The component fluxes are out of phase by $\frac{1}{4}$ period. The maximum value of flux due to coil BB' is only one-half as great as the maximum flux due to AA'. Draw vectors as in Fig. 247 to represent the resultant or total flux at equal intervals ($\frac{1}{24}$ period) of time, and draw a curve through the ends of these vectors.

Prob. 81–7. Solve Prob. 80, but on the basis that the component fluxes are out of phase by $\frac{1}{6}$ period.

Prob. 82–7. The coils AA', BB' and CC' of Fig. 248 are connected in Y. A terminal of AA' and a terminal of BB' are connected to a single-phase line. The remaining terminal (belonging to CC') is connected to the same line through a phase-splitting device like Fig. 267, which causes the current in the coil CC' to be $\frac{1}{8}$ period out of phase with the current in BB', and half as large in value. Assume fluxes to be directly proportional to currents. Draw vectors and curve as specified in Prob. 80.

Prob. 83–7. Repeat solution of Prob. 82, on the basis that flux due to coil CC' is only one-quarter as large as the flux due to AA' and BB' together.

Prob. 84–7. Show that when we start the rotor of Fig. 264 turning in either direction the magnetic field due to the speed e.m.f.'s and speed currents has such phase relation to the main field that the resultant flux rotates in such direction as to maintain the rotor movement which produced it. What would be the result if it were opposed to the initial movement of the rotor?

111. The Repulsion Motor. In the "repulsion motor" we have a rotor with a winding quite similar to that employed on the armature of a direct-current machine. At uniform intervals along this winding, taps are connected to bars in a commutator. The brushes which bear upon this commutator are short-circuited together. By shifting these brushes into various positions, we may cause the motor to turn in either direction, or to stand still, when the stator windings are connected to a source of single-phase power. The operating characteristics of this motor are similar to those of a series d-c. motor. At zero load, the speed goes indefinitely high, and as the load increases the speed decreases in approximately inverse proportion to the torque. The starting torque is high.

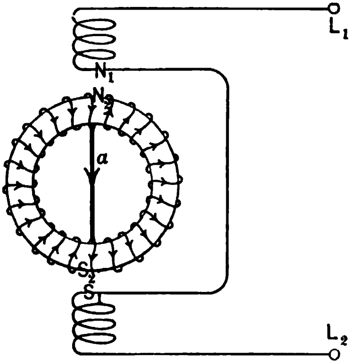

FIG. 270. The currents produced in the short-circuited rotor windings, as marked, cannot produce a torque with the stator field on account of their relative positions.

To understand the operation of the repulsion motor, first consider Fig. 270. The single-phase stator winding connected to line wires L_1L_2 produces two poles, let us say, at N_1 and S_1. Although the rotor is actually drum-wound, a ring winding is shown for simplicity in tracing circuits. A short-circuit (a) is connected between two definite coils which are in line with the stator poles. The flux due to the stator is in fact alternating, and the polarities marked correspond only to a particular part of each cycle. The variations of flux from N_1 to S_1 induces e.m.f.'s and currents in the rotor windings short-circuited at a, and these currents produce poles on the rotor in line with the short-circuit — or at N_2S_2 in Fig. 270. For this position (a) of rotor, there can be no torque between

N_1S_1 and N_2S_2, regardless of the strength of stator flux or rotor currents, since the torques developed under each half of any pole are equal and in opposite directions.

If the rotor be turned by hand into the position (*b*) shown in Fig. 271, there will still be zero torque. In this case, the rotor is in most favorable position to produce torque by interaction between rotor currents and stator flux. But it may easily be seen that the e.m.f.'s induced in each path of the rotor winding neutralize each other, so that no rotor currents and no rotor poles can be produced.

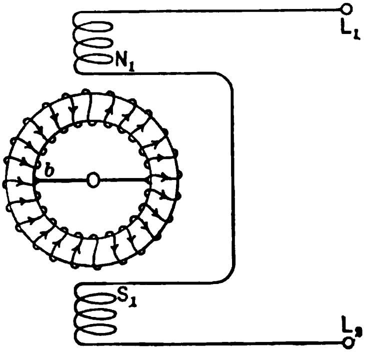

FIG. 271. When the short-circuited rotor is in this position, the e.m.f.'s induced by the alternating stator magnetism neutralize one another. Thus there are no rotor currents and no torque, although the rotor is in the most favorable position for developing torque if there were any current.

However, if the rotor be moved to a position somewhere between those shown in Fig. 270 and 271, the resultant e.m.f induced in each rotor path will be greater than zero, and the rotor currents will produce poles on the rotor somewhere between the stator poles, as shown in Fig. 272. Here, if the rotor is initially in the position (*a*), a clockwise torque will be exerted on N_2S_2, and in the position (*b*) a counter-clockwise torque will be produced on $N_2'S_2'$. The torque will not reverse as the current alternates, because both stator and rotor poles reverse simultaneously. In either case, however, this torque will be reduced to zero as soon as the rotor has moved enough to bring the short-circuit into position *cc*, midway between stator poles.

To maintain the torque steadily, it is necessary to adopt means to keep stationary the points on the rotor winding

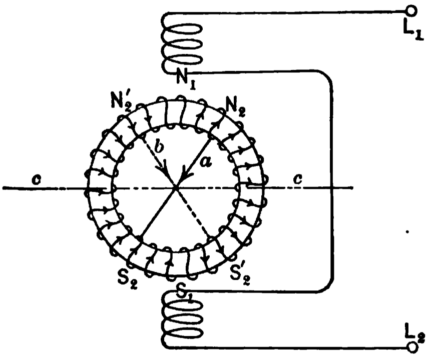

FIG. 272. When the short-circuited rotor is in the position *a*, a clockwise torque is exerted upon the currents induced in it. When the rotor is in position *b*, a counter-clockwise torque is exerted on it. In either case, the torque lasts only until the rotor has moved into the portion *cc*, when the torque becomes zero.

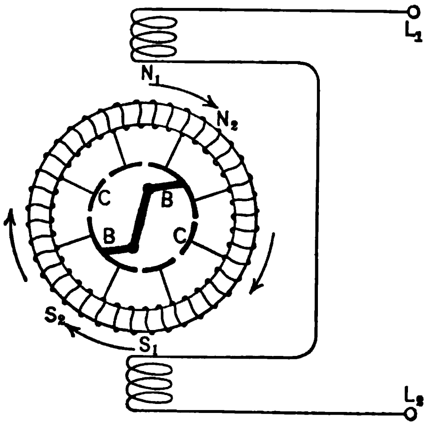

FIG. 273. The repulsion motor. It has a rotor wound like the armature of a direct-current motor, the coils being connected to the commutator *CC*, upon which bear the short-circuited brushes *BB*. If the brushes are set so as to produce the rotor poles N_2 and S_2 which are neither in line with the stator poles N_1 and S_1 nor at right angles, a continuous torque will be exerted upon the rotor.

between which the short-circuit is applied. For this purpose, the winding is connected as shown in Fig. 273 to a commutator *CC*, upon which bear the brushes *BB* with a short-circuit between them. The brushes are shifted out of line with the main stator poles N_1S_1, whereupon there are induced in the rotor, by transformer action, currents which produce rotor poles at N_2 and S_2. The stator poles exercise a **repulsive force** upon these rotor poles and produce thereby a torque. By shifting these brushes *BB*, we may have a torque in either direction, or zero torque. In reality, the internal actions become quite highly complicated by the speed e.m.f.'s and currents that arise as soon as the motor begins to turn, but this explanation has been made as simple as possible. Fig. 274 shows a conventional sketch for a simple repulsion motor.

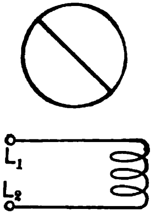

FIG. 274. A conventional sketch to represent the connections and arrangement shown in Fig. 273.

111(a). Operating Characteristics of Repulsion Motors. The straight repulsion motor, which has the characteristics of a series motor, has been applied to various purposes for which the latter would be suitable — such as driving of railroad cars and fans. Its widest application, however, has been as an auxiliary to the single-phase induction motor, to supply the starting torque which the latter inherently lacks. Fig. 275 shows a single-phase induction motor with wound rotor, the rotor winding being tapped to a (radial) commutator upon which bear brushes controlled by a centrifugal governor on the shaft. The brushes are short-circuited together and when the motor is at standstill they bear upon the commutator, being set so as to produce torque by repulsion motor action when the stator is excited. This torque accelerates the motor to nearly synchronous speed, at which point the governor connects all the commutator bars together, and at the same time throws the brushes out

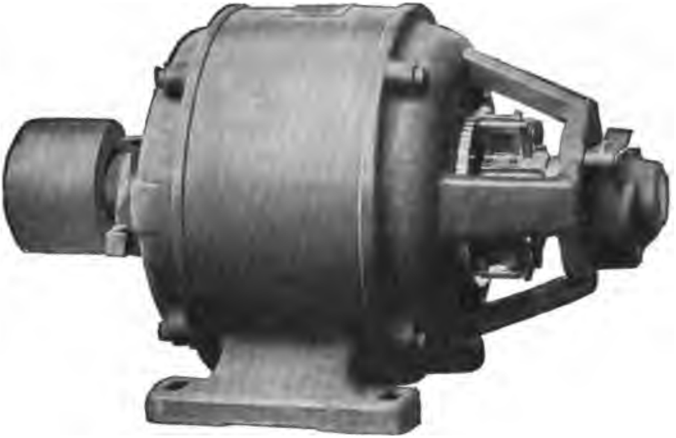

FIG. 275. A single-phase induction motor with commutator and brushes for starting as a repulsion motor. *The Westinghouse Electric and Mfg. Co.*

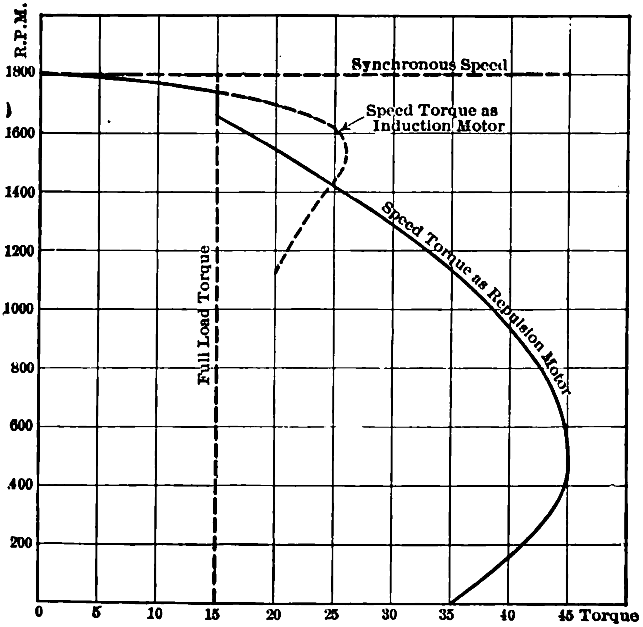

FIG. 276. Speed torque curves for single-phase induction motor of Fig. 275, with repulsion motor method of starting. *The Westinghouse Electric and Mfg. Co.*

of contact with the commutator, producing practically a squirrel-cage rotor. The motor then operates as a straight single-phase induction motor, having operating characteristics such as are shown in Fig. 265. Fig. 276 shows the speed-torque curve of such a motor both before and after the governor has operated.

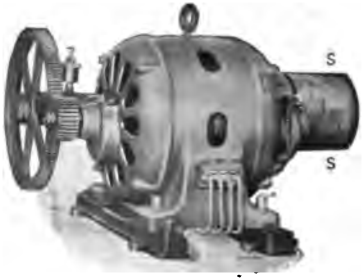

FIG. 277. Single-phase induction motor designed for starting as a repulsion motor and for operation at high power-factor. *The Wagner Electric Mfg. Co.*

An interesting type of single-phase induction motor having excellent operating characteristics is shown in Fig. 277 to 281. The rotor slots contain two distinct windings, a squirrel-cage winding of copper bars at the bottom, and a coil winding at the top connected to a commutator. These two windings are separated by "magnetic separators," as illustrated in Fig. 278. The general appearance of the motor is shown in Fig. 277, and the electrical connections (with the normal operating characteristics) in Fig. 279. As indicated in Fig. 279, there is placed in the same slots with the main field winding (M.F.) on the stator an auxiliary "compensating winding" (2), the function of which is to improve the power-factor of the motor. The brushes 5 and 6, Fig. 279, in line with the stator poles are short-circuited together, while another pair of brushes (7, 8) fixed midway between the stator poles is connected in series with the main field. The compensating winding (2) is shunted across the latter brushes (7, 8) and

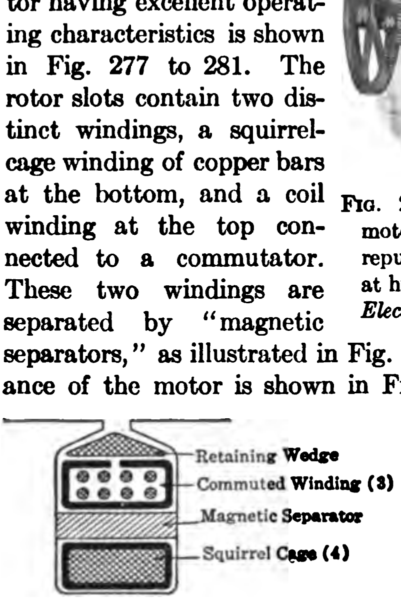

FIG. 278. The cross section of one slot in the rotor of the motor shown in Fig. 277. *The Wagner Electric Mfg. Co.*

there is included in this circuit (2) a switch (*S*, Fig. 277) operated by centrifugal force which closes the compensating field only after the motor has reached synchronous speed. Between any two brushes there is of course an alternating e.m.f.

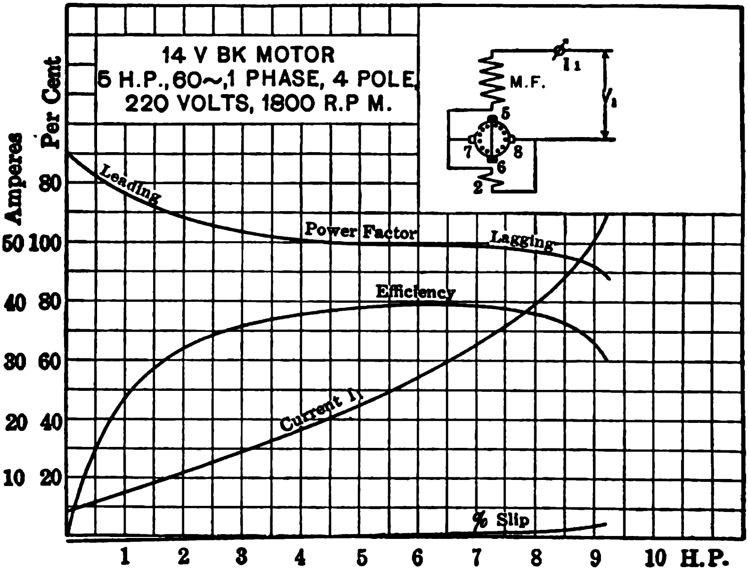

FIG. 279. Curves showing the performance under load of the Wagner single-phase induction motor of Fig. 277. *The Wagner Electric Mfg. Co.*

The normal operating curves for this motor (Fig. 279) show that at zero load the slip is negative (speed slightly above synchronism), and the power-factor is about 70 per cent **leading**. As the power output increases the speed falls and the power-factor rises, the slip being zero and the power-factor unity at about rated load. It should be explained that the power-factor may be adjusted by shifting connections on the compensating winding, and the direction of rotation may be reversed by reversing connections between the main field (M.F.) and the brushes (7, 8). The motor cannot race under any circumstances, because of the squirrel-cage winding in the bottoms of the slots; in this respect it is superior to some other motors which lack the squirrel-cage winding and which will race if some of the brushes become disconnected.

The effect of the compensating winding is illustrated by Fig. 280, which shows how the operating characteristics are

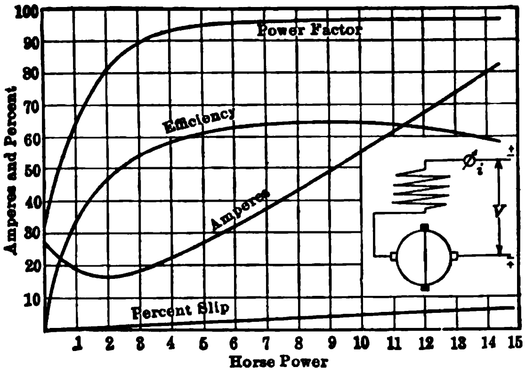

FIG. 280. Operating characteristics of the motor shown in Fig. 277, with the compensating winding disconnected. Note the change in power-factor and efficiency due to the lack of the compensating winding. *The Wagner Electric Mfg. Co.*

changed when the compensating winding is disconnected. Note that both the power-factor and the efficiency are markedly reduced, and the current increased at light loads. Fig. 281 shows the load characteristics of the same motor after all brushes have been disconnected from one another and from the stator windings, which may be done after starting the motor. Note that the current is relatively high for any given load, the power-factor is low, and the load capacity of the motor is greatly reduced.

112. The Series Motor for A-C. Circuits. When the direction of current through a direct-current series motor is reversed without altering the connections between its field and armature windings, the direction of torque and of rotation remain unchanged, because the magnetic poles on both field and armature have their polarity reversed at the same time by the reversed current which flows through both of them. Even if the reversals of current occur rapidly we should expect to find that the torque remains unidirectional; in other words, the series motor should produce a torque tending to turn it in the same direction, when either direct or alternating current is sent through it.

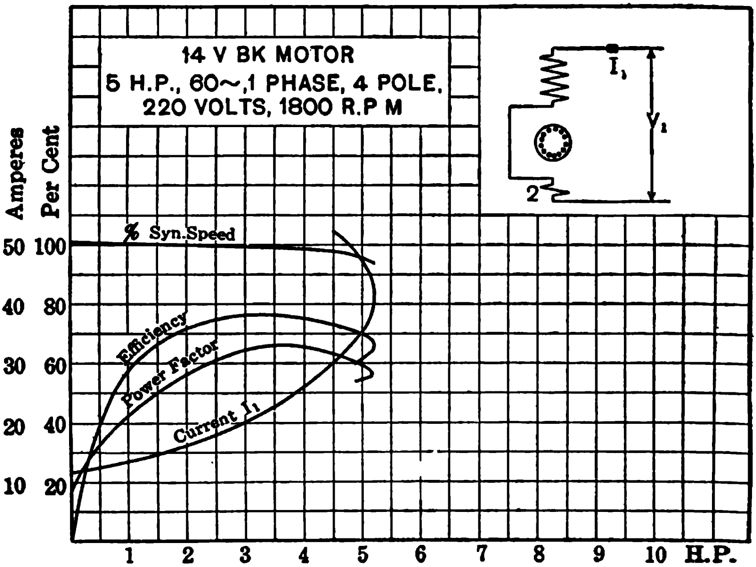

FIG. 281. Operation characteristics of the same motor of Fig. 277–280 but with all the brushes removed (after attaining full speed). Note the reduced capacity of the motor. *The Wagner Electric Mfg. Co.*

This is in fact the case; but the operation of the motor on alternating-current circuits is decidedly inferior to its performance on direct-current circuits, in the following respects:

First. The series motor designed for d-c. circuits takes alternating current at a very low power-factor, on account of the large amount of inductance in field and armature windings. This is objectionable because with the limiting current passing, the power developed will be much lower than for the same value of direct current and voltage.

Second. There would be excessive heating of the field cores of a d-c. series motor operated on an a-c. circuit, involving low efficiency and either damage to insulation or reduction of power capacity. This is due to large eddy currents induced in the solid pole-cores. The armature core is laminated even in a d-c. machine.

Third. The d-c. series motor would spark excessively at the brushes if operated on an a-c. circuit. This is due principally to alternating e.m.f.'s and currents induced in the coils that are short-circuited through each brush by the alternating flux which links with such coils in its path from one field pole to another.

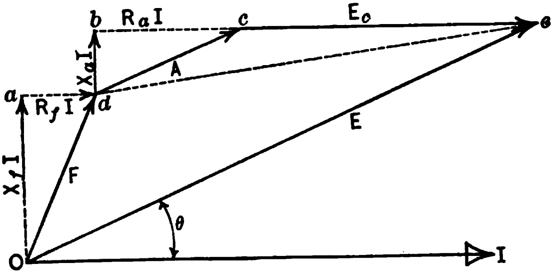

FIG. 282. Vector diagram for a series a-c. motor.

OI = current.
F = voltage drop over field winding.
A = voltage drop over armature impedance.
E_c = counter e.m.f. induced in the armature by the speed.
E = total e.m.f. applied to terminals of the motor.

Let us now examine how these difficulties are overcome in adapting the series motor to the alternating-current power supply. In discussing power-factor, first, we shall be assisted by drawing the vector diagram for a-c. series motor, as in Fig. 282. The current has the same phase throughout both field and armature, and is taken as the reference vector OI. The e.m.f. X_fI, required to overcome inductive reactance of the field winding, leads OI by one-quarter period or 90°, while the e.m.f. R_fI to overcome resistance of field winding is in phase with OI. The total e.m.f. consumed by field impedance, which would be indicated by a voltmeter across the field terminals, is marked F. Similarly, the e.m.f. required to overcome impedance of the armature winding is A. The counter e.m.f. induced by continuous rotation of the armature in the alternating flux from the field winding is marked E_c. It is in phase with the field flux, and therefore also with the current OI which produces the flux (neglecting the very slight phase difference between current and flux, due to hysteresis). A voltmeter tapped across the armature terminals would indicate the vector sum of A and E_c, or *de*. The total e.m.f. between terminals of the entire motor is E, or the vector sum of F, A and E_c. The power-factor of the motor is equal to $\cos \theta$.

Now, in order to increase the power-factor or reduce θ, we must reduce X_f and X_a. This demands first of all that the frequency of the circuit be low; series motors are rarely operated at more than 25 cycles per second, and for heavy series railway motors a frequency of 15 is advocated. Furthermore, the inductance of field windings is reduced by making the number of turns in the field coils as small as possible. This requires that the reluctance of the magnetic circuit be kept low, which is accomplished by making the air gap short, and the cross section and amount of iron in the magnetic circuit relatively large so as to reduce densities and increase permeabilities.

The most practicable way of reducing armature inductance, on the other hand, is by means of a "compensating winding" sunk into slots in the pole-faces. In Fig. 283, the conductors on the stator which are joined by full lines represent the main field winding producing poles at N_1 and S_1. The brushes *BB* are in such position on the commutator *CC* that the direction of current in the various rotor conductors is as indicated, producing poles at N_2 and S_2 on the armature, which act upon the stator poles N_1 and S_1 to produce torque and rotation. As demonstrated in Chapter IX of First Course, it is not economical to spread a single-phase winding over the entire periphery of the stator. The space not occupied, therefore, by the main field winding is filled with conductors (joined as shown by dotted lines) which form the "compensating winding."

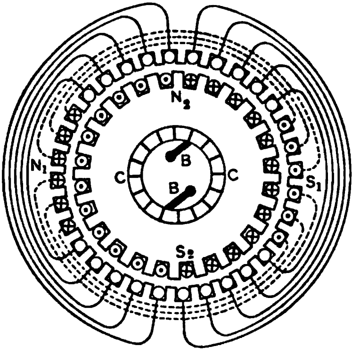

FIG. 283. The winding diagram for a single-phase series a-c. motor. The full lines show the main field winding. The dotted lines represent compensating windings used to reduce the armature inductance. *From Morecroft's "Continuous and Alternating-Current Machinery."*

This compensating winding is designed to have enough ampere-turns so that it tends to produce poles upon the surface of the stator equal and opposite to N_2 and S_2, in consequence of which the armature winding is actually unable to produce any appreciable amount of flux linking with its own turns — that is, its inductance is reduced to practically zero. However, the local flux linking the conductors in each single slot is not and cannot be compensated. The compensating winding is sometimes connected in series with the armature (conductive compensation), and sometimes it is designed to be short-circuited upon itself (inductive compensation) so that the necessary current is induced in it by transformer action of the flux from the armature poles N_2S_2. In the latter case the armature flux (from N_2 to S_2) limits itself to the small amount necessary to induce an e.m.f. just sufficient to overcome the impedance of the compensating winding.

The power-factor of the series motor is also increased by making E_c (Fig. 282) large in comparison with X_fI and X_aI. This induced counter e.m.f. is increased by using a relatively large amount of copper (turns) in the armature of the series a-c. motor. This large number of armature turns does not increase X_a, on account of the compensating windings. Of course the values of R_fI and R_aI are kept as small as practicable, by increasing the cross section of the conductors, because they represent energy losses which reduce the efficiency of the motor.

In order to reduce the heating in the a-c. series motor, and to keep the efficiency as high as possible, it is necessary to laminate the iron in the field cores as well as in the armature core.* Low flux densities must be used, so that this motor is characterized by a relatively large amount of iron, as well as of copper, for a given horse power. This makes the motor more expensive than a d-c. series motor of the same rated horse power and voltage. In general it may be said that a series motor designed for alternating current will have larger capacity and better operating characteristics when operated on direct current, whereas the d-c. series motor behaves very badly on a-c. circuits.

The most serious difficulty in operating series motors on a-c. power circuits is in regard to sparking. We have to contend with all the difficulties which arise in the d-c. machine, plus some which are peculiar to the a-c. machine. It is the latter and the special devices that must be used on account of them that we shall here consider. As shown in Fig. 284, the coils C_1C_2, which are short-circuited by the brushes B_1 and B_2 respectively while undergoing commutation, are threaded by the flux ϕ which passes from one field pole to another through the armature core. In the a-c. series motor, this stator flux is alternating, therefore each short-circuited coil (as C_1,C_2) has induced in it an alternating e.m.f. by transformer action, in addition to such e.m.f.

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* See Timbie, "Elements of Electricity," Art. 124, page 189.

as may be induced by movement of the armature. These transformer e.m.f.'s in the coils C_1C_2 produce excessive currents (limited only by the impedance of the individual coils), which heat the armature winding, the commutator and the brushes, reduce the efficiency of the motor and cause bad sparking in operation. The coils C_1C_2 might even be burned out if the motor were too long in starting, as then the currents are large and these coils have no chance to cool off.

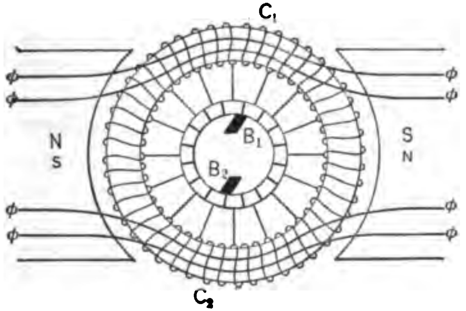

FIG. 284. Diagram of a single-phase series a-c. motor showing how the alternations of the flux ϕ, threading the short-circuited armature coils C_1C_2, induce in them heavy currents by transformer action.

The amount of e.m.f. and of current in the short-circuited coils may be reduced by designing the armature winding so as to have a larger number of coils with a smaller number of turns in each coil, and by using as little flux per pole as possible. Thus, a-c. series motors are characterized by a large number of thin commutator bars, and by being wound for rather numerous poles. Even these efforts are inadequate, however, and such motors are usually equipped with what are known as "resistance leads" or "preventive leads." Thus, in Fig. 285, the leads r_1, r_2, . . . , r_{11} between the armature coils and the commutator bars are designed to have not only a sufficient current-carrying capacity but also enough resistance to limit to a safe value the current induced in the short-circuited coils (C_1C_2) by transformer action of the main field flux. These resistances need not be large, because two of them (as r_1 and r_2) act in series to limit the current in any single short-circuited coil (as C_1), and the e.m.f. induced in each coil by transformer action is relatively small. Therefore the I^2R loss due to the flowing of the main current through them is not excessive, except when the motor is stalled or takes too much time in starting. Nevertheless, these preventive resistances are sometimes replaced by inductive reactances * in order to reduce the heating and improve efficiency. It is to be noted that the only preventive leads which carry current at any instant are those connected to the commutator bars which are under brushes.

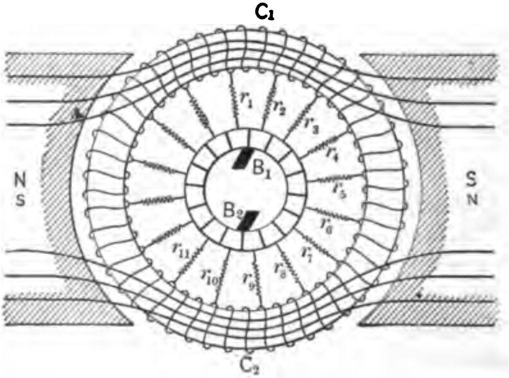

FIG. 285. A single-phase series a-c. motor showing the resistance leads (r) between coils and commutating bars, to limit the short-circuit currents in coils C_1C_2 due to the transformer action by the main field flux.

The series alternating-current motor is peculiarly useful and adaptable where we desire the speed to vary automatically with changes of load, or where we desire to control the speed of the motor at any given load. In the former respect, the series motor has practically the same characteristic on either d-c. or a-c. circuits; when the motor is required to pull a heavy torque it automatically slows down, and when its torque is small it runs at high speed. This makes it

* See Trans. A.I.E.E., Vol. XXIX, page 28.

suitable for driving railway trains, because the heaviest torque is needed while the train is starting, and then also the speed should be low. When the accelerating period is over, the torque is much reduced and then high speed is usually desired. The speed of the series a-c. motor is easily and efficiently controlled by feeding power to it through an autotransformer equipped with numerous taps, so that a number of different voltages may be applied to the motor terminals. The autotransformer may be wound, also, so as to take power directly from a high-tension line while the motor receives its input at a moderate pressure, thus economizing the expense and space for insulation in the motor. Corresponding reduction of voltage for speed control of the d-c. series motor may be had only by means of resistors, which waste large amounts of power and lower the efficiency of the whole outfit at reduced speeds to a value much lower than may be obtained by the a-c. motor with its autotransformer.

SUMMARY OF CHAPTER VII

INDUCTION MOTORS differ from other types in that the currents in the rotor or secondary windings are induced by the magnetic action of the currents in the stator or primary windings. These motors are inherently approximately constant-speed motors.

THE ROTOR may be of the " squirrel-cage " type, consisting of insulated heavy copper bars with short-circuited ends, laid in slots in a core of laminated iron; or it may be of the " wound " type, consisting of insulated windings laid in slots and brought out to slip rings mounted on the shaft.

THE STATOR is practically identical with the armature of any alternating-current generator of the stationary armature type.

A REVOLVING FLUX is set up by the alternating current in the stator windings of a polyphase motor. This revolving flux cutting the conductors of the rotor sets up currents in them, in such direction as to produce a continuous torque. If unloaded, the rotor will revolve at nearly the same speed as the stator flux. This is called the SYNCHRONOUS SPEED.

THE SYNCHRONOUS SPEED in revolutions per minute equals 60 times the frequency divided by the number of pairs of poles, as in any synchronous machine.

THE ROTOR OF A LOADED INDUCTION MOTOR does not rotate at synchronous speed but " slips," so that the revolving stator flux cuts the rotor windings and produces the necessary rotor e.m.f., current and torque to carry the load. The " slip " is stated in percentage and equals

$$\frac{\text{synchronous speed} - \text{actual rotor speed}}{\text{synchronous speed}}.$$

THE INDUCTANCE OF THE ROTOR windings causes a time lag of rotor currents behind rotor e.m.f.'s, and causes the field due to the rotor currents to occupy such a position with regard to the rotating flux that the torque is reduced thereby. It also cuts down the value of the rotor currents, especially at starting, when the e.m.f. induced in the rotor has the same frequency as the line. It would have no effect at synchronous speed as the frequency of the induced e.m.f. would be zero.

THE RESISTANCE OF THE ROTOR windings reduces the torque merely in that it limits the value of the rotor currents, as it does not produce an unfavorable effect upon the position of the rotor flux with relation to the stator poles.

THE ROTATING FIELD IN THE STATOR OF A POLYPHASE MOTOR is formed by the combined fluxes of the several phases. The dying fluxes of some phases always combine with the growing fluxes of the others so that the total flux remains the same value and sweeps around the axis of the rotor.

THE STARTING CURRENT OF A SQUIRREL-CAGE MOTOR is large, being usually from 3.5 to 7 times the full-load current if started at full voltage, because the ampere-turns of the stator must balance the large value of the ampere-turns in the rotor in which heavy currents are set up by the revolving flux. As the rotor is at rest, its conductors are cut by the stator flux at synchronous speed and a relatively high e.m.f. is induced in them.

THE POWER-FACTOR OF THE STARTING CURRENT of this motor is low. It is approximately the power-factor of the rotor current, being about 55 per cent, due to the large value of the reactance of the rotor in relation to its resistance. It depends upon the value of the slip which is maximum, and of the leakage reactance which is large on account of the air gap.

THE STARTING TORQUE IS LOW, depending upon the relative values of the reactance and resistance of the rotor and averaging about 1.5 to 2.5 times the full-load torque. This is due largely to the unfavorable position of the rotor currents and poles relative to the stator poles. The starting torque is greatest when the resistance of the rotor equals its reactance. Under these conditions, it is:

Directly proportional to the square of the e.m.f. induced per turn of the rotor circuit when at rest or the e.m.f. impressed upon the stator;

Inversely proportional to the angular velocity of the rotating field and to the resistance per circuit of the rotor.

A WOUND ROTOR CAN BE SUPPLIED with extra external starting resistance in order to make the resistance equal to the reactance. This resistance is cut out when rotor attains full-load speed, in order to increase the efficiency.

THE SPEED OF AN INDUCTION MOTOR falls as load is added to it. The increased slip causes a greater cutting of the rotating flux by the rotor conductors, consequently a greater e.m.f. is induced in the rotor, which increases the rotor currents and torque sufficiently to carry the added load. The torque may be increased to a maximum value known as the pull-out point, usually about 1.5 times the starting torque. The motor then comes rapidly to standstill, because of:

(a) The stator impedance drop, which decreases the mutual flux.

(b) The magnetic leakage, on account of which the increase in stator current does not produce a proportional increase in mutual flux. It also increases the rotor reactance, causing the rotor poles to occupy an unfavorable position.

INCREASING THE TORQUE (LOAD) ON A MOTOR causes:

(a) A falling off in speed. At given per cent slip the larger motors have higher power-factors and efficiencies.

(b) An increase in power-factor, the maximum power-factor being obtained at a considerable overload. High-speed motors have the best power-factor.

(c) An increase in efficiency to about ¾ load and only a slight change from there to 1¼ load.

THE EFFECT OF ROTOR RESISTANCE UPON THE PERFORMANCE OF AN INDUCTION MOTOR:

(a) Increasing the rotor resistance improves the power-factor and reduces the current at starting, and up to a certain point increases the starting torque.

(b) Increasing the rotor resistance reduces the speed, or increases the slip and the speed regulation corresponding to a given torque, in direct proportion to r_2. In this manner we may obtain any desired speed from synchronous speed to zero, at any given value of torque or load. Under these conditions, the current, power and power-factor of input are unchanged, but

(c) When the speed is thus reduced, the $I_2^2 r_2$ loss in the rotor is increased, and the power output and the efficiency corresponding to a given torque are reduced, in direct proportion to the increase of slip and of r_2.

(d) The pull-out point or maximum load capacity of the motor is independent of the value of secondary resistance.

SYNCHRONOUS WATTS signifies the amount of mechanical power (expressed in watts) which the machine could develop if it were to turn at synchronous speed while delivering a given torque.

Synchronous watts = 0.142 × rev. per min. × pound-feet.

AN INCREASE IN THE LENGTH OF THE AIR GAP:

(a) Greatly lowers the efficiency.

(b) Greatly lowers the power-factor for the same torque.

(c) Slightly lowers the speed.

(d) Slightly lowers the pull-out point.

EFFECT OF CHANGING THE IMPRESSED VOLTAGE:

(a) The torque varies as the square of the voltage, — for the same slip.

(b) The slip varies approximately inversely as the square of the voltage, — for the same torque.

EFFECT OF CHANGING THE FREQUENCY:

(a) The synchronous speed varies directly as the frequency, for constant voltage.

(b) The core loss and the exciting current increase as the frequency is lowered, with the same voltage.

(c) To operate a motor on other than the standard frequency, it is best to change the voltage in proportion to the square root of the frequency.

THE SPEED OF POLYPHASE INDUCTION MOTORS IS CONTROLLED COMMERCIALLY:

(a) By varying the resistance introduced into the rotor;

(b) By changing the number of the stator poles;

(c) By joining two or more motors in cascade or concatenation.

TWO INDUCTION MOTORS ARE SAID TO BE CONCATENATED when the rotors are rigidly connected by gears or other coupling, while the stator of one motor is connected to the line and its rotor is connected to the stator of the other motor. The first motor must have a wound rotor.

Such a combination rotates at a speed equal to that of a single motor having the sum or difference of the number of poles, according as direct or differential concatenation is used.

THE INDUCTION MOTOR MAY BE USED AS AN INDUCTION GENERATOR when its stator is connected to a source of alternating current and the rotor is turned above synchronous speed by a prime mover. When so used as an auxiliary in large power plants, it is usually run by a turbine driven by the exhaust steam from the reciprocating engines. The amount of power delivered is proportional to the (negative) slip. The machine ceases to generate when short-circuited. This generator effect is made use of for the " regenerative braking " of cars supplied with polyphase induction motors. As soon as the speed of a coasting car causes the motor to exceed the synchronous speed, the power returned to the line by the stator acts as a brake. Reactive power must be supplied to the stator for excitation.

A FREQUENCY CHANGER CAN BE ARRANGED by connecting the stator of an induction motor to a line of lower frequency (f_2) and the wound rotor to the line of higher frequency (f_1). The rotor is turned against the stator flux by a synchronous motor attached to the low-frequency supply mains, at a speed (n_r) equal to $\frac{f_1 - f_2}{f_2}$ times the synchronous speed (n_s) of the rotor.

The electrical power taken by the stator from the low-frequency line equals $\frac{n_s}{n_r + n_s}$ × total power generated in the rotor, plus the iron and copper losses in the stator.

The mechanical power taken by the rotor (through the synchronous motor) from the low-frequency line equals $\frac{n_r}{n_r + n_s}$ × the total electrical power generated in the rotor.

THE CIRCLE DIAGRAM FOR THE INDUCTION MOTOR. If vectors representing the currents taken by an induction motor (from zero load to locked-rotor condition) are all drawn to the same scale and from the same point on the axes of reference, the ends of these vectors will all lie in the circumference of a semicircle. The diameter of this semicircle can be determined from the no-load current and power-factor and locked-rotor current and power-factor, both being taken at rated voltage. From this semicircle, can be determined for any given value of stator current:

(a) The effective power, reactive power and power-factor.

(b) Copper loss in stator (by also measuring the stator resistance).

(c) Total fixed losses, as core and friction losses.

(d) Rotor copper losses.

(e) Mechanical output, torque and efficiency.

(f) Slip, speed and speed regulation.

SINGLE-PHASE INDUCTION MOTORS will operate if started and brought up to approximately full speed, but with smaller torque than a polyphase motor of the same general dimensions. The flux set up by the currents induced in the rotor combine with the flux set up by the current in the stator windings to form a revolving flux, when the rotor is revolving.

This resultant flux would be fairly constant in value if the rotor revolved at synchronous speed, but the slip causes it to vary in value, so that the end of a vector representing the direction and magnitude of resulting flux describes an ellipse, and not a circle as in the case of a polyphase motor. With the rotor at rest, this ellipse flattens into a straight line, so that under these conditions the field does not revolve, and thus no starting torque results.

SINGLE-PHASE INDUCTION MOTORS ARE STARTED:

(a) By hand, in the very small sizes.

(b) By supplying split-phase to starting-coils on the stator. A condenser or a coil with high inductance is put in series with these coils, so that the starting current through them sets up a flux which is out of phase with the flux in the main stator coils. These starting coils are cut out when the rotor attains full speed. A special phase-splitting device which is not built into the motor is shown in Fig. 268, for starting a three-phase motor on a single-phase circuit.

(c) By shading coils, which consist of short-circuited coils wound on the trailing tip of each pole. The e.m.f. set up by the changing flux in the pole causes a current to flow in these shading coils which sets up a flux out of phase with the flux in the remainder of the pole. This causes a resultant flux wave to sweep across the pole face, dragging the rotor with it.

(d) By repulsion motor action.

THE REPULSION MOTOR has a rotor similar to a direct-current armature. The brushes bearing on a commutator are short-circuited, and the currents induced in the armature coils set up poles on the armature surface. The brushes are so placed that the poles thus set up are slightly out of line with the stator poles, and the mutual repulsion between like poles on the stator and armature produces the torque. The general characteristics of a repulsion motor are those of a d-c. series motor. The starting torque is large and the speed indefinitely high at no load. Speed varies, inversely as the torque.

In using the repulsion motor effect for starting a single-phase induction motor, the commutator segments are automatically short-circuited together at full speed by centrifugal force, and the brushes lifted to reduce friction loss. By furnishing the stator with compensating field coils, excited from the armature by means of a second set of brushes, it is possible to produce a negative slip at no load, causing leading power-factor, and no slip at full load with the resulting unity power-factor. Additional squirrel-cage windings may be placed on the rotor to prevent racing at no load. This produces a single-phase motor of excellent characteristics but of somewhat reduced efficiency.

A SERIES MOTOR TO OPERATE SATISFACTORILY on alternating-current circuits must be designed somewhat differently from a direct-current series motor, which would operate inferiorly on an alternating-current circuit, on account of the following inherent characteristics:

(a) The large inductance of armature and field windings of the d-c. motor.

The field reactance of an a-c. series motor is reduced by operating the motor on low frequency and by using as few turns as possible in the field windings. The air gap must necessarily be made small to reduce the reluctance. The armature inductance is decreased by using compensating windings on the poles, the m.m.f. of which exactly neutralizes the armature m.m.f. and greatly reduces the flux produced by the armature currents. This practically reduces the armature inductance to zero. The power-factor of the motor is thus raised.

(b) The increased heating in the armature and field cores due to eddy currents.

This is avoided in the a-c. motor by laminating the field cores and yokes, as well as the core of the armature, and by using lower flux densities. An a-c. series motor is therefore heavier and more expensive than a d-c. series motor of the same horse power.

(c) Excessive sparking at the brushes, because the coils short-circuited by the brushes are cut by the constantly alternating flux. These short-circuit currents are limited by introducing resistance or inductive reactance into the leads connecting the commutator segments to the tapping points.

The operating characteristics of an alternating-current series motor are similar to those of a direct-current series motor. It has the advantage, however, over the direct-current machine of being easily and economically controlled as to speed, by taking its voltage from the taps of an autotransformer, instead of having the voltage controlled by series resistances, with the accompanying I^2R loss.

PROBLEMS ON CHAPTER VII

Prob. 85–7. (*a*) What percentage taps must be used on the compensator for the 7½-h.p. 1200-r.p.m. motor of Table I, to start it with a current equal to rated-load current? (*b*) What will be the starting torque under this condition, as percentage of rated-load torque?

Prob. 86–7. The 5-h.p. 1800-r.p.m. motor of Table I is started by means of a star-delta switch from a line whose pressure is equal to the rated voltage of the motor. Calculate, as percentages of the corresponding values at rated load, (*a*) starting torque; (*b*) starting current.

Prob. 87–7. The voltage of the supply line to the 40-h.p. 900-r.p.m. induction motor of Table I drops 20 per cent at the moment of starting the motor on account of the large value and low power-factor of the starting current. By what percentage are the values of starting torque and of starting current greater or less than they would have been if the line voltage had been unaffected?

Prob. 88–7. What effect would be produced upon the starting torque of a squirrel-cage induction motor if the end rings of the rotor are notched with a file or a hacksaw at frequent intervals between the points where the rotor bars connect to them?

Prob. 89–7. From the curves of Fig. 254, obtain data for calculating and drawing another curve having as abscissas the rotor circuit resistance, and as ordinates the total I^2R loss in the motor at starting as percentage of the total I^2R loss at rated load and maximum speed.

Prob. 90–7. From the results of Prob. 89 and from Fig. 254 construct a curve having as abscissas the starting torque as per cent of full-load torque, and as ordinates the I^2R loss at starting as per cent of I^2R loss at rated load. Discuss the form of this curve.

Prob. 91–7. Notice from Fig. 254 that we may develop the same starting torque (of 200 per cent rated torque, let us say) with either of two different values of resistance per rotor circuit. For each of these values calculate the total I^2R loss in rotor as per cent of I^2R at rated load, and on the basis of these figures state which adjustment of the rotor resistance is preferable.

Prob. 92–7. The motor of Fig. 254 is adjusted to produce its maximum starting torque, and the motor of Fig. 255 is adjusted to produce an equal starting torque. The two motors are rated exactly the same for full load. Calculate: (*a*) Ratio of starting current for wound-rotor motor to starting current for squirrel-cage motor; (*b*) similar ratio, between I^2R losses at moment of starting.

Prob. 93–7. To what value would the efficiency at rated-load torque of the motor in Fig. 258 be raised if the air gap were lengthened so as to have a power-factor 10 per cent lower than shown for this load by the power-factor curve, other things being equal?

Prob. 94–7. From the data of Table II for a 5-h.p. 1140-r.p.m. motor calculate the reactive volt-amperes supplied to the motor at $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, 1, $1\frac{1}{4}$ times rated load. Draw a curve setting forth these results, and discuss the significance of the curve.

Prob. 95–7. From the data of Table II determine how many kv-a. of generator capacity, and how many kw., would be required to drive: (*a*) A 10-h.p. 850-r.p.m. motor at half load; (*b*) a 5-h.p. 850-r.p.m. motor at full load. Discuss the significance of these figures, in view of the fact that both motors are delivering the same power at approximately the same speed — that is, performing identically the same service.

Prob. 96–7. Solve Prob. 95–7 with relation to: (*a*) A 2-h.p. 1140-r.p.m. motor at half load; (*b*) a 1-h.p. 1140-r.p.m. motor at full load.

Prob. 97–7. Table II gives the efficiency at various fractions of full-load horse-power output. Assuming that the slip varies in direct proportion to the torque, calculate from the data of full load in Table II the values of efficiency at $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, 1 and $1\frac{1}{4}$ times rated-load **torque** for a 20-h.p. 1140-r.p.m. motor. Is the efficiency curve appreciably different whether drawn with respect .to power or torque output as abscissas?

Prob. 98–7. For the 20-h.p. 1140-r.p.m. motor of Table II wound 3-phase 250 volts, calculate the values of amperes input (per line wire) and of torque (pound-feet) per ampere, at $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, 1 and $1\frac{1}{4}$ times rated-load power output.

Prob. 99–7. From the data of Table II, determine how many kw. and kv-a. generator capacity would be required to deliver 5-h.p. from an induction motor operating at full load: (*a*) At 1140 r.p.m.; (*b*) at 850 r.p.m. Discuss the significance of these figures.

Prob. 100–7. From the data of Table II calculate the overall efficiency of motor and mechanical transmission to a machine driven by: (*a*) A 10-h.p. 850-r.p.m. motor coupled directly to the machine and operating at full load; (*b*) a 10-h.p. 1140-r.p.m. motor delivering same power to the machine, but through gearing or belting which absorbs 10 per cent of the power of the motor. Discuss the results.

Prob. 101–7. Calculate the kw. and kv-a. of generator capacity required per horse power available at the shaft of the driven machine, for cases (*a*) and (*b*) of Prob. 100–7. Discuss the results.

Prob. 102–7. The motor of Fig. 258 is required to operate at a speed as near as practicable to 500 rev. per min. against a torque of 150 pound-feet. Determine: (*a*) At what point must the controller be set? (*b*) What will be the actual speed of the motor at the given torque? (*c*) What is the horse-power output of the motor under this condition? (*d*) What will be the speed regulation in per cent, if this load be suddenly removed?

Prob. 103–7. What should be the efficiency of the motor of Fig. 258 operating under the conditions stated in Prob. 102–7? Note that the efficiency curve of Fig. 258 corresponds to the speed-torque curve shown, in dotted line, as "inherent speed-torque characteristic."

Prob. 104–7. The efficiency curve of Fig. 258 corresponds to that amount of resistance in the rotor circuit which gives the speed-torque relation shown by the dotted line marked "speed-torque characteristic" — that is, with the rotor short-circuited. Calculate what efficiency (per cent) would be obtained when the motor produces 50, 100, 150, 200, 250 and 300 pound-feet of torque while operating on the last step (No. 8) of the controller.

Prob. 105–7. An 8-pole wound-rotor induction motor rated 25 h.p., 1150 r.p.m., 60 cycles, 240 volts (3-phase), is operating against rated-load torque at rated voltage and frequency, but with the controller set so that the rotor circuit resistances have four times their normal (inherent) value. If the efficiency at full rated load, without external resistance in rotor circuits, is 88 per cent, calculate: (*a*) Speed at rated-load torque with increased resistance; (*b*) efficiency under these conditions.

Prob. 106–7. For the frequency changer of Prob. 70–7, calculate and draw curves having as abscissas the rotor speed expressed as percentage of synchronous speed (of stator flux), and as ordinates the following: (*a*) Electrical power input to stator as per cent of total electrical power generated in rotor; (*b*) mechanical power input to rotor as per cent of total electrical power generated in rotor; (*c*) voltage generated in each rotor circuit as percentage of voltage generated at 60 cycles per second; (*d*) reactance of each rotor circuit as percentage of its reactance at 60 cycles per second. Neglect all losses in the induction generator, and assume that the current output and leakage inductance are constant. Consider the delivered frequency to vary from 0 to 60 cycles per second.

CHAPTER VIII

THE SYNCHRONOUS MOTOR

In studying the parallel operation of alternators, we noted that if the power is shut off from an alternator which is operating in parallel with other similar machines, the alternator will not stop, but will draw power from the other alternators and continue to run at synchronous speed. Such a generator has thus become a **synchronous motor,** so-called because at all loads from no load to the maximum load, it operates in synchronism with the frequency of line. Why the rotor always revolves in synchronism with the line frequency will be shown in the next articles.

113. Running Torque and Speed of a Synchronous Motor. In Fig. 286, the rotating field is assumed to be revolving counter-clockwise at synchronous speed. The four poles are excited by a direct current supplied to the coils through the rings. In the armature, the ends of the conductors composing the two phases *A* and *B* are shown, — *A* by hollow circles and *B* by solid circles. The current has a maximum value in phase *A* at the instant shown and therefore is zero in phase *B*, which has a phase difference of 90° with *A*. Assume the current in phase *A* to be **in** at the top and the bottom and **out** at the sides. Considering pole I, which is a **North** pole, we see that the action of the flux on the conductors at the top (which are carrying a current **in**) is such as to tend to push the conductors to the right.*

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* Left-hand rule for motors. Extend the thumb, forefinger and middle finger of the left hand at right angles to one another. If the forefinger points in the direction of the field flux and the middle finger in direction of the current in the wire, the thumb will indicate the direction in which the conductor is urged.

But the conductors of phase *A* are imbedded in a stationary frame and cannot move to the right. Therefore the north pole I, being free to move, is urged to the left in a counter-clockwise direction. Similarly the other poles, being directly opposite the windings of phase *A* are being urged in a counter-clockwise direction. If each field pole is to rotate in synchronism with the alternations of the line currents, then

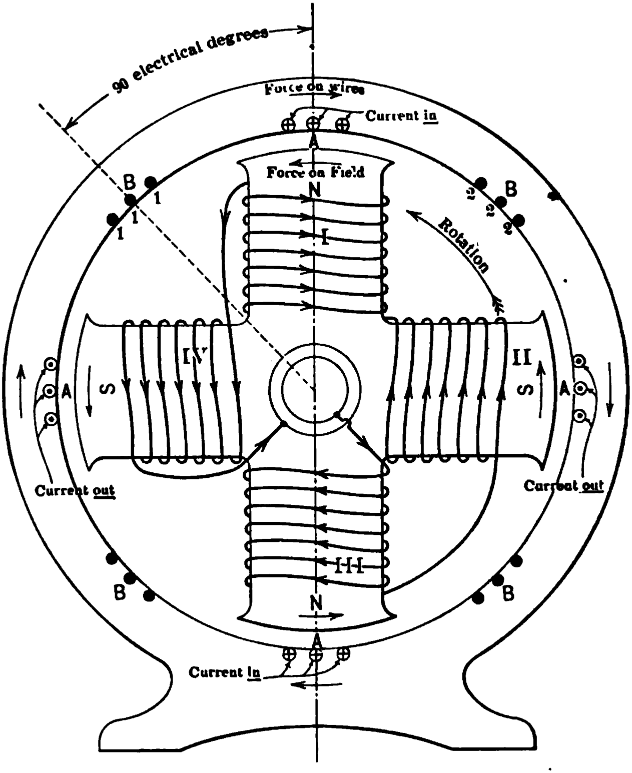

FIG. 286. Diagram of a two-phase four-pole synchronous motor of the revolving-field type. The reaction between the current in the stator winding and the field poles causes a torque which urges the field poles to rotate in a counter-clockwise direction.

in one-quarter of a cycle later (90 electrical degrees) the poles must all move from under the conductors of phase A to the corresponding position under the conductors of phase *B*. Thus pole I would move to the left to a position under the three wires of phase *B* marked 1, pole II to a position under the three wires of phase *B* marked 2, etc. But by this time the conductors 1, 1, 1, of phase *B* will be carrying a maximum current of **in**, and conductors 2, 2, 2, a maximum current of **out**, and the current in phase *A* will be zero. Thus the poles will receive a push in a counter-clockwise direction from phase *B*. Similarly, after another period of 90 electrical degrees, pole I will be opposite the left side conductors of phase *A*, but the current now will have reached a maximum value in the opposite direction and will be flowing **in**, as it is 180 degrees since pole I was under the top coils. Thus again the poles receive an impetus in the counter-clockwise direction.

Each pole is moved in this manner $\frac{1}{4}$ of a revolution in $\frac{1}{2}$ of a cycle, or $\frac{1}{2}$ of a revolution in 1 cycle. The rotor thus makes 1 revolution every two cycles. If the frequency is 60 cycles a second, the rotor makes $\frac{60}{2}$ or 30 revolutions a second or 1800 revolutions a minute.

The speed of any synchronous motor can be computed as above or from the equation

$$S = \frac{60f}{p},$$

where

S = speed in r.p.m.
f = frequency in cycles per sec.
p = number of pairs of poles.

Thus the speed of our 4-pole motor on a 60-cycle line would be found as follows:

$$S = \frac{60 \times 60}{2}$$
$$= 1800 \text{ r.p.m.}$$

Prob. 1–8. At what speed will a 12-pole 3-phase 25-cycle synchronous motor operate?

Prob. 2–8. How many poles must a synchronous motor have in order to operate at 514 r.p.m. on a 60-cycle line?

Prob. 3–8. At what speed will the motor of Prob. 2 operate on a 25-cycle line?

114. Counter E.M.F., Armature Current, and Synchronous Position. When in motion, the rotor of every motor always sets up a counter e.m.f. in the armature windings. Just as the e.m.f. impressed on the armature windings has approximately a sine wave-form, so the counter e.m.f. induced by the motion of the field poles has approximately a sine wave-form. The pole-face is so shaped and the armature conductors are so distributed that this form of wave is produced. Therefore the current which flows in the armature winding is due to the resultant of the impressed voltage and the counter e.m.f., both of sine wave-form. If the phase of the counter e.m.f. were exactly opposite that of the impressed e.m.f., the resultant e.m.f. would always be exactly their arithmetical difference, as in a direct-current motor.

But the counter e.m.f. can never be exactly opposite the impressed voltage in phase. That is, the armature conductors are not being cut at the greatest rate by the rotating field at exactly the instant at which the impressed e.m.f. reaches its maximum, as it would appear from Fig. 286.

We have said that at the instant shown in Fig. 286, the impressed e.m.f. in the conductors at the top is a maximum. As Fig. 286 is constructed, the induced counter e.m.f. must also be a maximum at this instant. This exact condition of affairs can never exist in a real machine. Such a condition would result in a power intake of almost zero and the machine would have to slow down. This can be seen from the following example in which we will compute the power intake on the assumption that the counter e.m.f. reaches its maximum at the same instant that the impressed e.m.f. is a maximum in the opposite direction.

Example 1. Let us assume that the capacity of the motor in Fig. 286 is 1500 kv-a., that the impressed voltage on phase *A* is 6500 volts, and that the counter e.m.f. is 6000 volts. Since we have assumed that the e.m.f.'s both reach their maximum values at the same instant the vector E (in Fig. 287) representing the 6000 volts counter e.m.f. will be exactly opposite in direction to E_x which represents the 6500 impressed volts. The resulting voltage, which tends to send a current through the armature will be 500 volts, represented by E_1, in phase with the impressed voltage E_x.

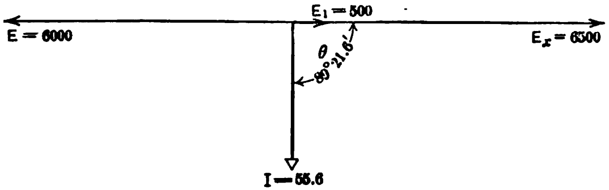

FIG. 287. Vector diagram of the current per phase, counter e.m.f. and impressed e.m.f. of the synchronous motor of Fig. 286. The counter e.m.f. E is exactly opposite in phase to the impressed e.m.f. E_x. The resultant e.m.f. E_1 sends the armature current I through the armature windings.

The amount of current which E_1 will send through the armature depends upon the synchronous impedance of the armature. The impedance of the armature of a synchronous motor is composed practically entirely of reactance, there being usually over fifty times as much reactance as resistance. Fair values for a 1500 kv-a. machine for this voltage would be 9 ohms reactance and 0.100 ohm resistance per phase.

$$\text{The impedance} = \sqrt{9^2 + 0.100^2}$$
$$= 9 \text{ ohms.}$$
$$\text{The armature current per phase} = \frac{500}{9}$$
$$= 55.6 \text{ amp.}$$

This current of 55.6 amp. would lag behind the 500 volts by an angle the tangent of which would equal $\frac{9}{0.100}$ or. 90. This angle is 89° 21.6′. The vector I in Fig. 287 thus represents this current of 55.6 amperes lagging 89° 21.6′ behind E_1.

The power received by the motor can now be found.

$$\begin{aligned} P &= E_x \cos\theta \times I_a \\ &= 6500 \times 55.6 \cos 89^\circ 21.6' \\ &= 6500 \times 55.6 \times 0.0111 \\ &= 4010 \text{ watts.} \end{aligned}$$

The two phases would be supplied with only 2 × 4.01, or 8.02 kw. Even running light, a machine of this size must take between 30 and 40 kilowatts to supply the various losses. Thus, if at the instant at which this motor was allowed to take power from the line, it was rotating in synchronism with its counter e.m.f. exactly opposite in phase to the impressed, it could not maintain this position, because it would not be drawing enough power from the line to overcome its own internal losses. It must therefore slow down, and if the power supplied to it does not then increase it must at length come to rest.

But let us now examine the conditions when as they are slowing down, the field poles have dropped back but 10° (in time) from the position shown in Fig. 286. Assuming that the values of the induced and counter e.m.f.'s remain the same as before, we can represent the impressed 6500 volts by the vector E_x, Fig. 288, and the counter e.m.f. of 6000 volts by the vector E, lagging 10° behind the position it had in Fig. 286 and 287.

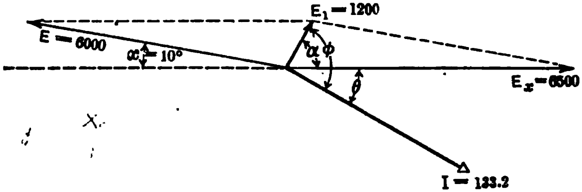

FIG. 288. Vector diagram for the conditions of the synchronous motor of Fig. 286 and 287, when the rotor has dropped back 10° from the position shown in Fig. 286 and 287. The resultant voltage E_1 has now become 1200 volts and forms 133.2 amperes through the armature windings. This current still lags 89° 21.6′ behind the resultant voltage E_1 but only 28° 54′ behind the impressed voltage E_x.

$$\begin{aligned} \text{The resulting voltage } E_1 &= \sqrt{E_x^2 + E^2 + 2\,E_x E \cos 170^\circ} \\ &= 1200 \text{ volts.} \\ \text{Current through armature} &= \frac{1200}{9} \\ &= 133.2 \text{ amp.} \end{aligned}$$

The angle α can be found from the cosine law for trangles.*

$$6000^2 = 1200^2 + 6500^2 - 2 \times 1200 \times 6500 \cos \alpha,$$

$$\cos \alpha = \frac{1200^2 + 6500^2 - 6000^2}{2 \times 1200 \times 6500}$$

$$= 0.493,$$

$$\alpha = 60^\circ\, 28'.$$

As before, $\phi = 89^\circ\, 21.6'$,

$$\theta = 89^\circ\, 21.6' - 60^\circ\, 28',$$

$= 28^\circ\, 54'$ **lagging.**

The power-factor, $\cos \theta = 0.875$.

Power received by phase A:

$$P_A = 6500 \times 133.2 \times \cos 28^\circ\, 54$$

$$= 757 \text{ kw.}$$

Power received by two phases

$$= 1514 \text{ kw.}$$

This power would be more than sufficient to supply all the losses. Thus the rotor would not only continue to turn in synchronism but could deliver some power to the shaft. In fact, if there were no load on the shaft, it would result in a dangerous condition of non-equilibrium for the rotor to get 10° behind the stator. The rotor would immediately surge ahead with great momentum and would be likely to start "hunting." (See Art. 120.) The unloaded motor would therefore not be likely to drop back as much as 10°, but only until it reached a position at which the power intake would exactly equal the losses in the motor. On the other hand, if the total load on the motor were greater than 1514 kw., the rotor would fall back still further in order to take in more power. When it reached a position such that the intake exactly equaled the output plus the losses, the rotor would continue to rotate in synchronism with the frequency of the line. This running position of the rotor is called the **synchronous position.**

115. Maximum Load for a Synchronous Motor. Constant Field. From the above it is seen that in order to carry an increased load, the rotor of a synchronous motor merely drops back with reference to the stator. But it must not be assumed this process of dropping back can proceed indefinitely and the motor carry an infinitely large load.

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* See page 510, First Course.

In order to determine the load limit of the motor, let us assume for simplicity that the field strength is such that the counter e.m.f. exactly equals the impressed e.m.f., 6500 volts, and compute the armature current and power delivered by one phase of this two-phase motor, as the rotor drops further back. Constructing diagrams similar to Fig. 288 and 289, we can compute the armature current and power input. The mechanical power which is developed in each phase can then be found by subtracting the I^2R loss in the armature. Tabulating the results as in Table A, we note that the mechanical power developed by the motor continues to increase as the rotor drops back until there is a phase difference of approximately 90° between the rotor and the stator. At this position, the armature current is 1021 amperes and the power input, 4745 kw. per phase. The armature I^2R loss is 104 kw., and the power developed, 4641 kw. Any further dropping back causes the armature current to increase, but the increase in the armature I^2R loss and the large decrease in the power-factor cause the total mechanical power developed to decrease.

Thus if we put a greater total load on this motor than 4641 kw. per phase, the rotor would drop back more than 90° in an attempt to carry it. This would cause a decrease in the power intake and therefore the load would be more than the power received by the motor. This would cause the rotor to stop. If the voltage were still maintained across the terminals, the excessive armature current would injure the windings.

This 90° position of the rotor is called the "pull-out" position for this motor and the load at this position, the "pull-out" load. Synchronous motors are usually designed to operate at about one-fifth or one-sixth of the pull-out load because the machine is unstable at this point. Accordingly, the windings of a commercial machine are not heavy enough to carry the maximum load current except for a second or so without injury to the insulation. By inspection of Table A, it will be seen that this motor, designed to carry $\frac{1500}{2}$ or 750 kw. per phase, "pulls-out" at 4641 kw. or about 6 times the normal load. Note that the armature I^2R loss is 104 kw. at the pull-out load but less than 1.6 kw. at the normal load. It is easy to see how rapidly a machine designed for an I^2R loss of less than 1.6 kw. would heat up when this loss became 104 kw.

It must not be assumed that the "pull-out" position of every synchronous motor must occur when the rotor lags 90° with respect to the stator, with the induced e.m.f. equal to the impressed e.m.f. This is true only when the synchronous armature reactance X is so much greater than the armature resistance R, that the armature current I lags approximately 90° behind the resultant voltage E_1. Since this is the case in practically all modern synchronous motors, it is unnecessary to discuss examples in which the armature resistance is a larger fraction of the reactance. It can be proved, however, that the maximum load always occurs when the rotor has dropped back through an angle whose value is arc tan $\frac{X}{R}$ or arc cos $\frac{R}{Z}$. See Art. 119 and Fig. 293a.

The effect of changing the field strength, and thereby changing the induced voltage, is explained in the succeeding articles. In general, a strong field increases the "pull-out" load while a weak field may greatly lessen it, because of the unstable condition which arises.

Prob. 4–8. What would be the "pull-out" load (output per phase) on a synchronous motor if the field strength were such that the counter e.m.f. per phase was 3000 volts when the impressed e.m.f. was 3300 volts. Armature resistance is 0.15 ohm per phase synchronous reactance, 7.5 ohms per phase. Plot a curve between output and angle $X°$ (Table A) using as values of $X°$, 70°, 75°, 80°, 82°, 84°, 86°, 88°, 90°, 92°, 94°, 96°, 98°, 100°, 105°.

Prob. 5–8. What input does each phase of the motor in Prob. 4 take when the power-factor is 90 per cent lagging with the same field excitation as in Prob. 4?

Prob. 6–8. What will be the synchronous position of the rotor of the motor in Prob. 5 under the conditions of that problem?

TABLE A

Angle through which rotor has dropped back $Y°$.	Resultant voltage in armature, E_1.	Current per phase, I.	Angle between impressed voltage and current $\theta°$.	Power-factor.	Power delivered to motor per phase, kw.	Armature I^2R loss per phase, kw.	Total mechanical power developed per phase, kw.
0°	0	0	0	0	0	....	......
5°	567	63.0	1° 51.6′	0.9995	409.3	0.4	408.9
10°	1,133	125.9	4° 21.6′	0.997	815.9	1.6	814.3
15°	1,695	188.3	6° 51.6′	0.993	1215	3.5	1211.5
30°	3,365	373.9	14° 21.6′	0.969	2355	14	2341
60°	6,500	722.2	29° 21.6′	0.872	3830	52	3778
80°	8,356	928.4	39° 21.6′	0.773	4666	86	4580
90°	9,191	1021	44° 21.6′	0.715	4745	104	**4641**
100°	9,960	1107	49° 21.6′	0.651	4693	123	4570
110°	10,648	1183	54° 21.6′	0.583	4423	140	4283
120°	11,358	1262	59° 21.6′	0.510	4180	159	4021
150°	12,557	1395	74° 21.6′	0.270	2445	195	2250

116. General Effect of Varying the Field Strength of a Synchronous Motor. It thus is evident that although the rotor of a synchronous motor rotates in synchronism with the alternations of the line e.m.f. and current, still its position relative to the poles produced by the current in the armature windings depends upon the load, — the greater the load the further the induced e.m.f. lags behind the impressed e.m.f. This action is very unlike that of a direct-current shunt motor which diminishes in speed as the load is increased and thus produces less counter e.m.f., so that a greater part of the impressed e.m.f. can be used in sending current through the armature windings. Since the speed of a synchronous motor is constant, it cannot cause any decrease in the counter e.m.f., but must produce a resultant voltage by a change in the phase relations between the impressed e.m.f. and the counter e.m.f. In fact, the value of the counter e.m.f. is determined entirely by the strength of the field current, which can be controlled independently of the load on the motor. In Example 1, we have assumed that the field strength was great enough to produce a counter e.m.f. of 6000 volts at synchronous speed of the rotor when the impressed e.m.f. was 6500 volts. By increasing the field strength, the counter e.m.f. may be made exactly equal to the impressed e.m.f., since the rotor cannot fall off in speed as the rotor of a direct-current motor does.

Let us assume that the phase relation of the counter e.m.f. remains the same as before, 180°+ 10° behind the impressed e.m.f., and see the effect of making the counter e.m.f. equal the impressed e.m.f.

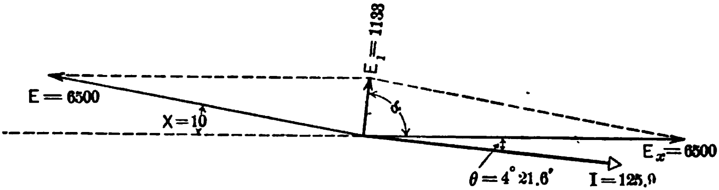

FIG. 289. Vector diagram of current and voltage conditions of the synchronous motor shown in Fig. 286, 287 and 288, where the counter e.m.f. is made equal to the impressed e.m.f. and has dropped back 10° from the position shown in Fig. 287.

We construct Fig. 289, similar to Fig. 288, except that the counter e.m.f. E has a value of 6500 volts equal to the impressed e.m.f. E_x.

The resultant voltage across the armature,

$$E_1 = \sqrt{6500^2 + 6500^2 + 2 \times 6500^2 \cos 170^\circ}$$
$$= 1133 \text{ volts.}$$
$$I = \frac{1133}{9}$$
$$= 125.9 \text{ amp}$$
$$\alpha = \frac{170^\circ}{2} = 85^\circ.$$
$$\theta = 89^\circ\ 21.6' - 85^\circ$$
$$= 4^\circ\ 21.6' \textbf{ lagging.}$$

The power-factor:

$$\cos\theta = \cos 4^\circ\ 21.6'$$
$$= 0.997.$$

The power received by phase A from the line wires at 6500 volts:

$$\begin{aligned} P_A &= 6500 \times 125.9 \times 0.997 \\ &= 815.9 \text{ kw.} \end{aligned}$$

Note that the power received per phase, 815.9 kw., is somewhat more than the 739 kw. it received when the field was weaker, and that the power-factor is greater, being now 0.997 against 0.875 for the weaker field. The current has swung around a little nearer to the impressed voltage.

By increasing the field strength still more, the counter e.m.f. may be further increased. Let us assume that we increase the field strength enough to raise the counter e.m.f. to 7500 volts, having the same phase difference with the impressed e.m.f. as before. The resulting voltage E_1, Fig. 290, now becomes

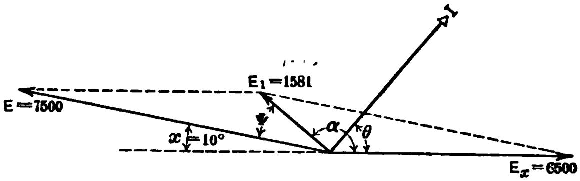

Fig. 290. Vector diagram showing the conditions when the counter e.m.f. has been made 7500 volts, but the rotor position remains 10° behind the phase of the stator flux.

$$\begin{aligned} E_1 &= \sqrt{7500^2 + 6500^2 + 2 \times 7500 \times 6500 \times \cos 170^\circ} \\ &= 1576 \text{ volts.} \\ I &= \frac{1576}{9} \\ &= 175.1 \text{ amp.} \\ \cos \psi &= \frac{7500^2 + 1576^2 - 6500^2}{2 \times 7500 \times 1576} \\ &= 0.697. \\ \text{Angle } \psi &= 45^\circ\ 50'. \\ \alpha &= 170^\circ - 45^\circ\ 50' \\ &= 124^\circ\ 10'. \end{aligned}$$

$$\theta = 124° \ 10' - 89° \ 21.6'$$
$$= 34° \ 48.4' \text{ leading.}$$

Power-factor, $\cos \theta = 0.821$.

Power delivered per phase to motor,

$$P = 6500 \times 175.1 \times 0.821$$
$$= 934 \text{ kw.}$$

Note that the power delivered to the motor has increased somewhat, but also that the motor is now taking a **leading current.**

The power-factor of a synchronous motor can be controlled by means of the field, — a weak field producing a lagging current and a strong field a leading current. Of course the field may be so adjusted that the current is exactly in phase with the impressed voltage, and the power-factor is unity. In later articles this effect will be taken up in detail.

117. Most Economical Field Excitation for a Given Load. V-Curves. In the above example we have allowed the field and load to change, keeping only the impressed voltage constant. We will now study the effect of keeping both the load and impressed voltage constant, and changing the field.

Let us assume that the load on the motor is such that it has to draw 1500 kw. from the line in order to operate at full load, i.e., 750 kw. per phase. As before, the impressed voltage is 6500 volts. We will start with such a setting of the field rheostat that the field has the proper value to cause the armature current to have a lagging power-factor of 70 per cent, — that is, the armature current lags practically 45° 34′ behind the impressed voltage. In order to draw 750 kw. from the 6500 volt mains at 70 per cent power-factor the armature must draw $\dfrac{750{,}000}{6500 \times 0.70} = 165$ amperes per phase. Thus in Fig. 291, draw vector OI to represent the armature current of 165 amperes, and E_x, 45° 34′ ahead of I, to represent 6500 volts impressed on armature. In order to force

165 amperes through the 9 ohms impedance of the armature, the resultant voltage across the armature must be 9 × 165 or 1485 volts. We have seen that the relation of the reactance to resistance is such in the armature that the armature current always lags 89° 21.6′ behind the resultant voltage. In order to simplify our computations we will consider this angle to be 89° 22′. No appreciable error is caused by so doing.

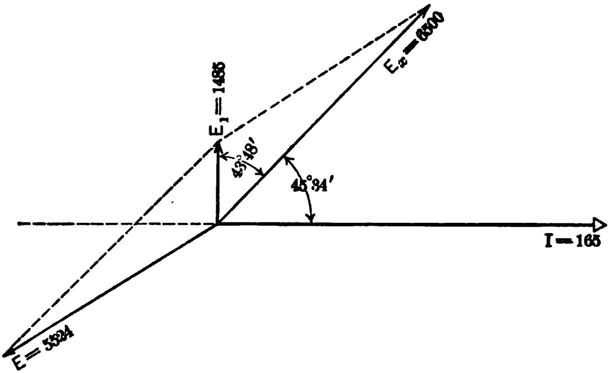

FIG. 291. Vector diagram for determining the field strength (counter e.m.f. E) necessary to cause the armature to take a current of 165 amperes lagging 45° 34′ behind the impressed voltage E_x.

Therefore draw E_x, 89° 22′ ahead of I, to represent the resultant voltage across the armature. Since IZ or E_1 must be the resultant of the impressed voltage E_x and the counter e.m.f., complete the parallelogram and find the counter e.m.f., E.

To find the value of E, note that the line E_1E_x must be equal in length to E.* The angle between E_1 and E_x must equal 89° 22′ − 45° 34′ or 43° 48′. Writing the equation for the triangle E_1OE_x, we have

$$\begin{aligned} E_1E_x &= \sqrt{E_1^2 + E_x^2 - 2\,E_1E_x \cos 43^\circ\,48'} \\ &= \sqrt{1485^2 + 6500^2 - 2 \times 1485 \times 6500 \cos 43^\circ\,48'} \\ &= 5524. \\ E &= 5524 \text{ volts.} \end{aligned}$$

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* The opposite sides of a parallelogram are always equal in length.

Therefore when we make the field strength such that the counter e.m.f. equals 5524 volts, the motor, operating under constant load (power input) and voltage, draws 165 amperes from the line at 70 per cent lagging power-factor.

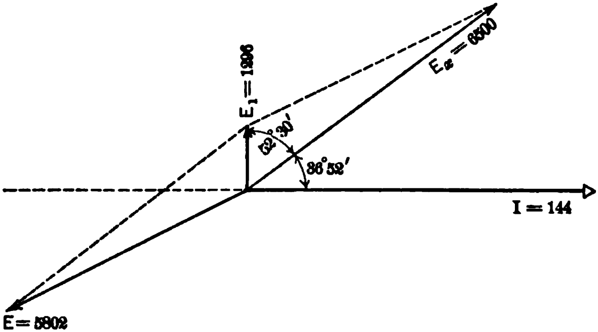

FIG. 291a. Vector diagram for determining the field strength (counter e.m.f. E) when the motor takes the same input as in Fig. 291, but at a power-factor of 80% lagging.

In a similar manner, let us determine the counter e.m.f. necessary to produce a power-factor of 80 per cent lagging with the same load and impressed voltage.

$$\text{Armature current} = \frac{750{,}000}{6500 \times 0.80} = 144 \text{ amp.}$$

$$\text{Resultant e.m.f.} = 9 \times 144 = 1296 \text{ volts.}$$

In Fig. 291a, draw

$I = 144$ amperes,
$E_x = 6500$ volts, leading I by 36° 52′ (arc cos 0.80),
$E_1 = 1296$ volts, leading I by 89° 22′.
Complete parallelogram and find E.

$$\begin{aligned} E &= \sqrt{E_1^2 + E_x^2 - 2\,E_1E_x \cos 52^\circ\, 30'} \\ &= \sqrt{1296^2 + 6500^2 - 2 \times 1296 \times 6500 \times 0.609} \\ &= 5802 \text{ volts.} \end{aligned}$$

Thus, when we strengthen the field so that the counter e.m.f. rises from 5524 to 5802 volts, the armature draws only 144 amperes at a lagging power-factor of 80 per cent, instead of 165 at a lagging power-factor of 70 per cent with the motor running under the same load (power input) and voltage.

In the same way, the field strength has been worked out for power-factor of 90 per cent lagging, unity power-factor, and 90, 80 and 70 per cent leading, the load (power input) and voltage of the motor being kept constant. The results are shown in the accompanying Table B. Note that as the field strength increases the current decreases and the power-factor continues to become greater. When the field strength has reached such a value that the power-factor becomes unity, the armature current becomes a minimum. Any further increase in the field strength causes the armature current to lead and reduces the power-factor.

TABLE B

Field strength, counter e.m.f.	Armature current per phase.	Power-factor.	
5524 volts	165 amp.	70%	lag
5802 "	144 "	80%	
6080 "	128 "	90%	
6580 "	115.4 "	100%	
7070 "	128 "	90%	lead
7340 "	144 "	80%	
7610 "	165 "	70%	

The curve in Fig. 292, plotted between the armature current and field strength, has a general V-shape, and is known as the **V-curve of the synchronous motor.** This curve shows that **for a given load there is a certain field strength which will produce a minimum armature current** called the **normal current,** when the motor has its full load. **This strength of field is reached when the power-factor of the motor is unity,** and is called the **normal field excitation** when the motor has its full load.

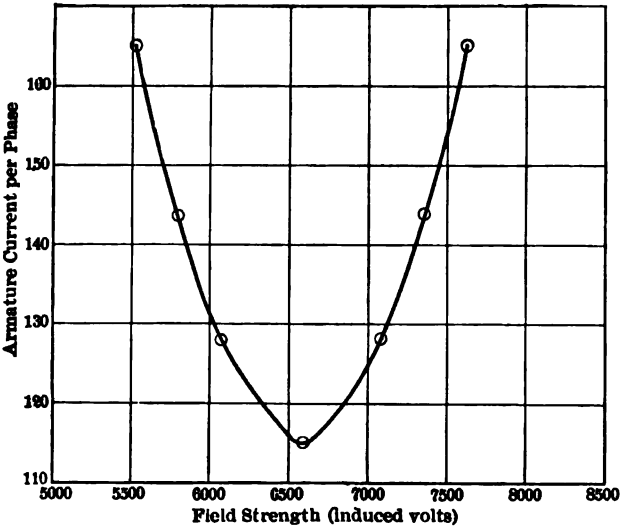

FIG. 292. The V-curve for the synchronous motor of Fig. 286–292, showing the relation between the armature current and the field strength at full load.

Prob. 7–8. Replot curve of Fig. 292. On same sheet and to same scale plot similar curves showing the relation between armature current (per phase) and field excitation (induced volts) in this motor for:

(*a*) No load total for both phases (stray power loss plus I^2R loss at no load) equals 60 kw.
(*b*) $\frac{1}{4}$ rated load (input).
(*c*) $\frac{1}{2}$ rated load (input).
(*d*) $\frac{3}{4}$ rated load (input).
(*e*) $1\frac{1}{4}$ rated load (input).
(*f*) $1\frac{1}{2}$ rated load (input).

118. Relation Between Reactive Armature Current and the Field Strength. Constant Load. We have seen in the preceding article that when the field is excited above a certain value, a leading reactive current in addition to the effective current flows in the armature, and causes the resulting armature current to lead the impressed voltage. Similarly, under-excitation produces a lagging reactive armature current which combines with the effective armature current and produces a lagging resultant current. Let us see what relation the strength of these reactive component currents bears to the amount of over- or under-excitation.

Table C is prepared from the data in Table B, the first column of the two tables being alike. Column 2 shows the percentage of over- or under-excitation at each power-factor. The field flux must be directly proportional to the induced counter e.m.f., because the speed is constant. Column 3 is found by multiplying the normal armature current by the tangent of the angle of lead or lag. Thus at 70 per cent power-factor, leading, the current leads the voltage by 45° 34′ (cos 45° 34′ = 0.70). The total current equals 165 amp. The reactive current equals 165 sin 45° 34′ or 118 amperes. Column 4 shows the percentage which the reactive current is of the normal current (which is 115.4 amperes). Note that when the field is 16 per cent under-excited the reactive current is approximately 102 per cent. When we make the under-excitation 12 per cent or ¾ as great, we likewise make the reactive current approximately 75 per cent or ¾ as great.

TABLE C

Field strength, counter e.m.f.	Per cent field strength of over or under strength for unity power-factor.		Reactive current, amperes.	Per cent reactive current is of full-load current.
5524	16	under	118	102.1
5802	12		86.5	74.9
6080	7.3		56.0	48.7
6580	0		0	0
7070	7.4	over	56.0	48.7
7340	12		86.5	74.9
7610	16		118	102.1

By inspection of the rest of the table we may draw the conclusion that:

The reactive armature current is approximately proportional to the amount of over- or under-excitation of the fields.

By the use of this rule, it is possible to estimate the armature currents and power-factors at all values of over- or under-excitations, if the power-factor is known for one value of over- or under-excitation.

Prob. 8–8. The following data apply to a typical three-phase Δ-connected 2500-kv-a. synchronous motor.

Voltage between terminals	6500 volts
Armature resistance (per phase)	0.16 ohm
Turns in field windings	1700 turns
Field current at full load, unity power-factor	5.68 amperes
Normal synchronous impedance (per phase)	13.3 ohms

Determine by means of a vector diagram the counter e.m.f. at full load, normal current and normal voltage.

Prob. 9–8. Determine from vector diagram the counter e.m.f. and reactive current at full load, normal voltage and 87 per cent lagging power-factor for the motor of Prob. 8 with it still receiving full-power-load input from the line.

Prob. 10–8. What will be the kv-a. and power-factor of the motor of Prob. 8 if the field current is reduced to 5 amperes and the power load remains constant? Assume that the field flux is proportional to the field current.

Prob. 11–8. Compute the power-factor and the kv-a. input of the motor in Prob. 8 if the field current is increased to 7 amperes and the power load remains constant.

119. The Circle Diagram for the Synchronous Motor.[*] By means of circle diagrams similar to that for the induction motor, it is possible readily to note the effect caused by changes in the power-factor, field excitation and total intake, and to determine the limiting values of these quantities.

Suppose that we wish to be able to see at a glance what effect will be produced upon the current and power-factor by changing the field excitation, assuming that the applied voltage and the effective load are to be kept constant. We will use the typical constants which we have used in previous examples for a 1500-kw. 6500-volt two-phase motor. Armature resistance = 0.100 ohm. Synchronous impedance = 9.00 ohms. To show the condition at full load, 750 kw. per phase, construct Fig. 293a, as follows:

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[*] This article is founded upon the discussion of the Blondel diagram in "The Standard Handbook for Electrical Engineers," Sec. 7–70.

Draw vector $OA = E_x = 6500$ volts impressed.

At unity power-factor, the current would be $\frac{750{,}000}{6500}$ or 117 amperes and in phase with OA. Draw vector OI to represent the current per phase. The pressure required to force 117 amperes through the 9 ohms impedance would be 117 × 9 or 1053 volts. Draw vector $OB = E_1$ at an angle to vector I of $89° \; 21.6' = \text{arc} \cos \frac{R}{Z}\cdot$ In Fig. 293a this angle is made less than $89° \; 21.6'$ in order to bring point C within the limits of the page. E_1, for the sake of clearness, is drawn larger in proportion to E_x than it actually is. For diagram drawn to scale, see Fig. 293b. The vector $AB = E$ will then represent topographically the counter e.m.f. or field excitation necessary to produce unity power-factor at full load. If we now lower the power-factor to $\cos \theta$, the current, in order to produce the same effective power intake, will be represented by the vector I_1 lagging θ behind the impressed voltage E_x. The voltage required to force this current through the armature will be represented by the vector OB_1, leading I_1 by $\text{arc} \cos \frac{R}{Z}\cdot$ The vector AB_1 will represent the counter e.m.f. (field excitation) which, in combination with the impressed voltage OA, will produce a resultant voltage of OB_1. Similarly may be found the field excitations of AB_2 and AB_3 necessary to produce other power-factors at the same kilowatt intake.

The points, B, B_1, B_2, B_3, etc., are found to lie in the circumference of a circle, the center of which is C. This center C may be found by drawing the line CA at an angle to OA,

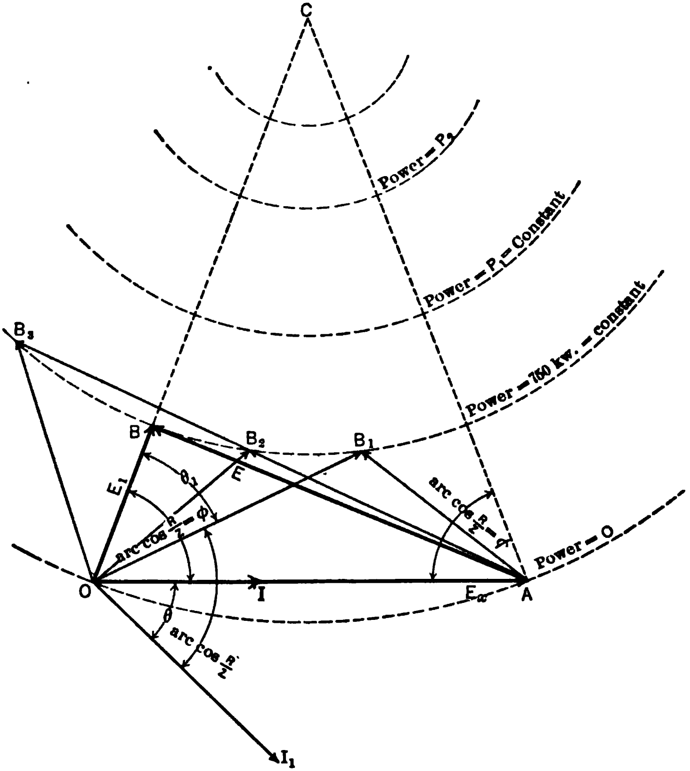

FIG. 293a. Circle diagram for synchronous motor. The vector OA represents the impressed voltage. The vectors AB, AB_1, AB_2 and AB_3 represent the field excitation necessary to produce the resultant voltages OB, OB_1, OB_2 and OB_3 and the necessary armature currents I, I_1, etc., to maintain a constant power intake of 750 kw. The points B, B_1, B_2, B_3, etc., all lie on a circle the radius of which is

$$CB = CO - OB = \frac{Z}{R}\left(\frac{E_x}{2} - \frac{PR}{E_x}\right).$$

E_x = impressed voltage.
P = constant power intake.
Z = synchronous impedance of armature.
R = effective armature resistance.
$\phi = \text{arc cos}\,\frac{R}{Z}.$

the cosine of which is $\frac{R}{Z}$ and extending it until it intersects at *C* the vector *OB*, which is drawn at an angle to *DA*, the cosine of which is also $\frac{R}{Z}$.

The head of all vectors for field excitation (E) and for resultant voltage (E_1) for a total input of 750 kw. per phase will lie on the circumference of this circle B_3, B, B_2, B_1. For any other constant input greater than 750 kw., the heads of these vectors would lie on the circumference of a circle of a smaller radius, such as the circles P_1 and P_2 in Fig. 293a, since the resultant voltage *OB* must increase as the load becomes larger. The greatest amount of power intake would be indicated when the resultant voltage vector *OB* had increased to the value *OC*, and the power circle became the point *C*. Of course this is an impractical condition, because the carrying capacity of the armature conductors would be surpassed long before the current value required for this amount of power had been reached. For any constant input smaller than 750 kw., these points will lie in the circumference of a circle of larger diameter, because the vector *OB* decreases as the load becomes smaller. When *Ob* becomes zero, the power intake is zero and the circle will pass through the points *O* and *A*. If the vector *OB* has a value less than zero (that is, a negative value), it must extend in the opposite direction (180° from the position of Fig. 293a) and the power intake becomes a minus quantity, — that is, the machine is giving out power rather than taking it in and is acting as a generator rather than as a motor. Thus all area outside of the zero circle represents generator power, and all area within the zero circle represents motor power. The radius for the zero power circle may be found from the equation

$$\text{Rad}_0 = \frac{ZE_x}{2R} = \frac{E_x}{2\cos\phi}.$$

Rad_0 = radius of power circle when there is no power intake or output.*

E_x = voltage per phase applied to motor.

* The value for the radius of circle of zero power may be found as follows: Construct Fig. *A*, which is a copy of the main outline of Fig. 293a, drawing the line *CM* from *C* perpendicular to *OA*. Since the angle at *O* equals the angle at *A* $\left(\text{both being equal to arc cos } \frac{R}{Z}\right)$, *CM* bisects *OA*. *OM* thus equals $\frac{E_x}{2}$.

$$\text{Cos } \phi = \frac{E_x}{2} \div OC$$
$$= \frac{E_x}{2\,OC},$$
$$OC = \frac{E_x}{2 \cos \phi};$$

but $$\cos \phi = \frac{R}{Z}.$$

Therefore *OC*, which is the radius of the circle for zero power,

$$= \frac{ZE_x}{2R} \quad \text{or} \quad \frac{E_x}{2 \cos \phi}.$$

Let Rad_P be the radius for any other power circle when the resultant armature voltage is E_1 and the power intake is *P* at unity power-factor.

Then $$\text{Rad}_P = BC = OC - E_1,$$
$$E_1 = IZ,$$
$$I = \frac{P}{E_x} \text{ (at unity power-factor).}$$

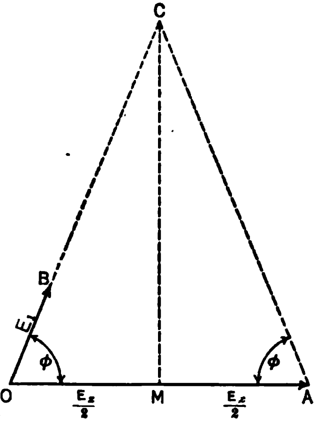

FIG. A. Copy of main outline of Fig. 293a and 293b. The radius of a power circle, when *OA* represents the impressed voltage and *OB* the resultant voltage across the armature, would be *OC* – *OB*.

Therefore $$E_1 = \frac{PZ}{E_x},$$

and $$\text{Rad}_P = OC - \frac{PZ}{E_x};$$

but $$OC = \frac{ZE}{2R}.$$

Therefore $$\text{Rad}_P = \frac{ZE_x}{2R} - \frac{PZ}{E_x}$$
$$= \frac{Z}{R}\left(\frac{E_x}{2} - \frac{PR}{E_x}\right).$$

Z = synchronous impedance per phase in ohms.
R = equivalent resistance per phase in ohms.
ϕ = angle of lag of armature current behind resultant voltage per phase.
$= \text{arc}\cos\frac{R}{Z}$.

The value of the radius for any power circle may be found from the equation

$$\text{Rad}_P = \frac{Z}{R}\left(\frac{E_x}{2} - \frac{PR}{E_x}\right),$$

in which Rad_P = radius of circle for constant power intake of P watts. When any vector of the resultant armature voltage, as OB, lies along the line OC, the current vector lies along the vector of the impressed voltage E_x and the power-factor is unity. When any resultant voltage vector as OB_1 and OB_2 falls to the right of OC, the current vector I_1 falls the same number of degrees below the vector of impressed voltage E_x and the power-factor is lagging and less than unity. Note that the angle θ_1, between OB, and OB_1, equals the angle θ between E_x and I_1. Thus $\cos\theta_1$ can be used as the power-factor when the field excitation has the value represented by the vector AB_1.

Similarly when any vector of the resultant voltage such as OB_3 falls to the left of the line OC, the power-factor is leading and equal to the cosine of the angle between OB_3 and OC.

In Fig. 293b, the lines are drawn to scale and the angles are correct. Note that the angle ϕ is so nearly 90° that the point C lies several feet off the page. The arcs of the power circles are accordingly very nearly straight lines.

The dotted circles show the effect of increasing the load while keeping the field excitation constant. The point B moves around in a circle about the point A. Thus for a constant field excitation producing 5000 volts counter e.m.f., as the load starts from zero and increases with a larger and larger intake, the path of the point B is along the arc of the circle B_2, B_3, B_4, B_5, B_6, B_7. . . . At any particular load as for 2000 kw. intake, the value of the resultant voltage can be found by drawing vector OB_4. Dividing the value of the vector by the synchronous impedance gives the current, and the cosine of the angle θ is the power-factor. Note that the maximum power is delivered at constant field excitation, when vector for the counter e.m.f. lies along the line CA, — that is, when the counter e.m.f. has dropped back from a phase position directly opposite to that of E_x by an angle whose cosine is $\frac{R}{Z}$. If the vector E is compelled to drop

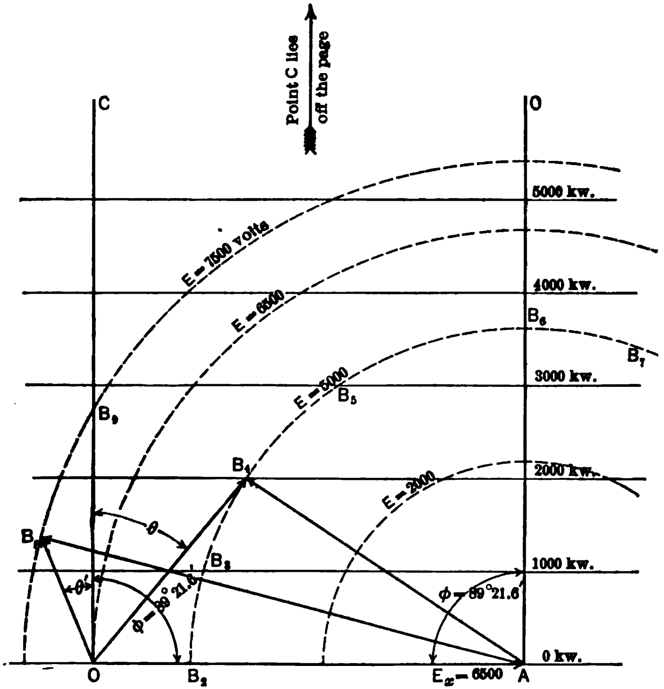

FIG. 293b. The circle diagram of Fig. 293a drawn to scale and with the correct angles. The arcs of circles drawn about A as center represent the paths of the ends of vectors of constant field excitation as the rotor drops back on account of greater loads being put on the rotor.

further back, that is, to pass to the right of the line *CA*, the point *B* will lie on smaller power circles. This shows that the motor can take in no more power than that indicated by power circle on which the point *B* falls when *E* lies along the line *AC*. Note that point B_7 lies on a smaller power circle than does B_6. Note also that the maximum load for a field excitation of 6500 volts is between 4000 and 5000 kw. This agrees closely with the results shown in Table A, Art. 115. The region, therefore, to the right of line *AC* represents a condition of unstable operation.

Note too, that for a given excitation the power-factor depends upon the load, although the excitation must attain a certain value before it is possible to produce unity power-factor or a leading power-factor. Thus, in this case, the counter e.m.f. must be more than 6500 volts before it is possible to produce unity power-factor at any load. But at a field excitation corresponding to 7500 volts counter e.m.f., the power-factor will be leading for all loads below 2700 kw. and lagging for all loads above that value.

It should also be noted that the vectors from *O* to the various *B* points not only represent the resultant voltage but may also represent the current, since the current is proportional to the resultant voltage. It merely requires that, to determine the value of the current, the vectors *OB* must be scaled off to a different scale than that used in scaling them to determine the value of the resultant voltage.

Prob. 12–8. Compute the value of the radius of the circle for zero power circle in Fig. 293b.

Prob. 13–8. Compute the value of the radius for the 1000-kw. power circle in Fig. 293b.

Prob. 14–8. The following measurements were made on a three-phase synchronous motor.

Synchronous reactance, 8.20 ohms per phase.

Effective resistance, 0.40 ohm per phase.

Applied voltage between terminals (Y-connected) 4400 volts. Construct circle diagram similar to Fig. 293b, using a scale of 1 inch = 1000 volts.

Prob. 15–8. Determine from circle diagram of Prob. 14–8.

(*a*) The pull-out load (intake) when the counter voltage equals the applied voltage.

(*b*) At the same counter voltage as in (*a*), the power-factor at $\frac{1}{3}$ the pull-out load. Call this the rated intake.

(*c*) The current at the load in (*b*).

Prob. 16–8. What must be the counter voltage in the motor of Prob. 14–8:

(*a*) To produce unity power-factor at the rated load?

(*b*) To produce leading power-factor of 95 per cent at rated load?

Prob. 17–8. Determine from the circle diagram the currents for the conditions of (*a*) and (*b*) in Prob. 16–8.

Prob. 18–8. Determine the total motor effect of the motor under conditions:

(*a*) Of Prob. 15 (*b*).

(*b*) Of Prob. 16 (*a*).

(*c*) Of Prob. 16 (*b*).

120. Phase Modifiers (Synchronous Condensers). The fact that the alternating current taken by a synchronous motor may be made either to lead or to lag with respect to the impressed voltage by means of the field control, is made use of in order to bring the current in transmission lines more nearly into phase with the voltage, thereby improving the power-factor and increasing the kilowatt capacity of the line.

Thus if the line is feeding induction motors, the line current will lag behind the line voltage, and the power-factor will be low. This results in a relatively large line current for a given kilowatt load. By replacing some of the induction motors with over-excited synchronous motors or by adding to the line over-excited synchronous motors of sufficient size, the lagging reactive currents for the induction motors may be neutralized by the leading reactive current taken by the synchronous motors. Stated in other words: When the synchronous condenser is used in parallel with induction motors to improve the power-factor of the line — to make it unity, let us say, — in reality **neither** the condenser **nor** the induction motors take **any** reactive current from the generator or from the transmission line between the generator and the point at which the condenser and the induction motors are placed. What really happens is that just at the instants in each cycle when the induction motor requires reactive power, the condenser is ready to give up reactive power and vice versa. So that the necessary reactive power circulates between the condenser and the induction motors without traveling through the generator or over the transmission line. Synchronous motors so used are called **phase modifiers** or **synchronous condensers.** They may be used wholly to better the power-factor of the system, or they may perform the double duty of bettering the power-factor and of delivering a mechanical load. This mechanical load may be used for the purpose of supplying power to shops, mills, etc., or for driving a direct-current generator in order to convert the alternating current supplied to the motor into direct current. When used for the latter purpose, the synchronous motor is usually direct-connected to the direct-current generator and the combination is called a motor-generator converter. For further details concerning this device see Chapter IX.

Again, if a generator is already supplying its maximum kilovolt-ampere load to induction motors, a synchronous motor can be used to add to the effective power supplied by the generator without increasing the kilovolt-ampere load. This is of particular advantage when it is desired to expand the plant without adding to the generator and distribution cost.

A synchronous condenser should be installed at the same point as the load in order that the reactive power shall have to circulate through as little of the conductors as possible and the whole line and all apparatus connected to it, between the load and the generator, will receive the benefit of the power-factor correction. Placing the condenser at the generator end economizes on generator capacity and losses, but not on transmission line capacity and losses. Furthermore, the motor may be used for mechanical purposes to greater advantage at the load end.

Prob. 19–8. A certain plant equipped with 60-cycle induction motors uses 1000 kw. at 80 per cent power-factor. What kv-a. synchronous condenser would be required to raise the power-factor of the plant to unity?

Prob. 20–8. The losses in synchronous motors may be assumed to decrease regularly from 8 per cent in 200 kv-a. motors to 4 per cent in 1000 kv-a. machines. (G. E. Review, June, 1914.)

(*a*) What would be the total kilowatt load in the plant in Prob. 19 before the condenser was installed?

(*b*) After the condenser was installed?

(*c*) What would be the total kilovolt-ampere load before the condenser was installed?

(*d*) After the condenser was installed?

(*e*) At what power-factor would the synchronous condenser be operating?

Prob. 21–8. If it had been desired to raise the power-factor of the plant in Prob. 20 to 90 per cent only, what kilovolt-ampere synchronous condenser would have been needed?

Prob. 22–8. Apply the questions in Prob. 20 to the conditions of Prob. 21.

Prob. 23–8. If it is desired to maintain the same kilovolt-ampere load at 90 per cent power-factor in Prob. 21–8 as when the induction motors only were running, the additional load being in the form of a mechanical load on the synchronous condenser, what size (kilovolt-ampere) condenser should be installed?

Prob. 24–8. (*a*) What effective power would be available as output from the synchronous motor in Prob. 23?

(*b*) At what power-factor would the synchronous motor operate?

Prob. 25–8. What kilovolt-ampere synchronous condenser should be installed in Prob. 19 so that the power plant would be delivering the maximum effective power and still not increase the total kilovolt-ampere output, the additional load being in the form of a mechanical load on the synchronous condenser?

Prob. 26–8. (*a*) At what power-factor would the synchronous motor in Prob. 25 be operating?

(*b*) How much would the effective power output of the plant be increased?

121. Constant-Voltage Transmission. Synchronous Reactor. A synchronous motor may be used at any point in a transmission line to keep the voltage at that point constant. When the line is carrying a large load with a low power-factor the motor is over-excited enough to produce a leading current large enough to counteract the inductive reactive component of the line current and thus reduce the resulting line current. When the line is carrying a small load, the synchronous motor is under-excited so that it draws lagging current enough to maintain a line drop between the generator and motor equal to the full load line drop. The changes in the field of motor are usually made automatically by a Tirrill regulator. A synchronous motor so used is often called a **synchronous reactor.** Several installations are already successfully using this method of line voltage regulation.*

The advantages of this scheme are summarized as follows in the "Electric Journal," September, 1915:

"While the use of the synchronous motor to improve the power-factor at full load makes possible an increase of load with the same regulation of the line from no load to full load, the elimination of this regulation and the substitution of a constant difference of voltage between the power house and the terminal station by use of a motor to supply lagging current, and so lower the power-factor at light load, is desirable from the operating point of view, for the following reasons:

1. Automatic compounding of the generating station voltage is done away with and constant voltage, automatically controlled, is substituted. Apparatus for the latter condition may be relied on.

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* See H. B. Dwight in the Electric Journal, Sept., 1914; L. A. Herdt and E. G. Burr in the Electric Journal, Sept., 1915; H. L. Unland in the General Electric Review, June, 1914; H. B. Dwight, "Constant Voltage Transmission."

2. Extreme overload capacity of lines in case of emergency, and control from the terminal station.

3. Telephoning between power house and terminal station is greatly reduced, as the voltage of the distribution bus is controlled in the terminal station.

4. Voltage of the line is steady and lightning arresters may be set closer with increased protection to insulators and apparatus. From this point of view, it is important to arrange the switching so that the motor is part of the line unit in case automatic relays are used on the line oil switches.

5. Loads may be supplied at intermediate points on the line with steady voltage."

If a synchronous motor so used is required to carry a mechanical load also, it must be protected from sudden increase of load when running on a weak field, as it is likely to be thrown out of step. If the field is very weak, a large mechanical load, even though applied gradually, may cause the rotor to slip beyond the pull-out position and cause it to drop out of step.

The following example, based on data given in the General Electric Review for June, 1914, illustrates the means by which these results are accomplished.

Example 2. The constants for a 30-mile, three-phase transmission line are as follows:

Copper cables, No. 000, effective spacing	36 inches
Reactance to neutral of each cable	19.55 ohms
Resistance of each cable	10.73
Reactance of each transformer referred to high-tension side	9 ohms
Resistance of each transformer referred to high-tension side	1.5 ohms
Voltage (to neutral)	30,000 volts, 60 cycles
Rated capacity of each transformer at each end of line	6000 kv-a. (line to neutral: transformers Y-connected)
Power-factor of load	75 per cent

What size synchronous motor is required at receiving end in order to maintain this end at constant voltage? Neglect capacitance of the line.

Total line reactance per conductor $= 9 + 9 + 19.55$
$= 37.55$ ohms.

Total line resistance per conductor $= 1.5 + 1.5 + 10.73$
$= 13.73$ ohms.

Line current at full load (75 per cent power-factor)

$$= \frac{6{,}000{,}000}{30{,}000}$$

$= 200$ amp.

Reactive component of current $= 200$ sine $41°\,25'$
$= 132.3$ amperes.

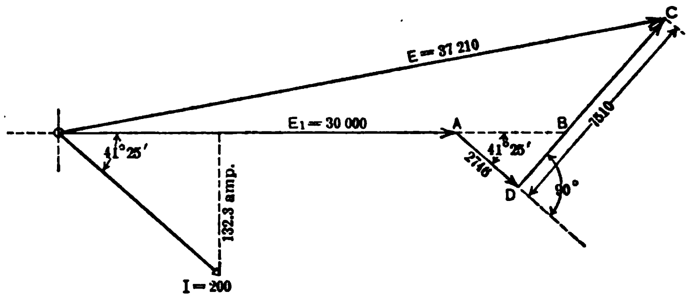

FIG. 294. The vector *OC* represents the voltage at the sending end of a transmission line, carrying 200 amperes with a pressure of *OA* at the receiving end. No synchronous reactor is used in this case.

Construct Fig. 294, drawing *OA* to represent E_1 or 30,000 volts (to neutral) at receiving end. *OI* represents the line current of 200 amperes, lagging 41° 25′ behind E_1. *AD* drawn parallel to *OI* represents the resistance drop of 13.73×200 or 2746 volts in phase with *OI*. *DC* drawn at right angles (leading) to *OI* represents the reactance drop of 37.55×200 or 7510 volts.

Since *OC* equals the voltage to neutral at the sending end,

$$OC = \sqrt{OB^2 + BC^2 - 2\,OB \times BC \cos \underline{|B}}.$$

$$OB = OA + AB.$$

$$AB = \frac{2746}{\cos 41°\,25'}$$

$$= 3660 \text{ volts.}$$

$$OB = 30{,}000 + 3660 = 33{,}660 \text{ volts.}$$

$$
\begin{aligned}
BC &= DC - DB.\\
DB &= 2746 \tan 41^\circ\ 25'\\
&= 2420.\\
BC &= 7510 - 2420\\
&= 5090 \text{ volts.}\\
\underline{|B} &= 90^\circ + 41^\circ\ 25'\\
&= 131^\circ\ 25'.\\
OC &= \sqrt{33{,}660^2 + 5090^2 - 2 \times 33{,}660 \times 5090 \cos 131^\circ\ 25'}\\
&= 37{,}210 \text{ volts.}
\end{aligned}
$$

The line drop is therefore 37,210 − 30,000 = 7210 volts when no synchronous condenser is used.

The voltage regulation thus equals $\frac{7210}{30{,}000} = 24.03$ per cent.

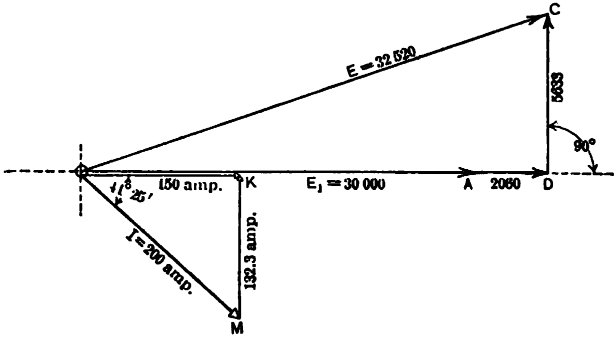

FIG. 295. The vector OC represents the voltage at the sending end of the line in Fig. 294, when a synchronous reactor is used to raise the full-load power-factor to unity.

Let us now add to the receiving end a synchronous motor which can be over-excited so that it takes a leading current of 132 amperes. This leading current will counteract the reactive component (132 amperes) of the load current. The line current will now be the resultant of the load current 200 amperes lagging 41° 25′ behind the voltage to neutral, and the leading reactive current of 132 amperes leading the voltage by 90°. In Fig. 295, this resultant is represented by the vector OK and is seen to be equal to 150 amperes.

The resistance line drop now equals the vector *AD*, in phase with the resulting current

$$AD = 13.73 \times 150$$
$$= 2060 \text{ volts.}$$

The reactance line drop $DC = 37.55 \times 150$

$$= 5633 \text{ volts.}$$

The voltage at the sending end will now have to be

$$OC = \sqrt{(OA + AD)^2 + DC^2}$$
$$= 32{,}520 \text{ volts.}$$

If the generator voltage is maintained constant the line drop thus becomes only 32,520 − 30,000 or 2520 volts. Assuming the field of the synchronous motor to be controlled so that it always supplies the reactive power of the load as the load and terminal voltage change, the regulation from no load to full load equals

$$\frac{2520}{30{,}000} = 8.4 \text{ per cent.}^*$$

We may, however, weaken the field of the synchronous reactor so that as the useful load on the transformer becomes zero, the current taken by the synchronous reactor lags and becomes just sufficient to cause a line drop of 2520 volts. The voltage at the receiving end will thus remain constant at 32,520 − 2520 or 30,000 volts.

To determine what lagging current the synchronous reactor must take, we have to remember that when the current lags approximately 90°, the greatest line drop is due to reactance. This reactance drop will lead the current by 90°. In Fig. 296,

OI_s = lagging reactive current taken by synchronous condenser when there is no load on the line (90° behind the terminal e.m.f.).

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* This result neglects losses in the synchronous motor, which do not appreciably affect the values.

AD = component of e.m.f. impressed on line, consumed in overcoming inductive reaction of the line, due to OI_s (leading OI_s by 90°).

DC = component of e.m.f. impressed on line, consumed in overcoming resistance reaction of the line, due to OI_s (in phase with OI_s).

OC = total e.m.f. that must be impressed upon line in order to give OA volts at end of line, after overcoming line reaction due to resistance and inductive reactance.

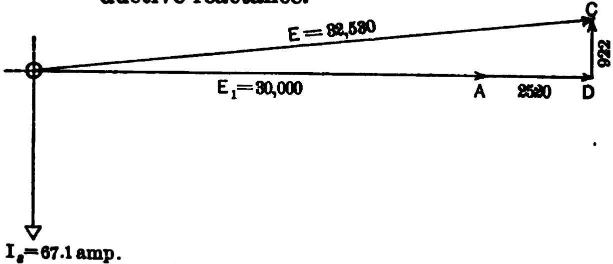

FIG. 296. At no load the synchronous reactor must draw from the line a current I_s of 67.1 amp. lagging 90° behind the impressed voltage E_1. The voltage to send this lagging current against the line reactance equals 2520 volts in phase with the impressed voltage. There are also 922 volts used to overcome the line resistance. The generator voltage E must be the vector sum of these quantities.

Since the current lags 90° behind the impressed voltage, as seen from Fig. 296, the reactance drop AD, due to this lagging current, neglecting the losses of the synchronous condenser, will be in phase with the impressed voltage. Since the resistance drop is but a small part of the total drop and at 90° to the line voltage, it does not appreciably affect the total drop OC.

Thus the lagging current for the reactor must be made great enough to produce approximately 2500 volts reactance drop, or

$$I_s = \frac{2520}{37.55}$$
$$= 67.1 \text{ amp.}$$

The resistance drop then equals

$$DC = 67.1 \times 13.73$$
$$= 922 \text{ volts.}$$

The generator voltage then checks:

$$OC = \sqrt{(30{,}000 + 2520)^2 + 922^2}$$
$$= 32{,}530 \text{ volts.}$$

Thus we see that the voltage at the load end of the line will have the same value (30,000 volts) at full load and at zero load while the e.m.f. of the generator remains constant at 32,520 volts, provided the field excitation of the synchronous condenser be changed in accordance with the load so as to produce a lagging reactive current of 67.1 amp. at zero load and a leading reactive current of 132 amperes at full load.

In order to produce this result, we must use a synchronous motor which will deliver a maximum leading current of 132 amp. and a maximum lagging current of 67.1 amp. It must therefore be able to supply per phase

$$132 \times 30{,}000 = 3960 \text{ kv-a.}$$

leading reactive power, or about 12,000 leading kv-a. for three phases.

$$67.1 \times 30{,}000 = 2013 \text{ kv-a.}$$

lagging reactive power, or about 6000 lagging kv-a. for three phases.

At the various points between full load, 75 per cent power-factor and zero load, the field of the synchronous motor can be adjusted either manually or automatically so that it draws sufficient current from the line at the proper power-factor to produce 2520 volts line drop, and thus maintain a constant voltage (30,000) at the receiving end.

It must not be assumed that the most economical equipment requires condenser capacity great enough to produce unity power-factor at full load. It is usually more economical to supply a smaller synchronous reactor and operate at less than unity-power at all loads. For methods of determining the most economical size of reactor, see articles referred to at bottom of page 570.

Prob. 27–8. What kv-a. will the synchronous reactor of Example 2 have to take in order to maintain a voltage of 30,000 volts at the receiving end, where the useful total load (three phase) is 2500 kw. at 80 per cent power-factor?

Prob. 28–8. Compute the kv-a. capacity of the synchronous reactor necessary to maintain a constant voltage at the receiving end of the line in Example 2, taking into account the capacitance of the line.

Prob. 29–8. Compute the kv-a. capacity of a synchronous reactor necessary to maintain a constant voltage at the load terminals of a three-phase line under the following conditions, which are fairly representative of good practice. The power-factor of the generator is to be 98 per cent lagging on full load. (Data from the Electric Journal, Sept., 1914.)

Length of line	200 miles
Conductor (copper cable)	400,000 cir. mils
Spacing (effective)	20 feet
Constant supply voltage	150,000 volts
Constant receiver voltage	110,000 volts
Resistance per conductor including transformers and protective coils	31.8 ohms
Reactance per conductor including transformers and protective coils	244.4 ohms
Full load delivered at receiver	50,000 kw.
Power-factor of load	85 per cent
Frequency	60 cycles

122. Hunting of Synchronous Motors. Damping Grids. We have seen that when a synchronous motor is in operation, the rotor always revolves at a speed in synchronism with the alternations of e.m.f. on the line. The induced e.m.f. and the impressed e.m.f. thus always maintain the same phase relation to each other, when the load remains unchanged,—the e.m.f. induced in the armature windings always being at such a phase angle with the impressed e.m.f. that the resulting e.m.f. sends sufficient current through the armature to produce the torque required to carry the load. In other words, for a constant load, the synchronous position remains unchanged. As the load is increased or decreased, the rotor drops back or surges forward to a new synchronous position. But as it takes time to start the rotor in its motion from one position to another, for a small fraction of a second it is out of step. Thus if a load has been added to the rotor, for a small fraction of a second it is ahead of the synchronous position and thus is not carrying enough current to maintain the torque required for the increased load. In dropping back, it has acquired a relative velocity backward and cannot stop the instant it reaches the desired synchronous position but is carried beyond by its own momentum. This causes it to take a current larger than necessary to carry the load, and it immediately surges ahead. Its acquired velocity in this direction again causes it to pass by the synchronous position. Thus it may surge back and forth across the synchronous position, even while the synchronous position is swinging around at synchronous speed. Any extra impulse, such as a slight periodic raising or lowering of the voltage, may be sufficient to increase these pulsations to such an extent that the rotor is thrown out of step. This oscillation about the synchronous position is called **Hunting** and unless special means are taken to stop it, it may prevent the machine from being operated as a synchronous motor. It is caused not only by sudden increase in load, but also by the pulsation in the motion of the engines driving the generators. These pulsations cause the frequency of the impressed e.m.f. to be uneven and thus change the phase relations between the impressed e.m.f. and the induced e.m.f. In general, the larger and heavier the rotor, the less likelihood there is of hunting, — the flywheel effect tending to hold the motion steady.

Hunting is effectually eliminated by means of damping grids, or squirrel-cage windings built into the pole faces of the motor. Fig. 241 and 297 show the construction of such a grid. A yoke, usually of bronze, surrounds the pole, while copper bars are sunk into the laminations of the face. In this way several paths of low resistance are formed around all portions of the pole. As long as the rotor revolves in synchronism with the frequency of the impressed e.m.f. there is no relative motion between the armature flux and the pole faces. Thus no current would be set up in the grid windings. But if the rotor hunts, the flux from the windings moves across the pole face in one direction or the other and currents are set up in these circuits, which produce a torque opposing the oscillations of the rotor.

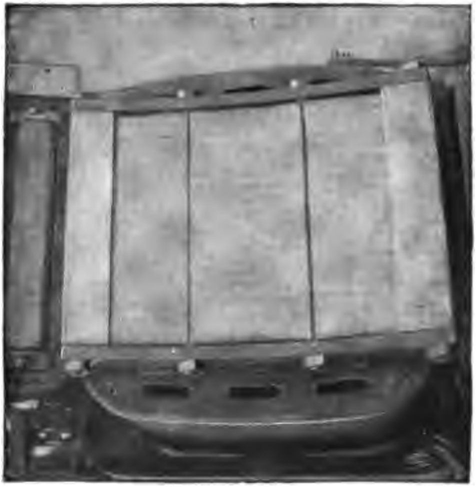

FIG. 297. The pole face of a synchronous motor showing the damping grid, which is used to prevent hunting. *The General Electric Co.*

122a. Starting Synchronous Motors. Synchronous motors are difficult to start. If a current is sent through the armature windings, the torque which it produces reverses its direction 50 or 120 times per second, depending on the frequency of the system. In order to receive the impulses from the armature in the proper direction, the rotor must start from rest and move the distance of the pole pitch in $\frac{1}{50}$ of a second, if the frequency of line currents is 25 cycles per second. It is impossible to construct a rotor of commercial size which will do this. Single-phase motors therefore are started by some outside source of power, usually a small induction motor, with a smaller number of poles, on the same shaft. If polyphase synchronous motors are fitted with damping grids, they may be started by the induction-motor action of the armature current on the circuits of these grids.* In this case the field is not excited and a reduced e.m.f. is impressed on the armature. The rotor acts exactly as the rotor of an induction motor. A slight torque is also produced in the same way by the eddy currents in the iron of the pole faces and pole pieces. For detailed direction for starting see Art. 143, on starting synchronous converters.

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* See Chapter VII, Art. 94.

SUMMARY OF CHAPTER VIII

THE SYNCHRONOUS MOTOR has the same construction as an alternating-current generator, and is usually of the revolving field type. If the rotor of such a machine is revolving at synchronous speed, and the alternating current is maintained in the armature, a torque is exerted by the interaction between the rotor fields and the revolving fields produced by the armature current, which keeps the speed up to synchronism until the machine is greatly overloaded.

THE ARMATURE CURRENT is proportional to the resultant of the impressed e.m.f. and the counter e.m.f. induced in the armature windings by the revolving field.

THE COUNTER E.M.F. depends in value upon the field excitation, since the speed of the field poles is constant at all loads. It does not, therefore, change in value as the load on the motor is increased. It merely comes more nearly into phase with the impressed e.m.f. so that the resultant voltage is increased and a greater armature current results.

THE ROTOR FALLS BACK as the load is increased and produces the necessary change in phase relation between the counter and the impressed e.m.f. in order to increase the resultant e.m.f. The position which the field poles occupy in space with regard to the revolving poles set up by the armature current is called the synchronous position.

THE MAXIMUM OR "PULL-OUT" LOAD occurs when the rotor has dropped back through an angle which has the value arc tan $\frac{X}{R}$. In this expression, X is the synchronous reactance of the armature and R the effective resistance. In modern machines the value of $\frac{X}{R}$ is usually between 50 and 100 and thus arc tan $\frac{X}{R}$ is approximately 90°. The normal load of such machines is usually about $\frac{1}{3}$ or $\frac{1}{2}$ of the maximum load, the machine being unstable in the neighborhood of maximum load.

VARYING THE FIELD EXCITATION changes the phase relation between the impressed voltage and the armature current, — a weak field causing the current to lag, and a strong field, to lead. For a given load, that field strength which produces unity power-factor results in the lowest armature current; the reactive armature current being approximately proportional to the amount of over- or under-excitation.

A CIRCLE DIAGRAM can be used for determining operating characteristics of a synchronous motor under varying conditions of load and field excitation.

SYNCHRONOUS MOTORS ARE USED TO MODIFY THE PHASE RELATION between the current and voltage of transmission lines. When performing this duty they are called synchronous condensers, and may also carry a motor load at the same time. An over-excited synchronous motor will supply the lagging reactive power to induction motors. The line has then to transmit the effective power only. Thus a saving in generator capacity, line loss, etc., is made.

SYNCHRONOUS MOTORS ARE USED TO MAINTAIN CONSTANT VOLTAGE at given points in a transmission line. The synchronous motor will produce nearly unity power-factor at full load, but, with a proper reduction of the field excitation, will draw sufficient lagging current from the line to produce the same line drop from zero load to full load. The field excitation may be controlled manually or automatically.

HUNTING IS THE TERM APPLIED TO AN OSCILLATING MOTION OF THE ROTOR back and forth across the synchronous position. It may be caused by a sudden increase of load on the rotor or by the pulsation of the prime movers, which produce an uneven frequency. It may become great enough to pull the machine out of step, if not stopped or damped. Hunting is effectively eliminated by damping grids inserted in the pole faces.

A POLYPHASE SYNCHRONOUS MOTOR MAY BE STARTED as an induction motor on low voltage, if equipped with damping grids. A single phase motor requires an auxiliary starting motor.

PROBLEMS ON CHAPTER VIII

Prob. 30–8. The following data may be taken as typical for a synchronous motor:

Rated capacity	2500 kv-a.
Normal voltage (between terminals)	2300 volts
Connection	Star
Armature resistance (per phase)	0.022 ohm
Field turns	1800 turns
Normal field current	4.35 amp.
Synchronous impedance per phase	0.92 ohm

Determine the counter e.m.f. at full load, unity power-factor and normal voltage.

Prob. 31–8. Compute the counter e.m.f., field current, and reactive current of the motor in Prob. 30–8 at half load (kv-a.) and normal voltage, when there is a leading power-factor of 80 per cent at the terminals. Assume the field flux to be proportional to the field current, and the kw. input to be the same in all three cases.

(*b*) When the field current of the motor in (*a*) is increased to 6 amperes, what will be the kilovolt-amperes intake and the power-factor?

(*c*) If the field current of the motor in (*a*) is reduced to 3.5 amperes, what will be the kilovolt-ampere input and the power-factor?

Prob. 32–8. How much reactive power will the motor of Prob. 31 (*c*) draw from the line?

Prob. 33–8. (*a*) What must be the field current of the motor in Prob. 31 in order that the motor draw 2000 kv-a. corrective leading reactive power from the line?

(*b*) How much total effective power (input) is then available for a motor load, when drawing 1000 kw. as in that problem? Note that to load (by power component) while still correcting 2000 kv-a. will require some additional field excitation.

Prob. 34–8. (*a*) Assuming that the field excitation of the motor of Prob. 30–8 can be increased to 150 per cent of the normal value, what will be the "pull-out" load at this excitation?

(*b*) What percentage is the rated full load of the pull-out load?

Prob. 35-8. Determine the proper capacity of synchronous motors which will maintain the voltage constant at the receiving end of a line having the following constants:

Length of line	120 miles
Power transmitted	32,100 kw.
Power-factor of load	80 per cent
Number of phases	3
Conductors (copper cables)	300,000 cir. mils
Spacing (effective)	14 feet
Frequency	60 cycles
Voltage at receiving end	95,000 volts
Voltage at generating end	110,000 volts
Reactive drop in transformers at each end at normal rated current	6 per cent each
Resistance drop in transformers at each end at normal rated current	0.75 per cent each
Power-factor of generators at full load	100 per cent

Prob. 36-8. A three-phase transmission line 140 miles long has the following constants:

Power transmitted	15,000 kw.
Pressure at receiving end	104,000 volts
Frequency	60 cycles

Conductors are No. 00 stranded copper, arranged in a vertical plane spaced 96 inches apart. Each transformer has 6 per cent reactance, and 1 per cent resistance drop. Total reactance of protective coils is 10 per cent; resistance, 0.5 per cent. Compute the regulation of this line at full load, 85 per cent power-factor.

Prob. 37-8. What size synchronous reactor is necessary to produce constant voltage at the receiving end of the line in Prob. 36-8, if the voltage at the generator is to be adjusted and held to a value to require equal kv-a. capacity of reactor at no load and at full load? Power-factor of generators at full load, 98.5 per cent. At what kv-a. input and power-factor must the synchronous reactor operate, (*a*) at full load of line? (*b*) at zero load of line?

Prob. 38-8. At half load (kilovolt-amperes) and 95 per cent power-factor on the line of Prob. 36, what reactive kv-a. must the synchronous reactor supply, if the voltage at the generator is to be adjusted and held to a value to require equal kv-a. capacity of reactor at no load and at full load?

Prob. 39–8. What would be the pull-out load per phase on the synchronous motor of Prob. 4–8, if the field strength were such that the counter e.m.f. became 2000 volts? Other conditions as in Prob. 4–8.

Prob. 40–8. Repeat Prob. 39–8 for a field strength producing 4400 volts induced e.m.f.

Prob. 41–8. The impressed voltage on the motor of Prob. 30–8 is increased to 3000 volts.

(*a*) What value must the field current have in order that the armature current shall not exceed the normal full-load value, when operating at unity-power factor?

(*b*) What will be the power intake of the motor in kilowatts and in kilovolt-amperes at full load?

Prob. 42–8. Construct circle diagram for the motor of Prob. 8–8.

Prob. 43–8. Determine the total motor effect of motor in Prob. 42–8 at rated intake, unity power-factor and rated full-load current in field coils.

CHAPTER IX

CONVERTERS AND RECTIFIERS

In many cities and towns electric systems were installed long before alternating-current machinery had been developed. The equipment of such places necessarily consists of direct-current machines. Even at the present time direct-current motors are better adapted to certain kinds of work — wherever adjustable speed is desired, for instance. Some commercial processes, such as electroplating and the charging of storage batteries, must be done by means of direct currents. If it is desired to take the power for any of these installations from an alternating-current system, it is necessary first to convert the alternating currents to direct currents.

For performing the conversion of large quantities of alternating-current power to direct-current power there are two machines in common use, — the motor-generator converter and the synchronous converter (sometimes called the rotary converter).

123. The Motor-generator Converter. This device consists of two separate machines, — an alternating-current motor and a direct-current generator, usually direct-connected to each other. The motor used for large sets, say over 100 kv-a., is generally the synchronous motor, on account of its constant speed and adjustable power-factor. This is connected to a compound direct-current generator.

In the motor-generator converter shown in Fig. 298, the motor (at the right) is a 10-pole 300-kv-a. synchronous motor operating on 2300 volts. The direct-current generator has a capacity of 275 kw., at 550 volts.

By varying the field current of the motor, the power-factor of the motor may be adjusted without affecting the voltage of the generator. By varying the field current of the generator, the voltage of the generator may be adjusted without disturbing the power-factor of the motor. For smaller sets, the power-factor of which is of much less importance, an induction motor is generally used because it can be started much more easily than the synchronous motor.

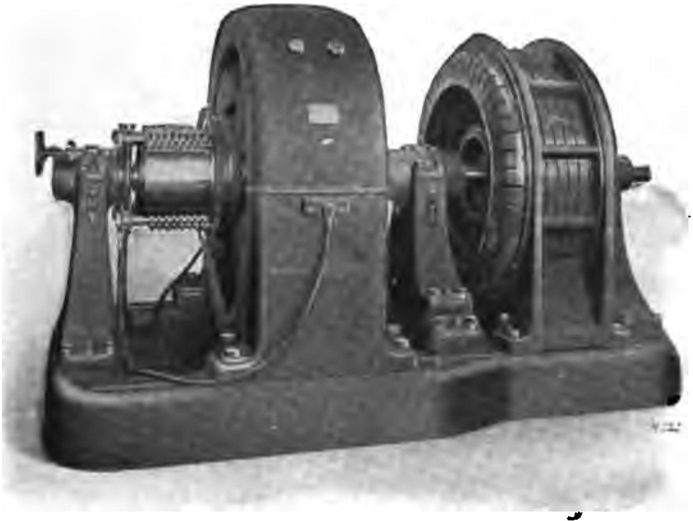

FIG. 298. Motor-generator converter. The motor on the right is a synchronous motor operating on 2300 volts. The direct-current generator on the left delivers 275 kw. at 550 volts. *The General Electric Co.*

124. The Synchronous Converter. Instead of using two separate machines for motor and generator, it is more general practice to make one machine, a synchronous motor of the revolving-armature type, perform the function of both motor and generator. The alternating current enters the closed winding of the armature through the collecting-rings and causes the rotor to turn in synchronism with the alternations of the current, as explained in Chapter VIII for the revolving field type. We thus have a revolving armature with alternating currents surging back and forth through the windings. We are already familiar * with the facts that the armature windings of a direct-current generator always carry such alternating currents, and that a commutator properly connected to the windings is all that is necessary to cause direct current to be delivered to a set of brushes. Thus we have only to tap the windings at the proper points and connect these taps to the proper segments of a commutator in order to deliver direct current to a set of brushes bearing on the commutator.

Thus the same armature fitted with collecting-rings and a commutator, revolving in a field separately excited from a source of direct-current supply, receives alternating current at the rings and delivers direct current at the commutator. Such a machine is called a **synchronous converter** or **a rotary converter** and is shown in Fig. 299. The simple scheme of winding and tapping an armature of such a converter has been indicated in Art. 72 of the First Course, but will be taken up in greater detail in this chapter. It is sufficient at present to know that the synchronous converter can be considered either as a synchronous motor having a revolving armature fitted with a commutator, or a direct-current generator having an armature fitted with collecting-rings.

125. The Synchronous Converter Versus the Motor-generator. The synchronous converter has the following advantages over a motor-generator converter:

First: The synchronous converter has a higher efficiency than a motor-generator of the same rating. The synchronous converter has but one field, thus the field loss is smaller. It has but one armature, thus the core loss is smaller. The currents in the armature windings are the resultants of the direct currents and the alternating currents which most of the time are opposing each other. Thus the I^2R loss in the armature of the synchronous converter is smaller than it would be in either a motor or a generator handling the same amount of power. Accordingly, it is much smaller than the I^2R loss in a motor-generator which has two armatures, each with a higher I^2R loss.

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* See First Course, Art. 65, 66, 72.

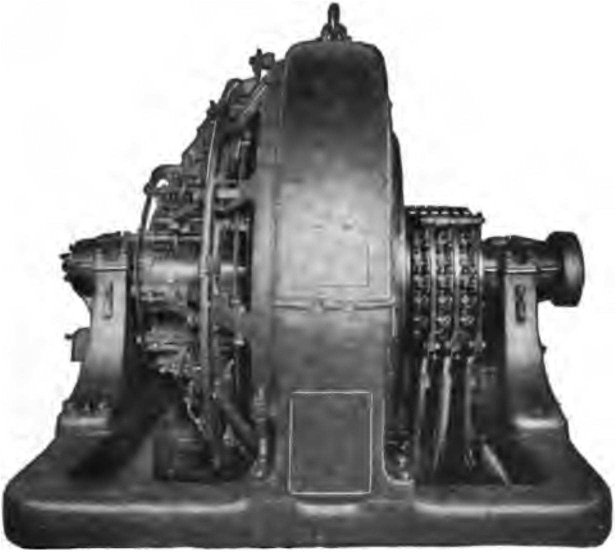

FIG. 299. Synchronous converter. Alternating current is received at the collecting-rings on the right and direct current is delivered at the brushes bearing on the commutator at the left. *The General Electric Co.*

These smaller losses all result in the synchronous converter having a higher efficiency than the motor-generator in the conversion of a given amount of power. The benefit of this higher efficiency of the synchronous converter, however, may be lost, because, in order to regulate the voltage on the direct-current side, it is frequently necessary to use auxiliary apparatus, the added losses of which lower the over-all efficiency.

Relative efficiencies of the two devices are given as follows in the American Handbook for Electrical Engineers.

	Per cent
Efficiency of converter	93
Efficiency of transformer (which generally has to be used with converter)	97
Efficiency of converter and transformer (0.97 × 0.93)	90
Efficiency of synchronous motor	93
Efficiency of direct-current generator	92
Efficiency of motor-generator converter (0.92×0.93)	85.5
Efficiency of motor-generator converter (if a transformer is necessary) (0.97 × 0.855)	83

Second: The synchronous converter weighs less and occupies less space per kilovolt-ampere capacity.

Third: The cost of a synchronous converter may fairly be taken to average about $11.00 per kilowatt for the larger units. The cost of a motor-generator is from 25 to 50 per cent greater than this, so that if no large amount of auxiliary apparatus is required, the synchronous converter is much cheaper than the motor-generator.

The motor-generator converter has the following advantages over the synchronous converter:

First: By using induction motors for the drive, motor-generator converters of small capacity (less than 100 kw.) start easily and operate satisfactorily. They are much used in central stations for the purpose of furnishing the direct current for the fields of the alternators. Small rotary converters are difficult to start and have the instability of small synchronous motors. Thus they do not operate well on lines where sudden changes in load occur.

Second: The voltage of the generator of a motor-generator can be controlled and regulated by compounding the field so that constant voltage may be maintained across the direct-current brushes, even though the alternating pressure varies through wide ranges, as it may at the end of long lines. The voltage across the direct-current brushes of a synchronous converter holds practically a fixed ratio to the alternating voltage across the rings. Thus transformers are usually necessary with synchronous converters in order to lower the alternating voltage so as to bring the direct voltage down to commercial value. Any change in the alternating voltage produces a corresponding change in the direct voltage. Changing the field excitation of a synchronous converter produces almost no change in the voltage across the direct-current brushes. It merely changes the angle of lead or lag of the alternating current taken by the machine, as explained in Chapter VIII in the case of synchronous motors. Thus wherever the direct-current voltage must be made to vary through wide ranges, a motor-generator converter is usually preferred.

There are various devices for regulating the direct voltage of a synchronous converter and maintaining it constant throughout a limited amount of change in the alternating voltage, but they all mean extra expense and many require expert attendants. They will be taken up in detail later.

Third: The synchronous motor-generator can be used to improve the power-factor of the load on a system. The direct-current generator may be run idle and the total kilovolt-ampere capacity of the synchronous motor may be used to supply only reactive power to the line, or part of its load may consist of effective power. A synchronous motor can in this way supply 70 per cent of its rated kilovolt-ampere capacity as reactive power, and still be able to supply at the same time another 70 per cent of real power (kilowatts) to the generator to be converted into direct-current power.

While it is true that the power-factor of the power taken by a synchronous converter can be varied just as that of a synchronous motor by means of changing the field current, still it will be shown later that the kilovolt-ampere capacity of the converter is lowered rapidly as its power-factor departs very much from unity, — a power-factor of 0.90 lowering the capacity to about 70 per cent of its unity power-factor rating. It is thus undesirable to depend upon the synchronous converter to supply much reactive power to the line load.

Example 1. A power station is supplying 7500 kw. at 80 per cent lagging power-factor, which puts the rated full load (in kilovolt-amperes) upon the generators.

(*a*) How many kilowatts can be added to the above load by the use of a synchronous motor if it is over-excited enough to produce unity power-factor in the load on the generators?

(*b*) At what power-factor must the synchronous motor be run?

(*c*) Assuming an efficiency of 90 per cent at this load, how many horse power mechanical load can the motor carry in addition to supplying the necessary reactive load?

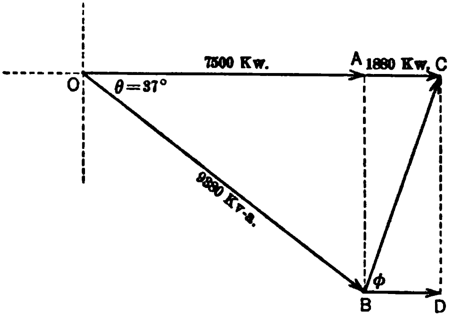

FIG. 300. The vector *OB* represents the load carried by the generators with no synchronous motor on the line. If a synchronous-motor load, represented by the vector *BC* is added, the generator must carry a load represented by the vector *OC*, which, however, is no greater than the vector *OB* which represented the original load, because the power-factor is improved.

Construct Fig. 300, drawing the vector *OA* to represent the 7500 kw., or effective power. Draw the vector *OB* at an angle of 37° (arc cos 0.80) to *OA* to represent the total apparent power which the generators must produce in order to deliver 7500 kw. at 80 per cent power-factor.

$$OB = \frac{7500}{0.80} = 9380\,\text{kv-a.}$$

If the generators were delivering 9380 kv-a. at unity power-factor they would be delivering 9380 kw. effective power. They would thus be delivering 9380 − 7500 or 1880 kw. more of real power.

Add vector AC, Fig. 300, to vector OA to represent this added effective power. Vector OC would then represent the same load at unity power-factor on the generators that vector OB does at 0.80 power-factor. Or OC represents the resultant load when a load of 9380 kv-a. at 0.80 lagging power-factor is combined with some other load to produce 9380 kv-a. at unity power-factor. This other load, which combined with the 9380 kv-a. at a lagging 0.80 power-factor will produce 9380 kw., must be represented by the vector BC, since OC is merely the resultant of OB and BC.

$$\begin{aligned} BC &= \sqrt{BA^2 + AC^2}. \\ BA &= 9380 \sin 37^\circ \\ &= 5630. \\ AC &= 1880. \\ BC &= \sqrt{5630^2 + 1880^2} \\ &= 5940 \text{ kv-a.} \end{aligned}$$

(*a*) Thus the synchronous motor may take 5940 kv-a. from the line and still not increase the load on the generators. But note that of this load, 5630 kv-a. (represented by the line AB) must be a leading reactive load, to counterbalance the 5630 kv-a. lagging reactive load already on the line. The effective part of the added load is 1880 kv-a. at unity power-factor. The power-factor of the synchronous motor load is thus

$$(b) \qquad \frac{\text{effective power}}{\text{apparent power}} = \frac{1880}{5940} = 0.316 \text{ leading.}$$

At 90 per cent efficiency, the motor could supply a mechanical load of,

$$(c) \qquad 0.90 \times 1880 \times \frac{1}{.746} = 2260 \text{ h.p.}$$

Thus by using an over-excited synchronous motor, 2260 more horse power can be taken from the line without adding to the load on the generators.

Prob. 1–9. If the motor of Example 1 were direct-connected to a direct-current generator having 92 per cent efficiency, how many kilowatts could it convert to direct-current power under the conditions of Example 1?

Prob. 2–9. (*a*) How many kilowatts, direct-current, could be delivered by the motor-generator converter of Prob. 1, if the motor is over-excited enough to raise the power-factor of the system of Example 1 to 0.90 lagging, and not add any kv-a. load to the generators? Generator efficiency, 92 per cent. Motor efficiency, 90 per cent. (*b*) At what power-factor must the synchronous motor operate in this case?

Prob. 3-9. (*a*) If a synchronous converter operating at 90 per cent leading power-factor were used to supply the same direct-current power as in Prob. 2, how many kilovolt-amperes would be added to the load on the generators? Assume an efficiency of 95 per cent for the converter. (*b*) At what power-factor will the generators now operate?

Prob. 4-9. It is fair to assume that the synchronous converter of Prob. 3, when operating at 90 per cent power-factor, can deliver not more than 70 per cent of its rated load (i.e., its load at unity power-factor). What must be the full-load rating (unity power-factor) of the converter of Prob. 3?

Prob. 5-9. How many revolutions per minute will the motor-generator converter of Fig. 298 make when operating on a 60-cycle system? The motor has 10 poles.

Prob. 6-9. How many poles would the motor of Prob. 5 have if it were intended to operate at 750 r.p.m. on a 25-cycle system?

126. Ratio of the Alternating E.M.F. to the Direct E.M.F. in a Synchronous Converter.

Single-phase. Consider the diagram of a simple single-phase synchronous converter shown in Fig. 301. The poles *N* and *S* are excited by direct current from an outside source. Alternating-current power is delivered to the collecting-rings *A* and *B* from an outside source. From these rings the lead wires *M* and *N* deliver the alternating current to the armature winding at the two tapping points *a* and *b*, situated 180 electrical degrees apart from each other. At the instant shown in Fig. 301 the alternating e.m.f. and current, at unity power-factor, would be at a maximum, and, considering the lead *M* positive at this instant, the armature current would cause the armature to rotate counter-clockwise as indicated. There would then be set up in the armature windings an induced e.m.f. as marked in the coils. Note carefully that whatever current the alternating e.m.f. may force through the armature windings at this instant must be forced against the induced e.m.f. and must then produce a motor effect tending to turn the armature in a counter-clockwise direction.

Note also that an alternating current at this instant can flow directly from the wires *M* and *N* through the neutral coils to the direct-current brushes without going through the armature. Any appliance attached to the brushes B_1 and B_2 would at this instant receive all its power directly from the alternating line.

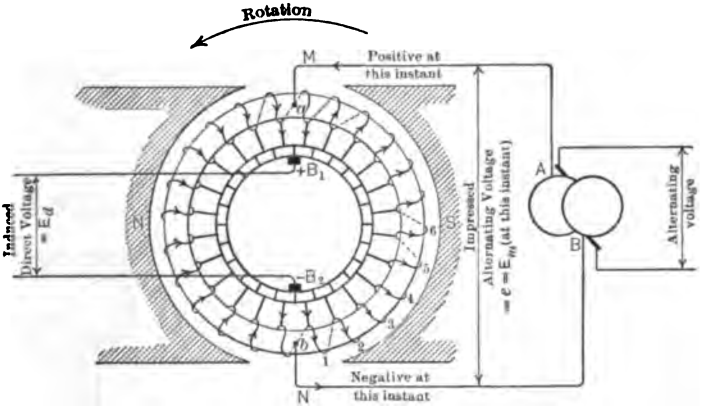

FIG. 301. Diagram of the armature windings and connections of a single-phase synchronous converter. At this instant the maximum value of the alternating e.m.f. is being delivered through the rings to the armature at the tapping points *a* and *b*, causing it to rotate as marked. The induced e.m.f. marked on the armature windings is being delivered to the brushes B_1 and B_2.

Let us assume, for the sake of simplicity, that the armature resistance and reactance are negligibly small, and that the losses and reactions can be neglected as is practically true when the converter is running idle. Under these conditions, the e.m.f. induced in the windings is practically equal to the impressed e.m.f. Now the induced e.m.f. is the e.m.f. which is delivered by the armature to the direct-current brushes, and the impressed e.m.f. is the maximum instantaneous value of the impressed alternating e.m.f.

When the armature has turned through 90°, the induced e.m.f. between the taps *a* and *b* becomes zero. But since the machine is in synchronism and in phase with the line voltage, the impressed e.m.f. between the rings *AB* at this instant has also become zero. There is thus no current in the wires *M* and *N*. The direct e.m.f. across the brushes, B_1 and B_2, however, will be the same as before.

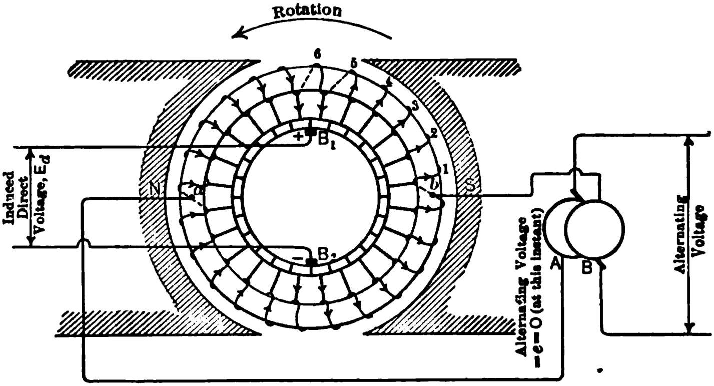

FIG. 302. The armature of Fig. 301 has turned through 90°. The impressed alternating voltage between the tapping points is zero at this instant. The induced e.m.f. between the brushes B_1 and B_2 is the same as in Fig. 301.

We thus have a direct e.m.f. at the brushes which is equal to the maximum value of the alternating e.m.f. at the rings, and we may write the equation

$$E_d = E_m,$$

in which

E_d = direct induced e.m.f.
E_m = maximum value of impressed alternating e.m.f.

Since the alternating e.m.f. is harmonic,

$$E = 0.707\,E_m.$$

Therefore

$$E = 0.707\ E_d;$$

where E = effective value of impressed alternating e.m.f.

E_d = direct induced e.m.f.

Thus for a single-phase converter, we may say that at no load the alternating voltage is 0.707 of the direct voltage.

Two-phase. In a two-phase (four-ring) converter, the second phase is tapped at points midway between the single-phase taps, the voltage across one phase being at a maximum when it is zero across the other phase. Thus the voltage across each phase of a two-phase tapping is the same as the voltage across a single-phase tapping. Accordingly, the alternating voltage in each phase of a two-phase converter, also, is 0.707 of the direct voltage.

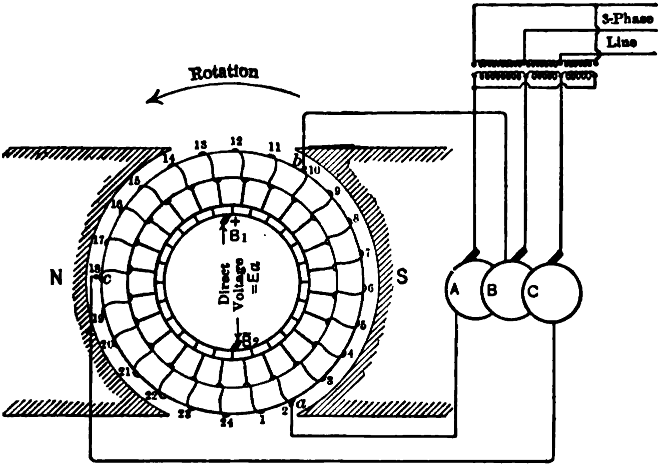

Fig. 303. Diagram of a three-phase three-ring converter. The impressed three-phase alternating voltage is brought to the three equidistant tapping points *abc* on the armature. The induced voltage is delivered as before to the brushes B_1 and B_2.

Three-phase. For the voltage relations in a three-phase converter, consider Fig. 303 and 304. The three lead wires are tapped into the armature windings at three equidistant points *a*, *b* and *c* (that is, with eight coils between any two taps), and brought out to their respective slip rings *A*, *B* and *C*. The alternating voltage impressed on the rings is thus applied to the armature at these three points and causes the armature to rotate as a synchronous motor. This induces an e.m.f. in the armature windings of practically a sine-wave form.*

Let the vector 1 in Fig. 304 represent the maximum e.m.f. induced in coil 1, and vector 2 the maximum e.m.f. induced in coil 2, etc. Note that the e.m.f.'s differ in phase with one another by $\frac{360°}{24}$ or 15°. Thus the e.m.f. in coil 2 is 15° ahead of the e.m.f. in coil 1, the e.m.f. of coil 3 is 15° ahead of the e.m.f. in coil 2, etc. The maximum induced e.m.f. through the twelve coils on one side or path of the armature, coils 1 to 12, and 24 to 13 inclusive, equals the vector sum of the maximum voltages across each coil. We have seen that this maximum e.m.f. across twelve coils is the voltage between the direct-current brushes.

The maximum induced e.m.f. between any two of the alternating-current taps equals the vector sum of the maximum e.m.f.'s in the eight coils composing the phase. Thus the vector *ab*, Fig. 304, is the vector sum of the maximum e.m.f.'s in coils 3 to 10, inclusive, and represents the maximum value of the induced e.m.f.'s across the phase *ab*, which is composed of these coils. This maximum value of the induced e.m.f. will take place when the phase *ab* is in the position shown in Fig. 303 and 304, because at that instant the instantaneous values of the voltages in the several coils composing the phase are nearest the maximum value. Note that in Fig. 303 at this instant the coils 3 to 10 are cutting lines of flux at the angle most nearly equal to a right angle and therefore at the greatest rate. Fig. 304 shows the same fact, in that the vectors 3 to 10 are so situated that their resultant *ab* makes a right angle with the horizontal,—this being the position of a vector corresponding to the maximum instantaneous value.

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* See First Course, Chap. VIII.

Assuming the armature resistance, reactance, etc., to be negligible, as they practically are at no load, the impressed alternating e.m.f. equals the induced alternating e.m.f. in each phase, at all instants. If the maximum induced e.m.f. *ab* occurs at the instant shown in Fig. 303 and 304, then the maximum impressed e.m.f. *ba* should occur at the same instant. Otherwise there would be instants when the induced e.m.f. was higher than the impressed e.m.f. and

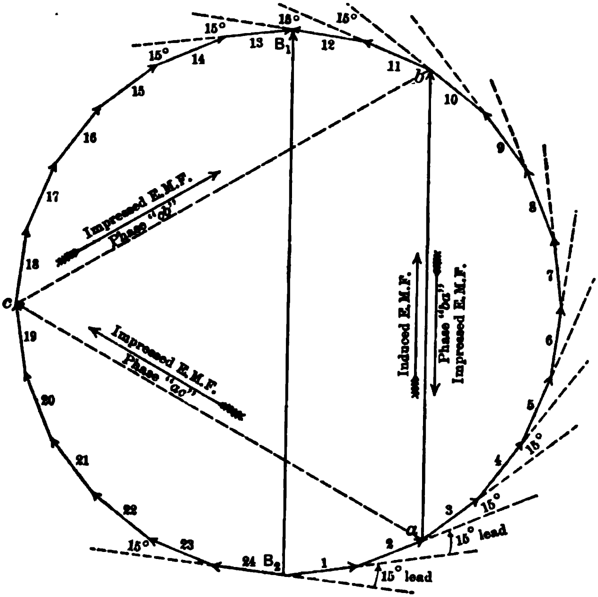

FIG. 304. Vector diagram of the induced and impressed voltage in the armature winding of a three-phase converter. The maximum induced voltage between the points *a*, *b* and *c* is equal and opposite to the impressed voltage at all times.

currents would be sent back from the armature windings into the source of supply. Thus the vector *ab* represents the maximum induced e.m.f. in the phase *ab* and also the maximum e.m.f. impressed on the phase *ba*, which of course is in the opposite direction to the induced e.m.f.

Accordingly, the ratio of the vector *ba* to the vector B_2B_1 equals the ratio of the maximum impressed alternating e.m.f. to the direct e.m.f., of a three-phase converter. This ratio is merely the ratio of a side of an equilateral triangle to the diameter of a circle circumscribed about the triangle. Thus in Fig. 305, the ratio

$$\frac{AB}{AD} = \frac{\text{maximum impressed alternating e.m.f., three-phase}}{\text{direct e.m.f.}}.$$

Since the angle ABD, Fig. 305, is a right angle,* and the angle ADB is 60°,

$$\frac{AB}{AD} = \sin 60° = 0.866.$$

Thus the maximum value of the alternating e.m.f. is 0.866 of the direct e.m.f.

But the effective value of alternating e.m.f. is the value generally used and this is 0.707 of the maximum value when the e.m.f. has a sine wave-form.

Therefore the effective value of the impressed alternating e.m.f. is 0.707×0.866 or 0.612 of the direct e.m.f.

The actual values of the ratio of alternating voltage to direct voltage varies but little from the ideal values of this article. Any variation may be due to any of the following causes:

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* An angle inscribed in a semicircle is a right angle.

Any angle at the circumference of a circle is equal to one-half of the angle at the center whose sides cut out the same arc on the circumference. The arc AB is 120°, therefore the angle ADB which subtends this arc is $\frac{120°}{2} = 60°$. Similarly the angle DBA is a right angle.

(*a*) The wave-form of the e.m.f. induced in the windings may not be that of a sine curve and thus the effective value would not be exactly 0.707 of the maximum. This depends largely upon what fraction of the pole pitch is covered by the pole-shoe. It has been found that if about 70 per cent of the pole pitch is covered by the pole-shoe, then the induced e.m.f. will have practically a sine wave-form.* As modern converters are designed very closely to this specification, the form of the e.m.f. is very close to that of a sine wave.

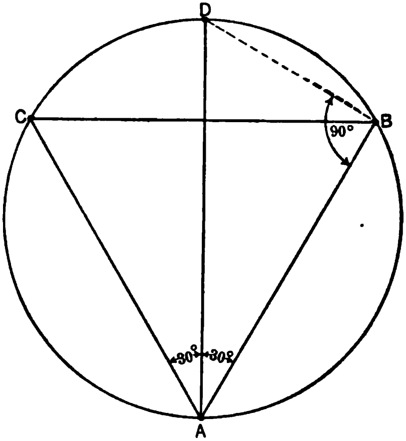

FIG. 305. The line AD represents the value of the voltage across the direct-current brushes. The lines AB, BC and CA represent the maximum value of the impressed alternating voltage $\frac{AC}{AD} = 0.866$.

The wave-form of the impressed e.m.f. also must be approximately a sine wave in order not to affect the ratio of voltages. It is especially important that the wave-form of impressed e.m.f. shall coincide exactly with that of the induced e.m.f., otherwise equalizing currents of considerable magnitude may circulate in the armature windings. See pages 85 and 87 of this volume.

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* See First Course, Chap. VIII.

(*b*) The direct-current brushes may not be set on the neutral axis and accordingly the direct voltage would not be the maximum induced voltage.

(*c*) The resistance of the armature must be overcome whenever the machine is delivering current. Voltage across the direct-current brushes would be a little lower than the induced e.m.f. when the machine is converting alternating current to direct current. The ratio of alternating voltage to direct voltage would thus be a little larger at full load than at no load. For the same reasons, if the machine is used to convert direct current to alternating current the ratio would be lower at full load. The converter is then said to be **inverted.** The following table is based on data published by the General Electric Co.

TABLE I

VOLTAGE RATIOS OF CONVERTERS

Ratio of Alternating E.M.F. to Direct E.M.F.

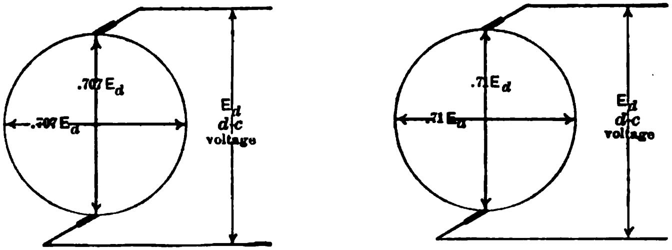
Number of phases.	Under ideal conditions.	Under actual conditions.	
		Full-load straight.	Full-load inverted.
One, two, six (diametral)	0.707	0.71	0.675
Three or six (double-delta)	0.612	0.62	0.580

Example 2. It is desired to maintain 550 volts between the direct-current brushes of a three-phase synchronous converter. What alternating e.m.f. must be applied between the rings?

For a three-ring converter,

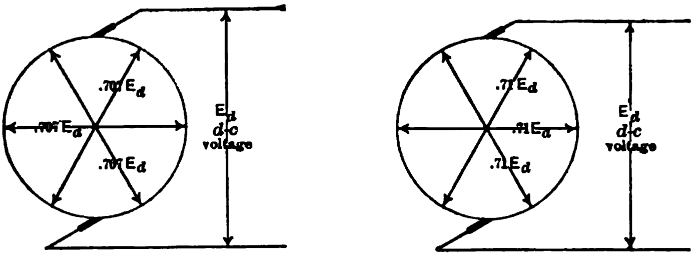
$$\begin{aligned} E &= 0.62\ E_d \\ &= 0.62 \times 550 \\ &= 341 \text{ volts (at full load).} \end{aligned}$$

The following figures will serve to help one remember the relation between the voltage at the direct-current brushes and the voltage at the alternating-current rings for the various schemes of tapping the armature and connecting the transformers.



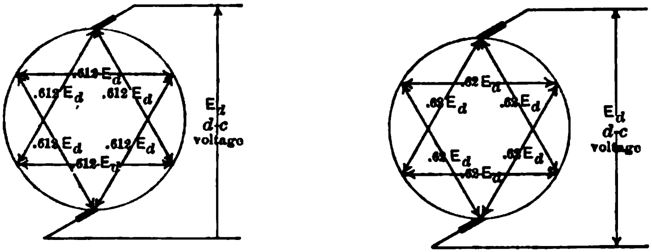
Single-phase and two-phase. (Ideal.)

Single-phase and two-phase. (Actual.)



Six-phase diametral. (Ideal.)

Six-phase diametral. (Actual.)



Six-phase double-delta. (Ideal.)

Six-phase double-delta. (Actual.)

Relation between the alternating voltage and the direct voltage of synchronous converters.

Prob. 7–9. If the e.m.f. of a 3-phase 2300-volt transmission line were applied directly to the rings of a 3-ring converter, what would be the voltage across the direct-current brushes?

Prob. 8–9. In Fig. 303 and 304 the voltage across each coil is harmonic and has a maximum value of 40 volts.

(*a*) What is the maximum e.m.f. induced in phase *ba*?

(*b*) What is the voltage across the direct-current brushes at no load?

(*c*) From (*a*) and (*b*), what is the ratio of maximum induced voltage in phase *ba* to the no-load voltage across the direct-current brushes?

Prob. 9–9. By addition of instantaneous coil e.m.f.'s, what is the instantaneous induced voltage across the phase *cb* at the instant shown in Fig. 303 and 304? Maximum e.m.f. in each coil = 40 volts.

Prob. 10–9. (*a*) From the value of the voltage impressed across phase *ba* as found in Prob. 8, compute the instantaneous impressed voltage across phase *cb* at the instant shown in Fig. 303 and 304.

(*b*) How does this value compare with the instantaneous induced e.m.f. of Prob. 9?

Prob. 11–9. If we regard a two-phase converter as a four-phase machine, what fraction of the direct voltage is the effective alternating e.m.f. between adjacent taps or rings, at no load? (Instead of stating the voltage of a two-phase alternator as that between adjacent rings, it is customary to state the voltage across one phase as the two-phase voltage.)

Prob. 12–9. Derive the ratio of the effective alternating e.m.f. impressed across adjacent rings of a six-phase converter to the e.m.f. across the direct-current brushes.

(It is customary to state as the alternating e.m.f. of a six-phase converter, either the voltage between rings which are tapped to points 180 electrical degrees apart on the armature, or the voltage between rings tapped to points which are 120 electrical degrees apart. This makes the e.m.f. ratios of a six-phase converter the same as that of a single-phase or that of a three-phase. See Art. 126 and 127.)

127. Ratio of Alternating Current per Ring to Direct Current.

Single-phase Converter. If we neglect the losses in the converter, which are actually very small, the total alternating-current power put in at the rings equals the total direct-current power taken out at the brushes.

For a **single-phase machine**, this may be written

$$(a) \qquad E_1 I_1 \cos\theta = E_d I_d,$$

in which

E_1 = effective value of single-phase alternating e.m.f.
I_1 = effective alternating current delivered to one ring.
E_d = voltage across direct-current brushes.
I_d = direct current delivered, total amperes.
$\cos\theta$ = power-factor.

Since a synchronous converter is designed to operate at unity power-factor, let us take $\cos\theta$ as 1.

We have seen that in a single-phase converter

$$E_1 = 0.707\, E_d.$$

Thus we may substitute these values in equation (*a*) and obtain

$$0.707\, E_d I_1 = E_d I_d,$$
$$I_1 = \frac{I_d}{0.707},$$
$$= 1.41\, I_d.$$

Due to the losses in the machine and the occasional lower power-factor, the ratio is a little greater than 1.41 in an actual converter.

Example 3. At 100 per cent efficiency and unity power-factor, what would be the alternating current per line wire and voltage between rings of a single-phase converter which was delivering 400 kw. direct-current power at 220 volts?

$$I_d = \frac{400{,}000}{220} = 1820 \text{ amperes.}$$
$$I_1 = 1.41\, I_d$$
$$= 1.41 \times 1820$$
$$= 2570 \text{ amperes.}$$
$$E_1 = 0.707 \times 220$$
$$= 155.6 \text{ volts.}$$

Example 4. If the converter in Example 3 is operated on 95 per cent power-factor at 90 per cent efficiency, what would be the alternating current and voltage when delivering 400 kw. direct current at 220 volts?

$$\begin{aligned}\text{Alternating-current power} &= E_1I_1 \cos\theta. \\ &= 0.95\, E_1I_1.\end{aligned}$$

$$\text{Direct-current power} = E_dI_d = 400{,}000 \text{ watts} = 0.90 \text{ of } 0.95\, E_1I_1.$$

$$\begin{aligned}E_1 &= 0.71\, E_d \\ &= 0.71 \times 220 \\ &= 156.2 \text{ volts.}\end{aligned}$$

Therefore, since

$$E_dI_d = 0.90 \times 0.95 \times 156.2 \times I_1 = 400{,}000,$$

$$\begin{aligned}I_1 &= \frac{400{,}000}{0.90 \times 0.95 \times 156.2} \\ &= 2995 \text{ amperes.}\end{aligned}$$

Three-phase Converter. The alternating-current power received at the rings of a three-phase converter running at unity power-factor, if we neglect the slight losses in the machine, would be equal to the direct-current power delivered at the direct-current brushes.

$$1.73\, E_3I_3 \cos\theta = E_dI_d,$$

where

E_3 = effective voltage between alternating-current rings.
I_3 = effective current delivered to each ring.

But $\cos\theta = 1.0$

and $E_3 = 0.612\, E_d.$

Thus we may write

$$1.73 \times 0.612\, E_dI_3 \times 1.0 = E_dI_d,$$

$$\begin{aligned}I_3 &= \frac{I_d}{1.73 \times 0.612} \\ &= 0.943\, I_d.\end{aligned}$$

As three-phase converters always operate at less than 100 per cent efficiency and sometimes at less than unity power-factor, the alternating current per ring is always somewhat greater than 0.943 of the direct current delivered by the machine.

Prob. 13–9. A three-phase three-ring converter is taking 1000 amperes per ring at a voltage of 700 between rings. If it were operating at 100 per cent efficiency and unity power-factor, what direct current would it be delivering, and at what voltage?

Prob. 14–9. What would be the direct current and voltage, if the converter of Prob. 13 were running with same alternating current and voltage at the rings, but were operating under practical conditions of 98 per cent power-factor and 92 per cent efficiency?

Prob. 15–9. Determine the ratio of the alternating current per ring of a six-ring converter to the direct current at the brushes for the ideal conditions of 100 per cent efficiency and unity power-factor.

Prob. 16–9. What would be the alternating current per ring and the alternating voltage between adjacent rings in a six-phase converter operating at unity power-factor and 93 per cent efficiency, if it were delivering 2000 amperes direct current at 550 volts?

128. The Operation of Six-ring Converters on Three-phase Systems.

On account of the resulting economy of design, two reasons for which are shown in §135, most converters are built with six collecting-rings. Since most transmission lines are three-wire systems, special transformer connections are employed to step down the three-phase power of the transmission line and change it into the six-phase power required by the converter. These special connections are discussed fully in §57 from the viewpoint of the transformers. In this article the discussion is from the converter viewpoint.

Diametral Connections. We may start with a three-ring converter tapped as in Fig. 303, and add three extra taps to it as in Fig. 306; that is, a tap at (a_1) diametrically opposite tap (a), one at (b_1) diametrically opposite tap (b) and one at (c_1) diametrically opposite tap (c). Each of these taps is now connected to a ring, necessitating six rings instead of the three of Fig. 303. This gives us a converter tapped at six points and supplied with six rings. We may now connect a transformer across each diametrical set of taps as in Fig. 306, where the secondary of the transformer A is connected across the tapping points a and a_1, the secondary of transformer B across the points b and b_1, and the secondary of transformer C across the points c and c_1. These connections are of course made by means of rings, which have been left out of Fig. 306 for the sake of clearness. The arrowheads in the transformers denote the positive direction internally

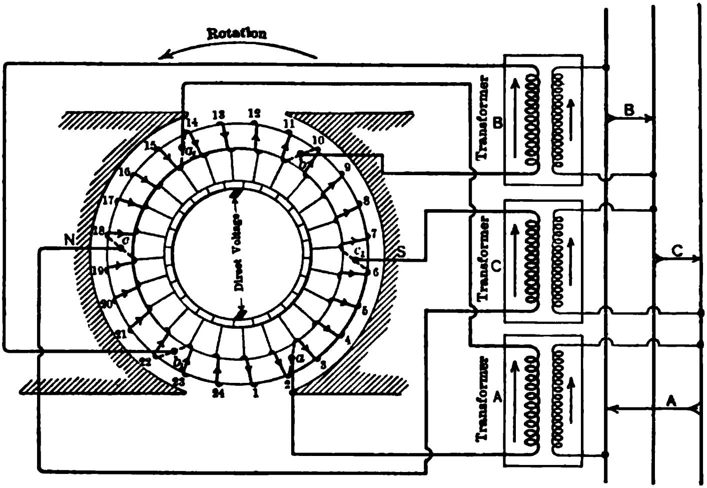


FIG. 306. Diagram showing the diametral method of connecting a six-phase converter to a three-phase line. Arrows on transformers and transmission lines show the positive direction through transformers and between wires of transmission line. Arrows on the armature conductors indicate instantaneous direction of induced e.m.f. in the windings. For the sake of clearness the collecting rings are omitted.

through the transformers, while the arrows on the armature windings denote the direction of the instantaneous e.m.f. induced in the windings for the position occupied by each inductor.

With the armature revolving counter-clockwise, note that the induced e.m.f. from b to b_1 reaches a maximum 120° ahead of the induced e.m.f. from a to a_1. This can be seen clearly from Fig. 307, in which the arrows represent the phase relations of e.m.f.'s between these points. Note that a–a_1 must move through 120° before it can be in the position of b–b_1, which in turn must move through 120° before it can be in the position of c–c_1.

Fig. 308 is a vector representation of the phase relations of induced e.m.f.'s between these points. These figures show that the induced e.m.f.'s between the diametral points b–b_1, a–a_1, c–c_1 are in the proper three-phase relation; thus, transformers of the proper voltage, frequency and phase sequence may be placed between these points, and the machine will then operate as a synchronous motor on a three-phase line. We merely have to be sure that the sequence of the positive direction through the secondaries of the transformers is the same as that of the induced e.m.f. in the armature, as shown in Fig. 307 and 308. Thus the vector diagram of the voltage relations in the three-phase line of Fig. 306 must be that of Fig. 309 in which phase *B* leads phase *A* by 120°, which in turn leads phase *C* by 120°. Then the secondary

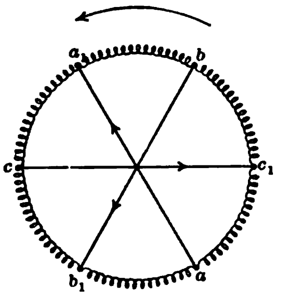


FIG. 307. Diagram showing the phase relations of the induced e.m.f.'s between diametral taps. b–b_1 is 120° ahead of a–a_1 which is 120° ahead c–c_1.

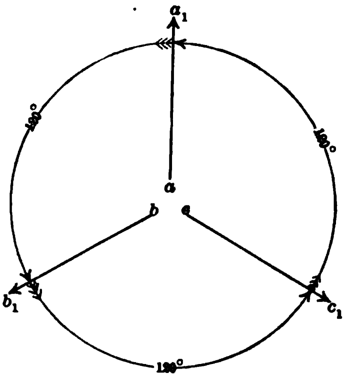


FIG. 308. Vector diagram of induced e.m.f.'s between diametral tapping points on the converters of Fig. 306. Vector b–b_1 is 120° ahead of vector a–a_1 which is 120° ahead of c–c_1.

terminals of the transformer can be connected diametrally to the armature taps, $b-b_1$, $a-a_1$, $c-c_1$, and the relation of the secondary e.m.f.'s also will be represented by Fig. 307 and 308. Care must be taken in making the transformer connections so that the sequence of phases in the line corresponds to the sequence of e.m.f.'s between the several pairs of points to which the transformers are attached. In other words, the transformer *B* can be connected to the points $b-b_1$, *A* to the points $a-a_1$, and *C* to the points $c-c_1$, only when the e.m.f.'s across the secondaries of transformers *B*, *A* and *C* have the same value and sequence as the induced e.m.f.'s between the points $b-b_1$, $a-a_1$, $c-c_1$.

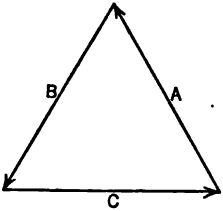


Fig. 309. Vector diagram showing the proper phase sequence of a three-phase line to be connected diametrally to converter of Fig. 306.

Note that this method of tapping the armature and connecting the transformers divides the armature into six phases and that six rings are required. Such a converter is called a six-phase or a six-ring converter. Fig. 310 shows the proper connections of the transformer secondary terminals to the rings. Note that the taps which are adjacent on the armature are brought out to adjacent rings. Each transformer must therefore be bridged across four rings; that is, when a terminal of a transformer is brought to a ring, the other terminal skips two rings and is brought to the fourth. Each transformer is connected to taps which are diametrally opposite to each other just as in a single-phase converter. We have seen that the maximum alternating voltage of the converter between these taps is exactly equal to the direct e.m.f. delivered by the direct-current brushes. Thus the maximum voltage of each transformer must equal the direct voltage of the converter. The effective voltage of each transformer must therefore be 0.707 of the voltage at the direct-current brushes at no load, or 0.71 at full load.

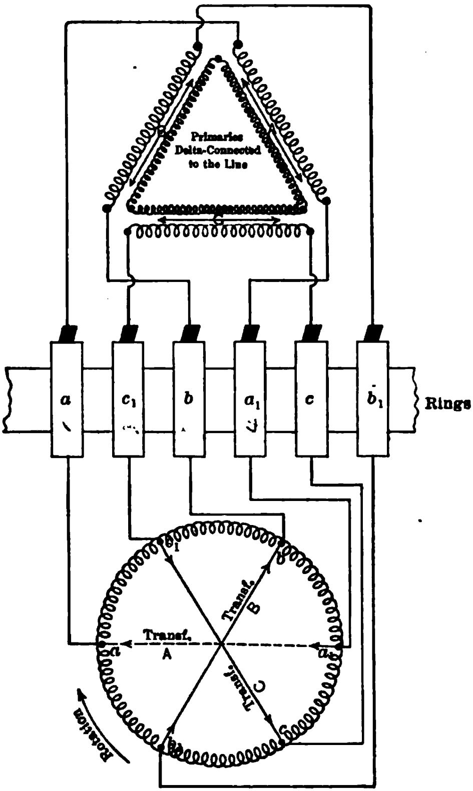


FIG. 310. Connection of tapping points of armature to rings and of transformers to rings for the six-phase diametral method. When one terminal of a transformer secondary has been brought to a ring, the other terminal skips two rings and is connected to the fourth.

Example 5. A 6-ring converter diametrally connected to three single-phase transformers at full load delivers 1000 kw. at 600 volts direct current. The converter has a full-load efficiency of 93 per cent, at unity power-factor. The line voltage is 11,000.

(a) What must be the full-load secondary voltage of each transformer?

(b) What must be the voltage ratio of each transformer if the primaries are delta-connected to the line?

(*c*) What current will each transformer terminal deliver under these conditions?

Solution: (*a*) The secondary voltage of the transformers must equal the effective value of the diametral induced alternating e.m.f.

$$\begin{aligned} E &= 0.71 \times 600 \\ &= 426 \text{ volts.} \end{aligned}$$

(*b*) The voltage ratio of each transformer must be

$$\frac{11{,}000}{426} = 25.8.$$

(*c*) At unity power-factor, each transformer must deliver

$$\tfrac{1}{3} \text{ of} \left(\frac{1000}{0.93}\right) \text{kw.} = 358.4 \text{ kw.}$$

At unity power-factor each transformer must deliver

$$\frac{358{,}400}{426} = 840 \text{ amp.}$$

Prob. 17–9. Draw a diagram similar to Fig. 310, except that the primaries of the transformers are to be star-connected to the transmission line.

Prob. 18–9. Solve Example 5 with the primaries of the transformers star-connected to the transmission line.

Prob. 19–9. Replace the 6-ring converter of Example 5 with the 3-ring converter and solve (*a*), (*b*) and (*c*) in Example 5. Connect the secondaries of the transformers in star and the primaries in delta.

Prob. 20–9. It is desired to deliver 500 kw. direct current at 220 volts from the brushes of a 6-ring converter. What must be the kv-a. capacity, voltage and voltage-ratio of the transformers if they are to be diametrally connected to the converter and delta-connected to a 3-phase 2300-volt line? Power-factor, 95 per cent, efficiency, 91 per cent.

Prob. 21–9. Solve Prob. 20 for a 3-ring converter and with the transformers star-connected to the line and delta-connected to the converter.

129. Double-delta Connection.

In the double-delta scheme of connection, advantage is taken of the common practice of constructing the secondary winding of a transformer in two equal coils. Leads are brought out from both terminals of each coil. Consequently one-half of the secondary e.m.f. can be impressed across part of the converter armature and the other half across another part. Since the e.m.f.'s of both coils are in phase with each other, the coils must not be connected across parts of the armature in which the induced e.m.f.'s differ in value or in phase. Otherwise, even though the armature were revolving in synchronism with the frequency of the line, one coil would continually be impressing on part of the armature, an e.m.f. which was out of phase with the induced e.m.f. of that part of the armature. Consequently, harmful local currents would flow through the transformer coil and part of the armature.

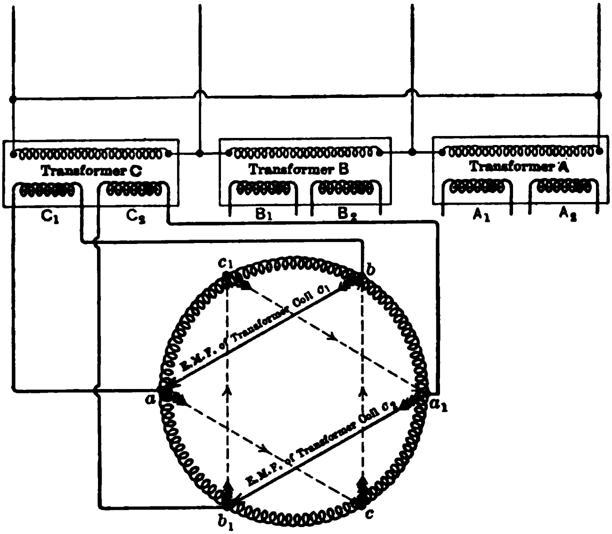


FIG. 311. Diagram showing the double-delta connection of the two secondary coils of transformer *C*. Coil C_2 is connected to the taps *b* and *a* in the direction reversed to that in which coil C_1 is connected to the taps a_1 and b_1.

Thus in Fig. 311, coil C_1, which is one-half of the secondary winding of transformer *C*, is impressed across the taps *a* and *b* of the converter armature, in such a way that the secondary e.m.f. tends to send a current from *b* to *a* through the armature windings.

Now coil C_2, the e.m.f. of which is exactly in phase with the e.m.f. of coil C_1, must be connected across that part of the armature in which the induced e.m.f. is the same in value and in phase as that part between the points *a* and *b*. But there is no part of the armature in which the induced e.m.f. is in phase and has the same value as the e.m.f. between the points *b* and *a*. However, the e.m.f. between the points b_1 and a_1 has the same value as that between *b* and *a*, but differs in phase with it by 180°, because when the induced e.m.f. tends to send a current from *a* to *b*, at that same instant it tends to send one from b_1 to a_1, which is in the reversed direction around the armature. We may then connect the coil C_2 in the reversed direction to the points b_1 and a_1, so that while coil C_1 tends to send a current from *a* to *b*, coil C_2 will tend to send a current from b_1 to a_1. Thus the e.m.f.'s of the two transformer coils tend to send currents in opposite directions around the armature. They are opposed in this tendency by the e.m.f. induced in the armature. This induced e.m.f. is at every instant opposite in phase to the impressed e.m.f.*

Similarly, coil B_1 of transformer *B* is put between the points *cb*, and coil B_2 is put in the reversed direction across the points b_1c_1. Coil A_1 of transformer *A* is connected between the points *ac*, and coil A_2, in the reversed direction between the points c_1a_1. Thus the coils A_1, B_1 and C_1 are connected to the armature in delta with the phase sequence $A_1B_1C_1$, and the coils $A_2B_2C_2$ are connected to the armature in the reversed direction in delta so that the phase sequence reads $C_2B_2A_2$. In this way we have a double-delta connection with the positive direction of the phases of one delta reversed with respect to the other. Fig. 312 shows the complete connections, minus the rings, for this method of connecting a six-ring converter to a three-phase line. Note that in connecting the coils $A_2B_2C_2$ in delta, it was necessary to make the connections such that the positive directions produced a phase sequence of $A_2B_2C_2$ in the transformers, while the phase sequence through the delta connection of the other coils was $C_1B_1A_1$. The voltage between any two taps in the same delta, as between *a* and *b*, *b* and *c*, *c* and *a*, a_1 and b_1, etc., is merely the voltage between the taps of a three-ring converter (see Art. 126, page 594).

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* If coil C_2 were not joined in the reversed direction, the e.m.f. of coil C_1 and C_2 would combine arithmetically to send a current through the parts of the armature a_1b and ab_1. The induced e.m.f. in these being at 90° to this impressed e.m.f. would offer no opposition to the current flow which would be excessive and produce little or no torque. Excess current would also flow in *ba* or b_1a_1.

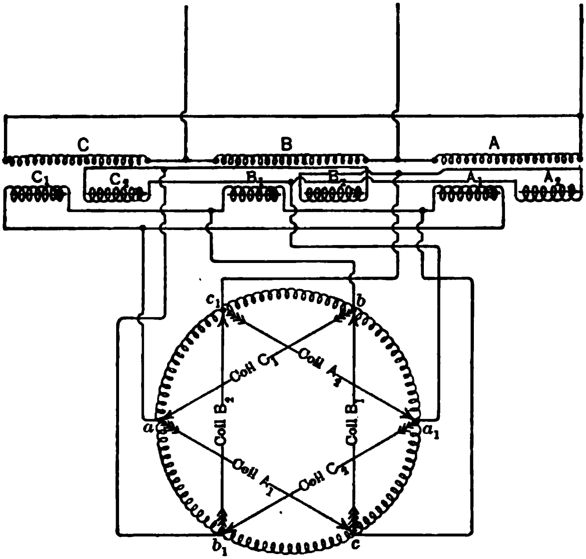

FIG. 312. Diagram showing the double-delta connection of three transformers to a six-ring converter. Note that the phase sequences of the two deltas are reversed with respect to each other. Thus the phase sequence of one delta is $A_1B_1C_1$ and of the other is $C_2B_2A_2$.

At full load the voltage between the taps on a three-ring converter is 0.62 of the voltage between direct-current brushes. Thus the voltage between taps *a* and *b*, that is, the voltage across the coil C_1, is 0.62 of the voltage between the direct-current brushes. Similarly, the voltage across each of the secondary coils is 0.62 of the voltage between the direct-current brushes.

The diametral and the double-delta are the two standard connections to six-phase converters. The "ring-connection," which is sometimes used, is described in Prob. 26–9. Of the two standard connections, the diametral is by far the more common, owing to its simplicity, certain advantages which it possesses in starting the converter on the alternating-current side, and the fact that the central points of the secondaries are all at the same neutral potential. This last advantage allows these points to be joined and a neutral wire to be brought out to form a three-wire system for the direct-current power.*

Example 6. A six-ring converter, connected double delta to three single-phase transformers, at full load delivers 1000 kw. at 600 volts. The converter has a full-load efficiency of 93 per cent while operating at unity power-factor. The line voltage is 11,000 volts. Compare with Example 5.

(*a*) What is the full-load secondary voltage of each transformer?

(*b*) The primaries are delta-connected to the line. What is the voltage ratio of each transformer?

(*c*) What current does each transformer coil deliver under these conditions?

Solution. (*a*) The voltage across a single coil of each transformer is 0.62 of the voltage between the direct-current brushes.

Voltage of one coil of transformer = $0.62 \times 600 = 372$ volts.

If the two coils of each secondary were joined in series, the secondary voltage would be $2 \times 372 = 744$ volts.

(*b*) The voltage ratio of each transformer with the two secondaries in series is,

$$\frac{11{,}000}{744} = 14.8.$$

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* For other important considerations in the choice of connections see "American Handbook for Electrical Engineers," under heading "Transformer Connections for Synchronous Converter."

(*c*) Input at unity power-factor and 93 per cent efficiency equals

$$\frac{1000}{0.93} = 1075 \text{ kw.}$$

Each coil delivers

$$\frac{1}{6} \text{ of } 1075 \text{ kw.} = 179.2 \text{ kw.}$$

Each coil must deliver

$$\frac{179{,}200}{372} = 482 \text{ amperes.}$$

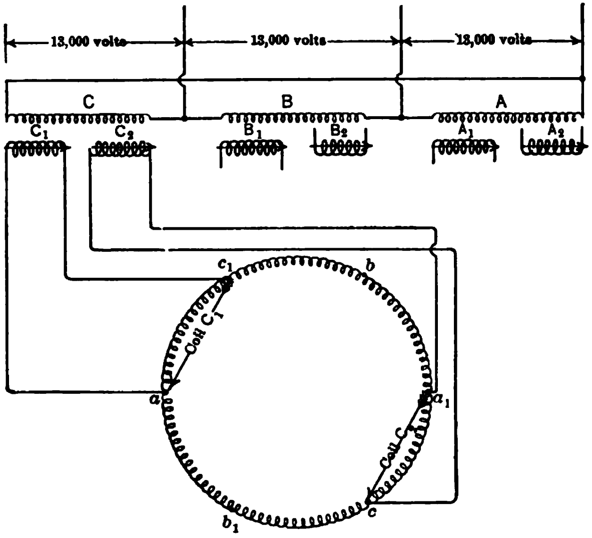

FIG. 313. Diagram showing the method of making the *ring* connection for operating a six-phase converter on a three-phase system.

Prob. 22–9. Construct a diagram similar to Fig. 312, except that the primaries of the transformers are to be joined in star to the main line.

Prob. 23–9. Solve Example 6 with the primaries connected in star to the line as in Prob. 22.

Prob. 24–9. What will be the voltage across the direct-current brushes of a six-ring converter double-delta connected to the secondaries of three transformers? The voltage ratio of the transformers is 30, with the secondaries in series, and the primaries are connected in delta to a three-phase 13,200-volt line.

Prob. 25–9. When the converter of Prob. 24 is operating at 0.95 power-factor and 90 per cent efficiency, it delivers 1000 amperes. What current will each transformer coil deliver under these conditions?

Prob. 26–9. Fig. 313 shows two coils of a transformer arranged for the ring-connection, which is a scheme sometimes used. Connect the remaining secondary coils to the converter.

Prob. 27–9. The full-load voltage between the direct-current brushes of Prob. 26 is 660 volts, when the converter is delivering 1000 amperes and running at unity power-factor. The alternating-current line voltage = 13,000 volts. The transformer primaries are delta-connected. Find:

(*a*) The voltage across each secondary coil.
(*b*) The voltage ratio of the transformers.
(*c*) Current delivered by each transformer coil, on a basis of 92 per cent efficiency for the converter.

130. Motor and Generator Actions in a Converter. At a given instant, current flows from an alternating-current source into the converter armature at tap *a* and out at tap *b*, Fig. 314. As the current enters at *a*, it divides, one part flowing through the upper half of the armature winding to tap *b* and the other part flowing down through the lower half of the armature winding to the tap *b*. While the current which flows through the upper half of the armature is passing from tap *a* to brush B_1, it is flowing in the same direction as the induced e.m.f. of that part of the circuit. The current from tap *a* to brush B_1, therefore, produces a generator action and opposes rotation of the armature. But as some of this current also flows on from B_1 to tap *b*, it goes in the direction opposite to the induced e.m.f. in this part of the armature and thus has a motor effect, assisting rotation of the armature. Similarly, that part of the current which enters the lower half of the armature is in the opposite direction to the induced e.m.f. as it flows from the tap *a* to the brush B_2 and therefore produces a motor action. But, some of it in flowing on from the brush B_2 to the tap *b* is going in the same direction as the induced e.m.f. and produces a generator

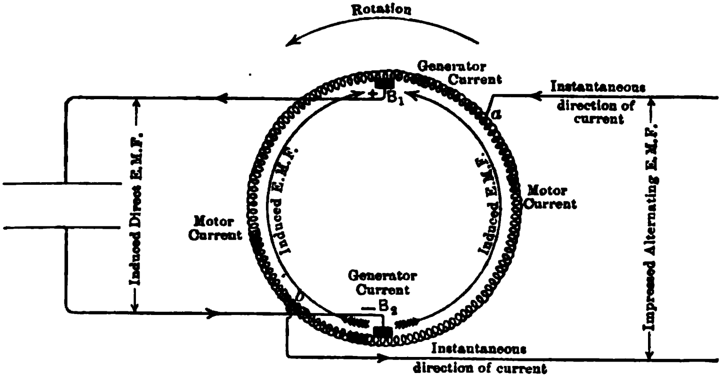

Fig. 314. The alternating current entering at *a* divides, part flowing up to B_1 in the direction of the induced voltage and thus producing a generator effect. Some of this part of the current flows on from B_1 to point *b* against the induced voltage, thus producing motor action. Similarly the other part of the current entering at *a* produces both motor and generator action.

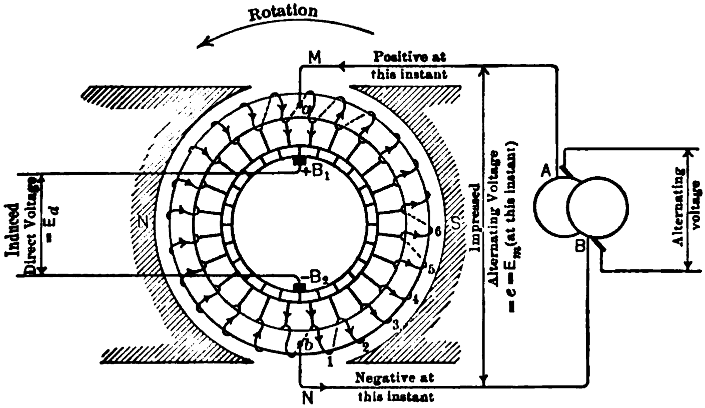

Fig. 315. The alternating current at this instant enters at point *a* and flows to point *b* in a direction everywhere opposing the induced e.m.f. The value of the direct current flowing through the armature in the direction is very small at this instant, since the direct-current brushes can take the current directly from the rings. Thus the motor effect is larger than the generator effect.

action. There is, therefore, both motor and generator action going on in various parts of the armature at this instant.

At a later instant, as shown in Fig. 315, the taps *a* and *b* will be directly under the brushes B_1 and B_2 respectively. Then, if the power-factor is unity, the alternating current supplied will be at its maximum value, and throughout the entire length of both branches of the armature circuit it must flow in the opposite direction to the induced e.m.f. Thus at this instant the total effect of the alternating current is to drive the machine as a motor, and the armature would tend to forge ahead because the motor effect exceeds the generator effect.

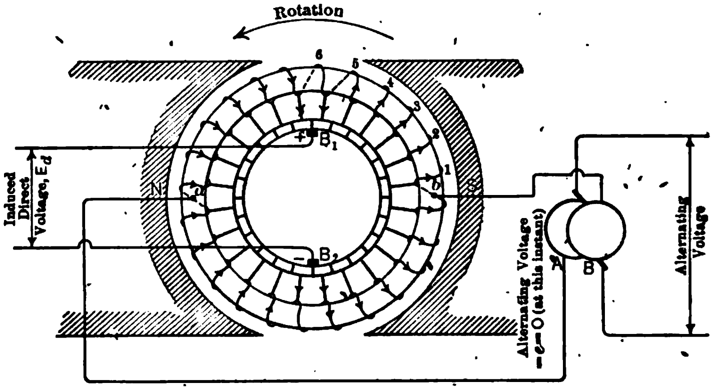
Rotation
6
5
4
3
2
1
+B1
−B2
N
S
a
b
A
B
Induced Direct Voltage, E_d
Alternating Voltage =e=O (at this instant)
Alternating Voltage

FIG. 316. At this instant there is no alternating current being delivered to the armature windings if the power-factor is unity. The generator action must greatly exceed the motor action and tend to lessen the speed of rotation.

At a quarter of a period later, as in Fig. 316, the voltage between taps *a* and *b* is zero, and the current received from the alternating-current line (at unity power-factor) is zero, and thus no motor or generator effect is supplied from the alternating-current line. At this instant, if the converter is supplying direct current, it tends to slow down, on account of the excess of generator action within the armature. For this reason, a single-phase converter is very unstable and tends to hunt somewhat after the fashion of a single-phase synchronous motor but to a greater degree. Synchronous converters are therefore usually polyphase, in order to receive and deliver a steady flow of power.

Of course the current flowing in any armature coil will be neither the alternating current supplied by the rings, nor the direct current supplied to the brushes, but a combination of the two. Since the amount of current in the coils determines the heating of the armature, and this in turn determines the kilovolt-ampere rating of the converter, it is essential that we investigate the heat produced in certain typical armature coils.

RELATIVE CAPACITY OF TWO-, THREE- AND SIX-RING CONVERTER

131. Capacity of a Two-ring Converter. It is customary to compare the capacity of a machine used as a converter with the capacity which it would have if used as a direct-current generator under the same conditions, such as speed and voltage.

The capacity of most electrical machines is limited only by the temperature to which the various parts of the machine may be allowed to rise with safety. In the converter, the heating effect of the current upon certain armature coils limits the capacity. Accordingly, in order to compare the capacity of a machine as a converter of a particular type with its capacity as a direct-current generator, it is only necessary to compare the respective loads which will produce the same effective currents or the same rate of heating in the hottest armature coils.

We will consider first the case of a two-ring converter as illustrated by Fig. 315, which is a copy of Fig. 301. Let us assume that as a two-ring converter, it is capable of delivering 1000 amperes direct current at 120 volts.

The voltage between the alternating-current rings then will be 0.707 × 120 or 84.8 volts. In order to simplify the computation, we will assume at first that there are no losses.

At unity power-factor, the alternating current received by the rings would be

$$I = \frac{1000 \times 120}{84.8}$$
$$= 1414 \text{ amp.}$$

We will now consider the current flowing under these conditions in typical coils of the armature.

Coil Midway between Taps. Consider first the current in a coil which is midway between the alternating-current taps *a* and *b*. This would be such a coil as No. 6 of Fig. 315, which represents this converter running at synchronous speed and delivering 1000 amperes from the direct-current brush B_1. Of this 1000 amperes direct current, each of the two paths of the armature contributes one-half or 500 amperes. Thus, while coil No. 6 was passing the *S*-pole it would be carrying a current of 500 amperes flowing in one direction, and while it was passing the *N*-pole it would be carrying 500 amperes flowing in the opposite direction. Therefore at all instants except at those two when the coil No. 6 was being short-circuited by the brushes it would be carrying 500 amperes. At these two instants, as shown in Fig. 316, the current in the coil drops to zero and is started in the reversed direction. Fig. 317 represents the current in coil No. 6 throughout a complete cycle. Note that at the instants the coil passes under the brushes B_1 and B_2 the current drops to zero and reverses its direction. At all other times it has a value of 500 amperes.

But in addition to this 500-ampere current which is delivered to the direct-current brushes, there is also the current which the coil receives from the alternating-current rings.

We have seen that when the machine is running at synchronous speed, the impressed alternating voltage is greatest whenever the alternating-current taps are under the brushes.

At unity power-factor the alternating current would also be greatest at this instant, since the current would be in phase with the pressure. Thus the alternating current in coil No. 6 would be greatest at the instant shown in Fig. 315. Note, however, that this alternating current which is driving the machine would be in the opposite direction to the induced e.m.f. and to the direct current which the coil is delivering. When the coil has passed through a quarter of a cycle, as shown in Fig. 316, the alternating voltage and current between the tapping points a and b has become zero. The alternating current in coil No. 6 would therefore be zero at this instant. Notice that just at this instant coil No. 6 is passing beneath brush B_1. As the armature continued to revolve, the alternating current in coil No. 6 would rise to a maximum in the opposite direction, and gradually decrease to zero again as the coil came under brush B_2. Note that the alternating current in coil No. 6 is zero at the same instant that the direct current is zero, and that at all other times the alternating current is opposite in direction to the direct current in the coil.

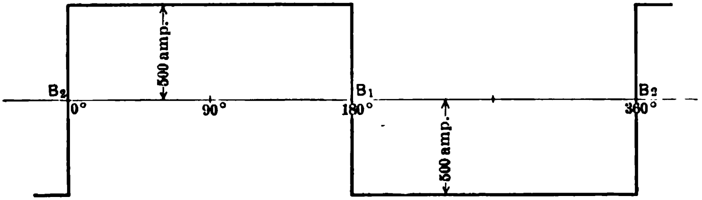

Fig. 317. The direct current in coil No. 6 of Fig. 315. It is zero as the coil passes under brush B_2, and then becomes 500 amperes and remains 500 amperes until the coil passes under brush B_1. Here the current again becomes zero, but rises to 500 amperes in the opposite direction as soon as the coil leaves the brush. It maintains this current until the coil again comes under brush B_2, where the current again becomes zero.

Fig. 318 shows the alternating current flowing in coil No. 6 throughout one cycle. Assuming no losses in the machine, we have seen that the effective value of the alternating current is 1414 amperes. Since this flows in two parallel paths in the armature, the effective current in each path and therefore in coil No. 6 equals $\frac{1414}{2} = 707$ amperes. The maximum value of this current equals $\frac{707}{0.707}$ or 1000 amperes. Thus, in Fig. 318, the alternating current in coil No. 6 is zero while the coil is under brush B_2, rises to 1000 amperes in a direction opposite to the direct current as the coil passes the S-pole and falls to zero when it passes under brush B_1. Here the current reverses, and then gradually increases in the opposite direction, until it becomes 1000 amperes as the coil passes the center of the N-pole, dropping to zero again as the coil comes under brush B_2.

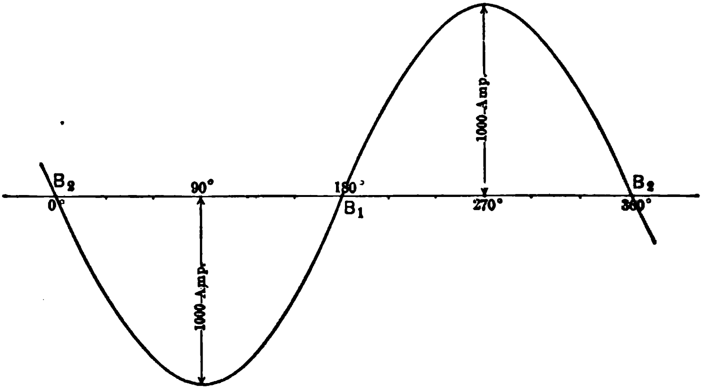

FIG. 318. When the power-factor is unity, the alternating current of sine wave-form flowing through coil No. 6 of Fig. 315, the midcoil, is zero as the coil passes under brush B_2, rises to a maximum of 1000 amperes as the coil passes the center of the S pole, and then decreases until it is zero again as the coil reaches brush B_1. The current now reverses and grows to a maximum value of 1000 amperes in the opposite direction as the coil passes the center of the N-pole and decreases to zero when the coil passes under brush B_2 again.

The resultant of this alternating current and the direct current must be the current flowing in the coil No. 6. Fig.

319 shows the peculiar shape that this resultant current curve will have. It is merely the direct-current curve of Fig. 317 with the alternating-current curve of Fig. 318 algebraically

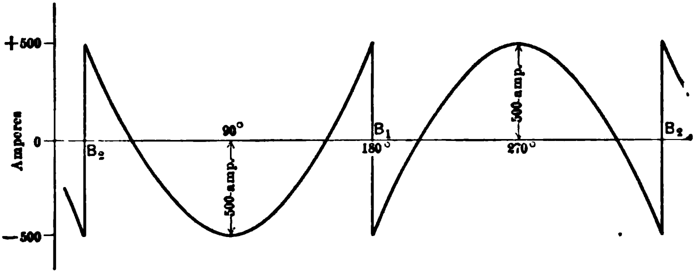

FIG. 319. The resultant current flowing in the midcoil of a two-ring converter is merely the sum of the direct current of Fig. 317 and the alternating current of Fig. 318.

added to it. The value of the resultant current at any instant can be found from the equation for this curve.

$$i_6 = I_d - I_m \sin \phi,$$

where i_6 = the instantaneous resultant current.
I_d = the direct current = 500 amperes.
I_m = the maximum value of the alternating current = 1000 amperes.
ϕ = time angle through which alternating current has passed, counted from instant when coil passes brush B_2.

Then $i_6 = 500 - 1000 \sin \phi.$

The heating of this armature coil would, of course, be proportional to the average of the squares of all the values of this resultant current, as shown by the curve in Fig. 320. This average value of the squares of the instantaneous values of the resultant current may be found by means of a planimeter if the curve is drawn to scale, and is equal to 113,000. The heating effect of the direct current alone would be proportional to $500^2 = 250{,}000$. Thus coil No. 6 would be heated $\frac{113,000}{250,000}$ or 0.452 as rapidly when the machine is used as a converter, as it would be when the machine is used as a direct-current generator, delivering the same load.

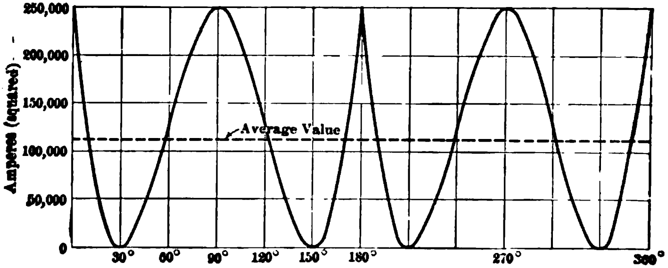

FIG. 320. The square of the curve of the resultant current in the mid-coil. The heating of the coil will be proportional to the average height of this curve.

132. Heating of Coil at Tapping Point.

If we examine the current conditions in a coil at one of the tapping points as, for instance, coil No. 1 of Fig. 315 and 316, we see that the alternating current supplied to it is greatest (1000 amperes) just as this coil comes under the brushes. The direct current at this instant is just changing its direction and is therefore zero, as Fig. 315 shows. A quarter of a revolution later, when coil No. 1 has reached the position shown in Fig. 316, the alternating current has become zero, though the direct current is 500 amperes.

Thus the direct and alternating currents in coil No. 1 can be said to be 90° out of phase and can be represented by the dash-line curves of Fig. 321. Note that in this coil the alternating current flows against the direct current one-half the time and flows with it the other half. This produces the peculiar resultant curve shown by the heavy line of Fig. 321. Note that it rises at instants to 1500 amperes.

If we square the instantaneous values of the current on this curve, we find that their average value is 750,000. The square of the direct current in this coil is 500^2 or 250,000.

This coil is therefore being heated $\frac{750,000}{250,000}$ or 3 times as much as though the machine were merely a direct-current generator of the same capacity.

Treating coils No. 2, No. 3, No. 4 and No. 5 in the same way and averaging their relative heating effects, together with those of coils No. 6 and No. 1, we find that the total or average heating effect of a machine used as a two-ring converter is approximately 1.38 times as much as when used as a direct-current generator delivering the same current and voltage. As a direct-current generator, it can thus deliver $\sqrt{1.38}$ or 1.17 times the current it can deliver as a two-ring

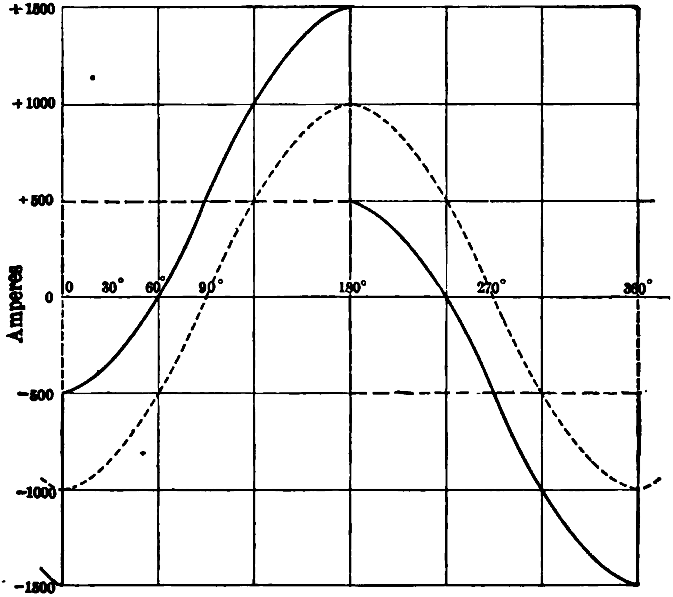

Fig. 321. The curve of resultant current in coil No. 1 at the tapping point of a two-ring converter is the heavy line and is the sum of the direct-current curve and the alternating-current curve shown in dash lines.

converter.* Or as a converter it can deliver only $\frac{1}{1.17}$ or 0.85 as much current it could be permitted to deliver if operated as an engine-driven direct-current generator.

That is to say, neglecting all losses, a two-ring synchronous converter has 0.85 as great power capacity as it would have if used as a direct-current generator of the same terminal voltage.

133. Capacity of a Three-ring Converter. We can investigate the three-ring converter in a similar manner. Referring to Fig. 303, we see that coil No. 6 is the midcoil between taps a and b. The applied alternating e.m.f. in this phase is at its maximum when coil No. 6 is in the position of Fig. 303, and since it furnishes the motor current, the impressed e.m.f. must be in the opposite direction to the induced e.m.f. The alternating current in coil No. 6 therefore opposes the direct current and reaches its maximum 90 electrical degrees (in this case 90 space degrees also) after the coil passes under the brush B_2. One-quarter period later, the alternating current in the coil will become zero, but during these 90 degrees the coil will have passed from its position in Fig. 303 to a position directly above brush B_1 and thus the direct current will also become zero at this instant. The alternating current and the direct current are thus always opposed to each other. Fig. 322 shows these two currents by the dash-line curves. The full-line curve represents the resultant current in coil No. 6 when the armature is tapped for three rings. Note that it has the same shape as the resultant current in the same coil when the armature was tapped for two rings as shown in Fig. 319. The loops and consequently the effective value are much smaller, however, due to the smaller current taken at each ring for the same direct-current output.

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* In other words, the average effective value of the resultant currents in all the coils is 1.17 as great as the direct current which the converter is delivering.

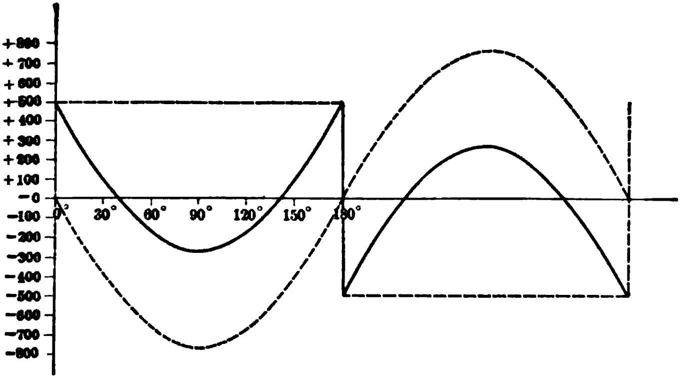

FIG. 322. The heavy line represents the resulting current in the midcoil of a three-phase converter. This current is made up of curves of the direct current and the alternating current shown as dotted lines. Compare with Fig. 319 which shows the current in the midcoil of a single-phase converter.

The output and the input (assuming no losses and unity power-factor) each equals

$$120 \times 1000 = 120{,}000 \text{ watts.}$$

Voltage between rings A and B or taps a and b

$$= 120 \times 0.612$$
$$= 73.4 \text{ volts.}$$

Since power in a three-phase circuit $= 1.73\, EI \cos\theta$, and $\cos\theta$ here equals 1.0,

$$\text{current per ring} = \frac{120{,}000}{1.73 \times 73.4}$$
$$= 943 \text{ amperes.}$$

The current in each phase of the delta-connected armature of Fig. 303

$$= \frac{943}{1.73}$$
$$= 545 \text{ amperes.}$$

The maximum value of the alternating current in coil No. 6 thus

$$= \frac{545}{0.707}$$
$$= 770 \text{ amperes.}$$

This is the value used for the alternating-current curve of Fig. 322. The direct current, of course, has the same value of 500 amperes as in the previous curves.

To find the heating effect of the resultant current in coil No. 6 we merely have to square the curve of Fig. 322 and find the average value as we did for the curve of squares in Fig. 320. This average value amounts to 56,000. The square of the direct current = 250,000. Thus coil No. 6 in this case develops heat only $\frac{56,000}{250,000}$ or 0.224 as fast as it would if the machine were used as a direct-current generator for the same output.

134. Heating of Coil at Tapping Point. Three-phase Converter. In the same way we can investigate coil No. 2 of Fig. 303, which is at a tapping point. The alternating current is at a maximum in this coil as it is situated in Fig. 303. Thus the maximum occurs 30 electrical degrees after it has passed under brush B_2.

The curves of the alternating current and of the direct current in coil No. 2 are shown by dash lines in Fig. 323, together with the resultant current curve in full line. Note that it has the same general shape of the resultant current in an armature coil at the tapping point of a two-ring converter, as shown in Fig. 321, except that its corresponding values are smaller, on account of the smaller alternating current in the coils of a three-ring converter.

To find the heating effect of this resultant current we square the instantaneous values of this current and average them as in previous cases.

This average of the square of the currents = 301,000. This coil therefore heats $\frac{301,000}{250,000}$ = or 1.20 times as fast as when the machine is used as a direct-current generator.

Treating the rest of the coils of phase *ab*, Fig. 303, in the same way, and averaging the relative heating effects of all the coils of this phase, we find that the average heating effect of a machine used as a three-ring converter is approximately 0.57 of what it would be if used as a direct-current generator delivering the same current. As a direct-current generator it can thus deliver $\sqrt{0.57}$ or 0.756 of the current it can deliver as a three-ring converter, since the heating is proportional to the square of the current.*

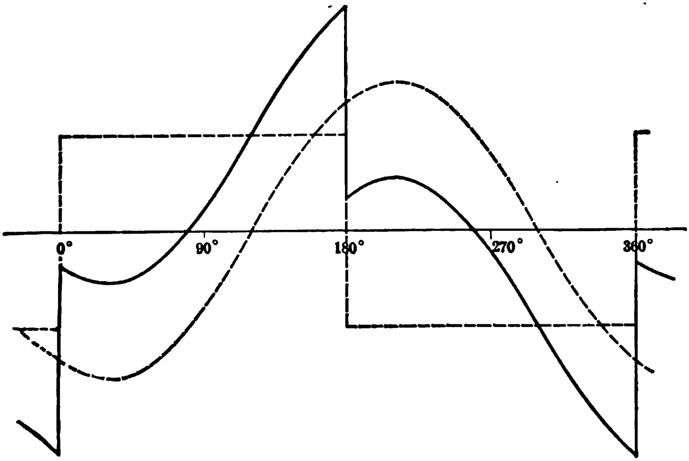

FIG. 323. The current curve in the tap coil of a three-ring three-phase converter. Compare with the current in the tap coil of a two-ring converter as shown in Fig. 321.

Therefore, as a three-ring converter, it can deliver $\frac{1}{0.756}$ or 1.32 times the current it can deliver as a direct-current generator.

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* In other words, the effective value of the resultant current in the armature is 0.756 of the direct current delivered from the terminals.

135. Capacity of a Six-ring Converter. Using the same methods, we find that if six rings are used, the machine can deliver 1.97 times the power that it can deliver as a direct-current generator.

Thus a machine fitted up as a six-ring converter has $\frac{1.97}{0.85}$ or 2.32 times the capacity that it would have as a two-ring converter, and $\frac{1.97}{1.32}$ or 1.49 times what it would have as a three-ring converter. Accordingly, if the converter in the example which we have been using for our investigation can deliver 1000 amperes at 120 volts when fitted with two rings, it could deliver $\frac{1.32}{0.85}$ times as much current, or 1550 amperes when fitted with three rings, and 2320 amperes when fitted with six rings, — as far as the heating of the armature coils is concerned. Of course a much larger commutator and more brush area would be required for the increased current output. Converters of 500-kw. capacity or over are commonly built with six rings; the smaller sizes, with three.

The effect of lower power-factor in decreasing the capacity of a converter can be determined by merely repeating the above curves and computations for the same machine operated at any power-factor less than unity. It will be seen from the results of Prob. 28–9 that when the power-factor on the converter of Fig. 303 is reduced to 87 per cent and the machine is compelled to deliver the same direct-current load of 1000 amperes at 120 volts, it greatly increases the effective current. A very slight decrease in the power-factor will cause a large increase in the current and I^2R loss in the conductors near one side of the tapping points, and a decrease in the current and I^2R loss in the corresponding conductors on the other side of the tapping points.

The following table, taken from the "American Handbook for Electrical Engineers," summarizes the current, voltage and output relations between the various converters and a direct-current generator.* The last two lines show the relative capacity of converters with different numbers of rings with the same total or average armature heating, when run at unity power-factor and at 87 per cent power-factor.

TABLE II

VOLTAGE, CURRENT AND OUTPUT RATIOS

	D-C. generator.	Converters.					
		2-ring.	3-ring.	4-ring.	6-ring diametral.	6-ring double delta.	12-ring.
D-C. volts........	100	100	100	100	100	100	100
A-C. volts between lines..........	....	71	61.2	71	71	61.2	71
A-C. volts between rings..........	....	71	61.2	50	35	35	18
D-C. amperes.....	100	100	100	100	100	100	100
A-C. amp. in line .	....	141	94	71	47	47	24
A-C. amp. in winding............	....	71	55	50	47	47	45
Relative I^2R loss.	100	137	55	37	26	26	20
Relative output, for same heating, unity power-factor	100	85	134	165	197	197	224
87% power-factor.	....	....	99	115	129	129	135

Prob. 28–9. Assume that the three-ring converter of Fig. 303 is delivering 1000 amperes at 120 volts, but that it is running at 87 per cent power-factor. Plot current curves for the middle coil No. 6 similar to those in Fig. 322, and the curve of squares similar to Fig. 320. Determine the average heating of coil No. 6. Compare with heating of coil when machine is used as a direct-current generator delivering same current.

Prob. 29–9. Repeat Prob. 28, for the tap coils No. 2 and No. 10, Fig. 303.

* The expression

$$\frac{1}{\sqrt{\dfrac{8}{n^2 \sin^2\left(\dfrac{180^\circ}{n}\right)} - 0.62}}$$

is the ratio of the capacity of a machine used as a converter at unity power-factor to the capacity the same machine used as a direct-current generator. The symbol n stands for the number of rings. It gives values slightly below those in the table.

Prob. 30–9. Repeat Prob. 28 for coils No. 4 and No. 5, and No. 3 and No. 11, Fig. 303.

Prob. 31–9. From data obtained in text and above problems, compare by averaging the heating effect of all coils in one phase capacity of this converter when running at 87 per cent power-factor, with its capacity at unity power-factor. Compare with data in Table II.

136. Rating of Converters. Overload. The rating of an electrical machine is the output marked on the rating plate, and is based on the maximum load which can be taken from the machine without exceeding the standard temperature rise under certain standard test conditions. Converters are rated on the same basis as alternating-current generators. See A.I.E.E. Standardization Rules as revised to July, 1915.

Converters built for railway substations must be able to stand momentarily a large overload. Accordingly, these converters, generally of the 25-cycle type, are rated so that they can deliver twice normal load for one minute, and one and one-half normal load for two hours without exceeding the specified standard temperature rise, and with no serious sparking at the brushes.

The fact that the alternating current and the direct current oppose, and, to a large extent, neutralize each other explains the moderate heating at these overloads. The excellence of commutation is explained as follows. In Art. 10 we learned that in polyphase alternators the shifting of the magnetic flux due to armature reaction was steady and toward the **trailing** pole-tip. A converter is merely an alternator used as a synchronous motor, the direction of current relative to induced e.m.f. and field poles being opposite to the direction in the generator. Thus the shift of the magnetic flux due to the alternating current would be steady and toward **leading** pole-tip. The shift due to the direct current, which the machine might be delivering, would be steady and toward the **trailing** pole-tip. Since the alternating current is always practically proportional to the direct current, each tends to shift the flux almost an equal amount, but in opposed directions. Thus the axis of sparkless commutation would be practically the same from zero load to the limiting overload.* Recent improvements have put the 60-cycle converter on a par with the 25-cycle in the matter of ruggedness, simplicity, cost of operating and ability to carry overload.

137. Voltage Variation in a Converter. Regulation. Owing to the resistance of the armature and brushes, the full-load voltage of a converter is somewhat less than the no-load voltage. But just as the heating of a converter is less than the heating of a direct-current generator delivering the same load, so the regulation is also less. This is due also to the opposing voltage drops in the armature and the opposing armature reactions of the direct and the alternating components of the armature current.

For example, the armature resistance of a certain 14-pole 250-volt 300-kw. 60-cycle converter was found to be 0.00264 ohm when measured on the direct-current side. When delivering full-load current of $\frac{300,000}{250}$ or 1200 amperes the drop due to armature resistance would be found as follows.

We have seen in Art. 133 that the effective values of the resultant currents carried by the armature coils in a three-ring converter equals 0.75 of the direct current delivered by the converter.

Effective armature current, thus

$$
\begin{aligned}
&= 0.75 \times 1200 \\
&= 900 \text{ amperes.}
\end{aligned}
$$

The average IR drop in armature, therefore,

$$
\begin{aligned}
&= 900 \times 0.00264 \text{ (approximately)} \\
&= 2.4 \text{ volts.}
\end{aligned}
$$

---

* As seen in Art. 9, there is a double frequency variation in the armature reaction of a single-hhase alternator. This causes poorer commutation in single-phase converters, and offers another reason why single-phase converters are seldom used.

The IR drop at the direct-current brushes, in modern converters, averages about 2 volts for the sets of both polarity taken together.*

The total drop at full load thus equals

$$2 + 2.4 = 4.4 \text{ volts.}$$

$$\text{Regulation} = \frac{4.4}{250} = 1.8 \text{ per cent.}$$

This is a fair value for the regulation in the best types of separately excited converters.

When the converter is used on a transmission system with step-down transformers, the drop at the direct-current brushes due to the transformer resistance and to the line resistance between the transformers and the converter is usually something over 3 per cent of the full-load voltage. Thus the usual regulation is about 5 per cent on standard substation converters.

138. Voltage Variation in Converters. Pulsation. The voltage between the direct-current brushes of a converter is never quite steady. In addition to the **regulation,** or the variation in voltage from no load to full load, there is also a periodic voltage variation which takes place with no changing of the load.

In Fig. 301, it is seen that at this instant the current goes directly from the rings to the brushes without going through the armature. The voltage across the brushes is thus the maximum alternating voltage minus, of course, the drop in the brushes. In Fig. 302, one-quarter of a cycle later, the rings are delivering no current, so the armature windings are carrying a direct current only, which flows from B_2 to B_1 through the two paths in the armature. The brush voltage has therefore become the induced voltage (equal to the maximum alternating voltage) minus the IR drop of the armature at this instant, as well as the drop in the brushes. Thus the voltage at the direct-current brushes has a pulsation back and forth twice during every cycle equal to the IR drop in the armature when the full direct current is flowing through it. Due to the armature reaction of double frequency and the alternate motor and generator effect, this pulsation is quite serious in a single-phase converter, but becomes less and less as more rings are added, being practically negligible in a six-ring converter. This fact is brought out very forcibly in Fig. 324, 325 and 326, which show actual oscillograms of the voltage at the direct-current brushes of a converter when operated as a single-phase, a two-phase and a three-phase machine.

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* Barr and Archibald, "Alternating Current Machinery," pages 441–451.

139. Voltage Control of Converters. It has been stated that the alternating characteristics of a converter are those of a synchronous motor. It will be remembered that strengthening the field current of a synchronous motor merely causes the armature to take a leading current. We would naturally suppose that the field flux would be increased by a larger field current, but the leading armature current so reacts on the field flux that it remains practically the same strength as before the field current was increased.* Similarly lessening the field current merely causes a component of the armature current to lag and the resulting armature reaction holds the field flux up to its former value. As the movement of the rotor is always in synchronism with the frequency of the line, and as changing the field current does not change the amount of the flux, it is evident that the induced e.m.f. in the armature must remain practically constant throughout any change in the field current. Thus the voltage of a converter cannot be controlled by any field control of machine.

But it is often desirable, especially in railway work, to raise the direct voltage at a converter substation when the load is heavy, in order to keep the voltage normal at the far end of the line.

There are, in common practice, five ways of accomplishing this, all of which, except one, depend upon increasing the voltage at the alternating-current rings. Since the ratio of the direct to the alternating voltage is practically constant, this increase of the alternating voltage raises the direct voltage.

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* See Chapter I and Chapter VIII on Synchronous Motors.

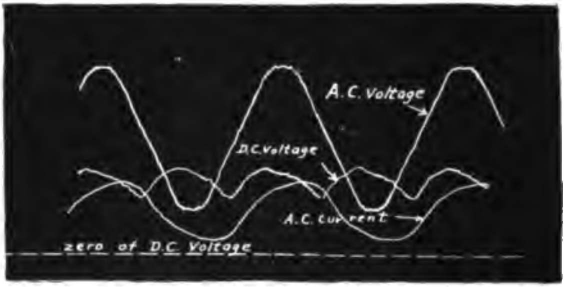

Fig. 324. Oscillograms of single-phase operation of a synchronous converter, 4 poles, 60 cycles, 10 kw., 110 volts (d-c.). *Oscillograms furnished by Mr. C. W. Bates.*

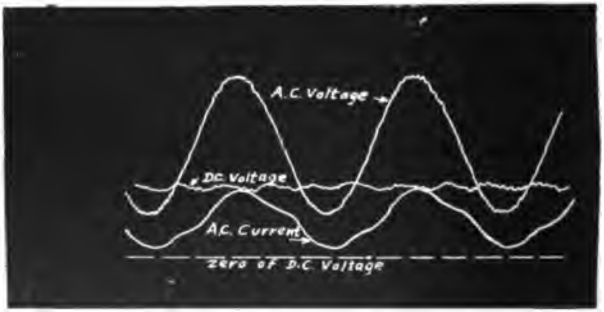

Fig. 325. Oscillograms of the converter of Fig. 324 operated as a two-phase converter. Note that the direct voltage is much less fluctuating in this case. *Oscillograms furnished by Mr. C. W. Bates.*

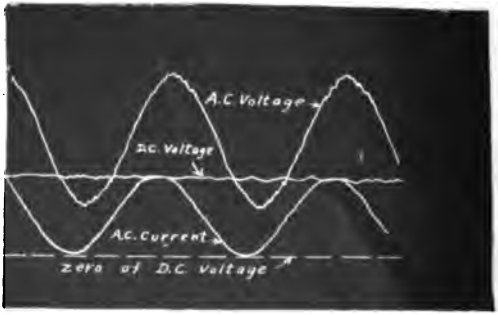

Fig. 326. Operation of the converter of Fig. 324 as a three-phase three-ring converter. Note that the direct voltage fluctuates even less than in the two-phase operation. *Oscillograms furnished by Mr. C. W. Bates.*

First. **Extra taps on the transformers.** A number of taps on the secondary of the transformers are brought out to switches. The converter runs on the intermediate taps at the smaller loads, but as the load increases and the brush voltage falls, the rings are switched to the higher-voltage taps.

Owing to the large currents to be switched there is great danger of fusing the contact points. For this reason, the taps are sometimes brought out of the primary coils of the transformer, although switching of the high-tension currents then requires somewhat expensive and complicated apparatus.

Second. **Induction regulator.** To avoid the switching difficulties mentioned above, it is more customary to use an induction regulator to control the alternating voltage at the rings. This device can be made automatic, as explained in Chapter IV.

Third. **Synchronous booster.** A small alternating-current generator is sometimes mounted on the same shaft as the converter in order to raise the alternating voltage at the rings of the converter. The small generator is called a synchronous booster and may be either of the revolving-field or of the revolving-armature type. The booster, marked S. G. B. in Fig. 327, is of the revolving-field type, the revolving-armature type usually being placed inside the block between the collector-rings and the armature windings of the converter proper. The armature of the booster is connected in series with the supply line, and since the booster and the converter have the same number of poles and are on the same shaft their frequencies are the same, and they are always in synchronism when the converter is running normally. Thus the voltage across the rings is the sum of the line voltage and the booster voltage.*

By sending the direct current delivered by the converter through the field coils of the booster, the ring voltage and accordingly the direct voltage may be made to regulate automatically. Although this method employs two separate machines it produces a very efficient combination, often as high as 95 or 96 per cent.*

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* The booster may be designed so that the polarity of the field coils may be reversed. In this case the ring voltage may be either the sum or the difference of the line and the booster voltages.

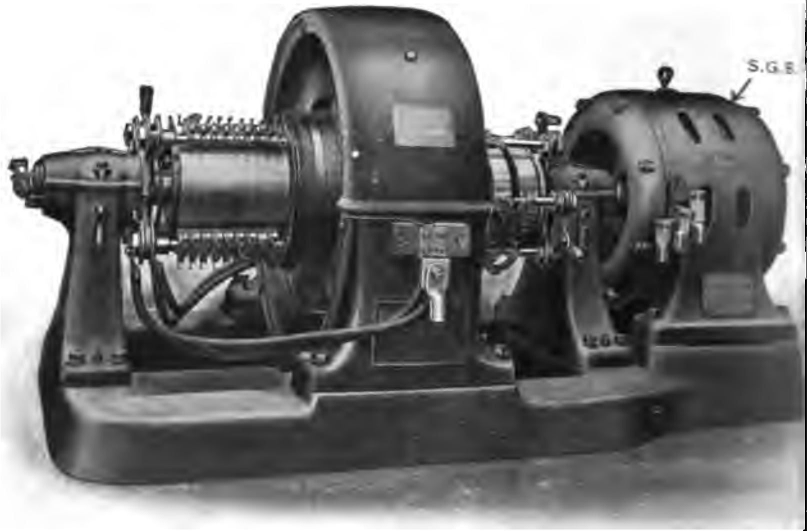

FIG. 327. Synchronous converter with synchronous booster. The booster is the small machine on the right direct-connected to the generator. The armature of the booster is placed in series with the line supplying the converter. *The General Electric Co.*

Fourth. **Regulating or split pole.**† The general construction of a regulating-pole converter is shown in Fig. 328a. Between the main poles *M*, smaller regulating poles *R* are placed, not in the middle of the space but slightly nearer the trailing pole-tip. These regulating poles are excited independently of the main poles, and may even be of opposite polarity to the main poles nearest to them.

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* Electric Journal, Feb., 1913.

† Adapted from General Electric Bulletin No. 4723.

In order to study the effect of these regulating poles, consider the action of the main pole and the regulating pole on a group of armature conductors between the direct-current brushes *a* and *b*. Let us assume that the regulating poles are not excited and that the direct e.m.f. produced by the conductors cutting the flux of the main poles is 250 volts. The alternating e.m.f. for a diametral connection would then be 0.71 × 250 or 178 volts. Thus when the e.m.f. at the direct-current brushes is 250 volts, the e.m.f. between any two diametrally connected rings would be 178 volts.

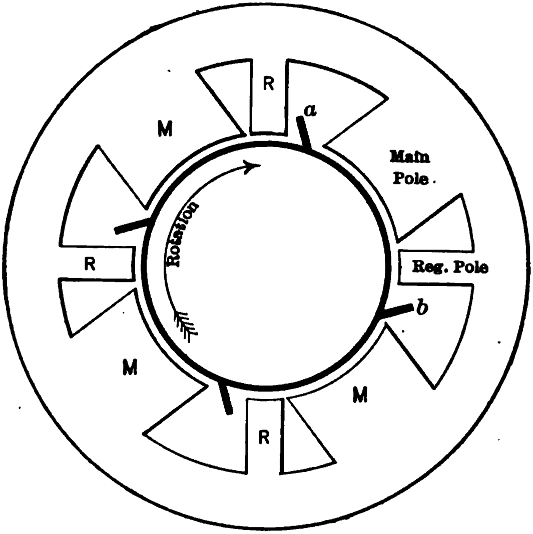

FIG. 328a. Diagram of a converter fitted with regulating poles *R*. Note that these poles are placed near the trailing tip of the main poles.

If now we keep the main pole excitation unchanged and excite the regulating poles so that each furnishes 20 per cent as much flux as the main pole nearest it and in the same direction, the direct voltage will become 120 per cent of 250 or 300 volts, since the conductors between each set of brushes are cutting 20 per cent more flux than before. Let us see what the alternating voltage has become between any two diametral taps. Although the wave-form of e.m.f. generated by the flux of the regulating poles is composed largely of the odd harmonics, for all practical purposes, it may be treated as a sine wave of the same frequency as that set up by the main poles. There are thus practically two sine waves of e.m.f. induced in each group of armature conductors as it passes each set of main and regulating poles. The resulting induced e.m.f. is therefore merely the sum of these two waves.

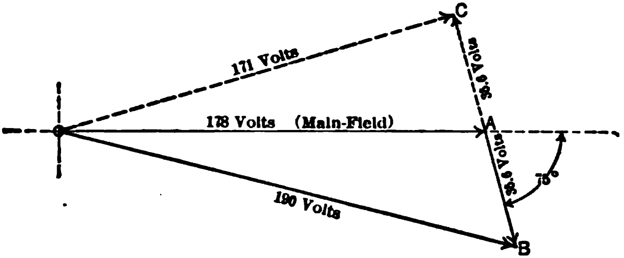

Fig. 328b. Topographic vector diagram of voltage between two diametral taps when the converter is supplied with regulating poles of 20 per cent of strength of main poles. The resulting voltage *OB*, when the regulating poles is excited in the same direction as the main poles, is the vector sum of the voltage *OA* due to the main poles, and the voltage *AB*, due to the regulating poles, so placed that the voltage due to them lags 75° behind the voltage induced by the main poles. Vector *OC* represents the diametral voltage when the regulating poles are reversed.

But note that the e.m.f. due to the regulating poles reaches its maximum value much later than the e.m.f. due to the main pole, in fact there is usually a phase difference of about 75° between the two. (If the regulating poles were exactly half way between the main poles, the phase difference would be 90°, but the regulating pole is about 15° nearer the earlier pole.) We may thus find the resultant e.m.f. between any two diametrally connected rings by means of a vector diagram as in Fig. 328b. Vector *OA* represents the effective e.m.f. of 178 volts produced by the flux from the main poles.

Vector *AB* lagging 75° behind *OA* represents the effective e.m.f. of 20 per cent of 178 or 35.6 volts produced by the flux from the regulating poles placed 75 time degrees later than the main poles and excited in the same direction. Vector *OB* represents the resultant effective e.m.f. of 190 volts. Thus the regulating pole has increased the diametral alternating e.m.f. 190 − 178 or 12 volts only, while it has raised the direct voltage 300 − 250 or 50 volts.

Similarly, if we reverse the flux in the regulating poles keeping it still equal to 20 per cent of the flux in the main poles, the resulting e.m.f. between two direct-current brushes such as *a* and *b*, Fig. 328a, would be 250 − 50 or 200 volts. The effective value of the alternating e.m.f. between rings connected to diametral taps would be represented by the vector *OC* of 171 volts in Fig. 328b. This e.m.f. is merely the resultant of the vector *OA* representing 178 effective volts of the main field and the vector *AC* representing 35.6 effective volts of the regulating field. The vector *AC* is drawn at 180° to vector *AB* because the field and therefore the induced e.m.f. has been reversed. Thus the reversed regulating pole produces a change of 178 − 171 or 7 volts in the alternating voltage, but a change of 250 − 200 or 50 volts in the direct voltage. The range of the voltage at the direct-current brushes is therefore from 200 to 300 volts if the main field is kept constant and the regulating poles changed 20 per cent in each direction. The change in the alternating-current rings would be from 171 to 190 volts at the same time, or about 18 per cent as great as the direct-current change.

But the effective value of the alternating voltage applied to a converter is practically constant. Accordingly, the effective value of the induced e.m.f. must be made constant. This is readily accomplished by changing the excitation of the main field enough to counteract the slight change in the alternating e.m.f. caused by the regulating pole. Thus in Fig. 328c, the vectors *OA*, *AB* and *OB* are merely reproduced from Fig. 328b, in which OA is the alternating e.m.f. due to the main field, AB the e.m.f. due to regulating pole 20 per cent as strong as the main poles and OB the resultant e.m.f. of 190 volts between rings diametrally connected. The direct voltage for this strength of main poles and regulating poles is 300 volts. If now we decrease the strength of the regulating poles to one-half their former value, so that the effective e.m.f. induced by them is only 17.8 volts as represented by the vector A_1B_1, and at the same time increase the strength of the main poles from OA to OA_1, the resulting alternating e.m.f. will have the value represented by the vector OB_1 which is equal to the vector OB in length because they are both radii of the same arc BB_1B_2 of a circle.

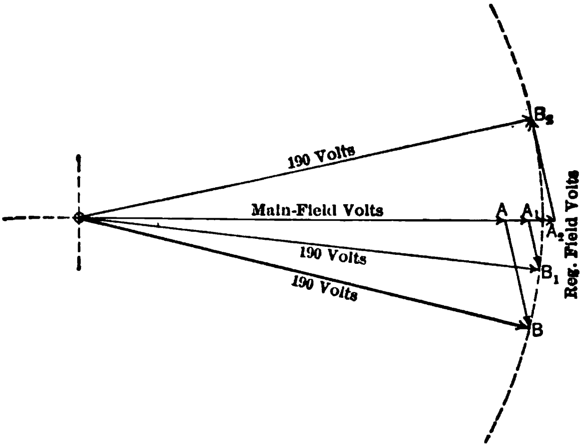

FIG. 328c. Diagram showing that the voltage OB, OB_1 and OB_2 between diametral taps can be kept constant at 190 volts by increasing the main-pole voltage to OA_1 and OA_2 as the regulating poles are weakened to A_1B_1 or even reversed to A_2B_2. Thus the vector OB is the resultant of the main-pole voltage OA and the regulating-pole voltage AB. The vector OB_1 equals the vector OB_1 and is the resultant of main-pole voltage OA_1 and the regulating-pole voltage A_1. OB_2 is the resultant of OA_2 and A_2B_2. The change in the regulating-pole voltage, however, changes the direct voltage proportionately.

The value of the vector OA_1 representing the strength of the main field may be found by the equation

$$OA_1^2 = \overline{OB_1}^2 - \overline{A_1B_1}^2 + OA_1 \times A_1B_1 \cos 105^\circ$$
$$OA_1 = 188 \text{ volts.}$$

If therefore we increase the main field so that the effective alternating e.m.f. is 188 volts instead of 178 volts, and at the same time decrease the regulating poles field so that the alternating e.m.f. is 17.8 volts instead of 35.6 volts, we shall maintain the same effective value of 190 volts for the induced alternating e.m.f. between rings diametrally connected.

This change in the main and the regulating fields causes the following change in the voltage across the direct-current brushes, which was 300 volts before the change, — 250 volts due to main field, and 50 volts due to regulating field. The regulating field volts have been reduced one-half to 25 volts and the main field volts increased to $\frac{188}{178}$ of 250 or 265 volts. The total direct voltage is therefore 265 + 25 or 290 volts. There is thus a reduction of 10 volts across the direct-current brushes with no change in voltage across the alternating-current rings. Even with the regulating poles reversed the alternating e.m.f. may be kept constant by increasing the main field, as is seen from vector OB_2 which is the resultant ring e.m.f. for a main-field strength of OA_2 and a reversed regulating-field strength of A_2B_2.

The practical limit to the variation of the voltage between the direct-current brushes is about 20 per cent in either direction of polarity of the regulating poles or 40 per cent total. By means of voltage regulators, the currents through the main-field coils and the regulating-field coils may be automatically controlled, thereby making the regulation of the converter automatic. Fig. 328d shows the appearance of a regulating-pole converter.

Fifth. **Compound-wound converters with series reactances.** Where a converter regulation of from 5 to 10 per cent is desired, the fields of the converter are often compounded, by having series field coils which carry the current delivered by the direct-current brushes. Of course as the current varies in the series windings, it can do no more than vary the power-factor of the alternating current supplied to the brushes. But an inductive reactance, X, is placed in series with the converter across the secondary of the transformer as shown in Fig. 329. The voltage between the rings will, therefore, equal the voltage of the transformer minus (vectorially) the voltage consumed by the reactance. The voltage required to overcome the reactance is always 90°

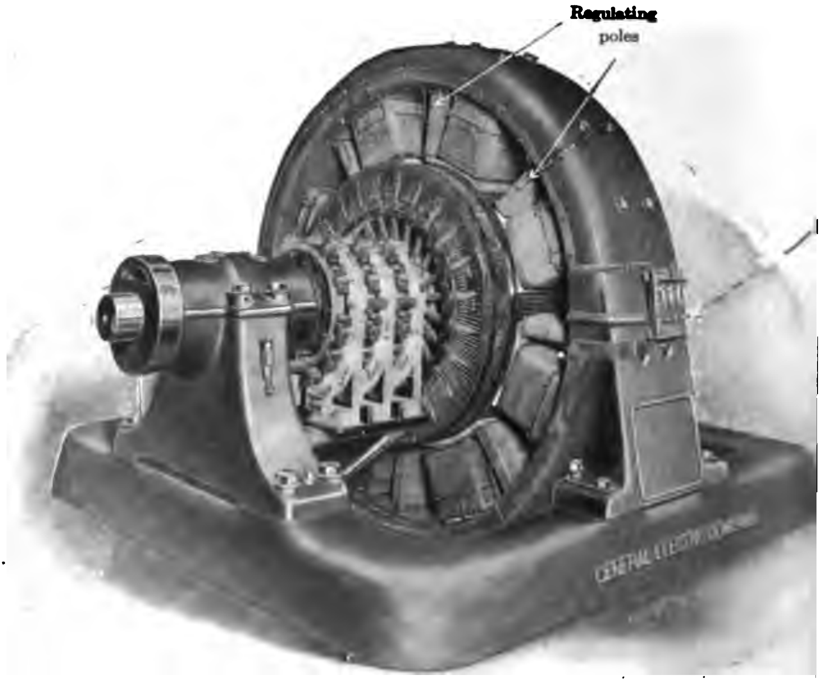

Fig. 328d. A regulating-pole converter. Note that each of the regulating poles is nearer one main pole than the other main pole.

ahead of the current through it. Thus, by causing the current to lead the ring voltage sufficiently, the voltage across the reactance will so combine with the transformer voltage as to cause the ring voltage to be higher than that which the transformer gives. This is very analogous to the rise in voltage at the end of a long line possessing considerable reactance when a leading current is sent over it.

The strength of the shunt fields is generally so adjusted that at the average load on the converter, the power-factor will be as nearly as possible unity, because the load-capacity and the efficiency are greatest at the highest power-factor. This load is usually about three-quarters of the rated load of the converter. At less load than this, the weakening of the series fields causes a lagging current, and at greater loads, the strengthening of the series field causes a leading current.

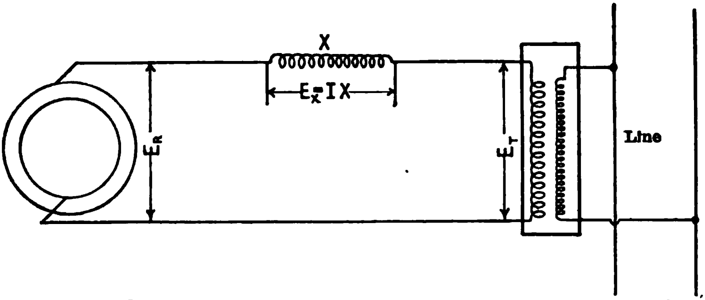

Fig. 329. The reactance X is placed in series with the converter rings across the secondary of the transformer. The voltage E_R between the rings is always the vector difference of the voltage E_T across the transformer and the voltage E_X across the reactance coil.

The rise in ring voltage as the strength of series field increases with the load and advances the phase of the current is seen from Fig. 330, 331 and 332.

In Fig. 330, the converter is delivering one-half rated load, and the series field combined with the shunt field is so weak that the alternating current I, which is supplied by the transformer, lags $\alpha°$ behind transformer voltage E_T. To find the ring voltage E_R, draw vector E_T to represent the transformer voltage and vector I to represent the transformer current at proper angle $\alpha°$ behind E_T. The drop across the reactance X will lead the current I by 90°. Accordingly

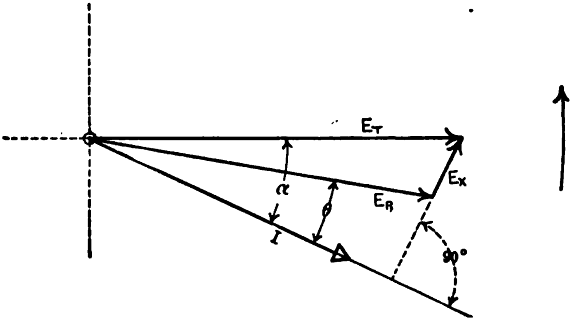

FIG. 330. Vector diagram showing current and voltage conditions for the arrangement of Fig. 329 when the converter carries one-half load. The weak field causes the current I to lag $\alpha°$ behind the transformer voltage E_T. The ring voltage E_R equals E_T minus (vectorially) the voltage E_X across the reactance coil. E_X must lead I by 90°.

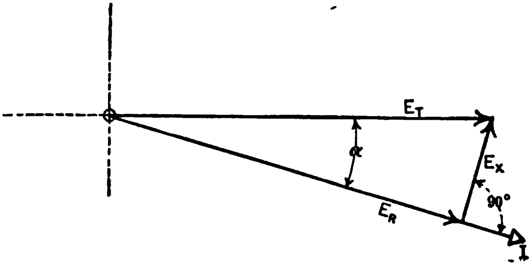

FIG. 331. Vector diagram when the converter carries three-quarters of full load. The current I has increased and has swung nearer into phase with E_T because the field has become stronger. E_R has increased slightly because E_X has become more out of phase with E_T. E_R and I are in phase and converter is operating at unity power-factor.

draw $E_X (= IX)$ at 90° to I, leading. Connect O and the tail of E_X with a vector, which will represent E_R in amount and in the proper phase relations to E_T and I, because the ring voltage plus (vectorially) the drop across the reactance equals the transformer voltage. Note that at one-half load, the current I lags a little, $\theta°$, behind the ring voltage E_R and that the ring voltage E_R is less than the transformer voltage E_T.

In Fig. 331, the load has increased to three-quarters full load, the current still lags α behind the transformer voltage, although by a smaller amount, but is in phase with the ring voltage E_R, which has increased slightly, due to the change in direction of I.

In Fig. 332, the full-load current through the series coils has caused the current I to lead the transformer voltage E_T.

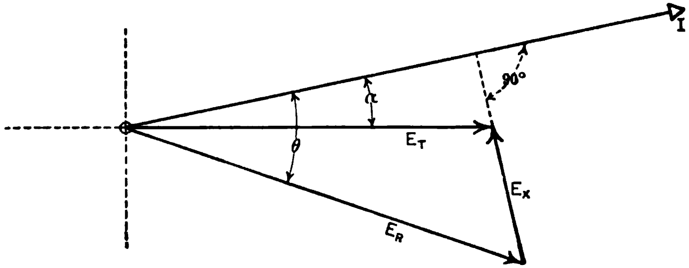

FIG. 332. Vector diagram of conditions when the converter of Fig. 329 is carrying full load. The increased current I now leads the voltage E_T because the field has become much stronger. The ring voltage E_R has now become greater than the transformer voltage E_T, although it still equals $E^T \ominus E_X$.

The combination of the drop across the reactance and the transformer voltage now causes the ring voltage E_R to be larger than the line voltage. Note, however, that the drop across the reactance E_X still leads the current by 90° and that the transformer voltage E_T minus (vectorially) this reactance drop still equals the ring voltage E_R.

It is the usual practice so to arrange this combination that the rise in the ring voltage, as the load increases, causes the direct voltage to rise just the amount it loses on account of the increase in armature and transformer drop. This produces practically constant direct voltage at the brushes from no load to full load. It is not good practice to over-compound a converter on account of the loss in capacity which would be caused by operating at less than unity power-factor. The necessary reactance is generally obtained by constructing the step-down transformers with the amount of leakage reactance required to flat-compound the converter used.

140. Voltage and Power-factor of Compound-wound Converters at Various Loads.* For modern railway converters, the following approximate method of computing the terminal voltage on the direct-current side and the power-factor at various loads is sufficiently correct. The voltage of a transmission line usually varies from time to time through values of several per cent, hence there is no practical need of a more accurate determination of the voltage variation at the converter under various load conditions. However, it may be said for the method here presented, that it is as precise as the determination of the resistance and the reactance of a converter circuit.

We have seen that an ohmic (or IR) drop of 5 per cent at full load is a fair value for a modern converter. The synchronous reactance drop of the same converter will also be about 5 per cent. It is good practice to design the transformers to be used for flat-compounding with such a converter with about 10 per cent leakage reactance. This makes a total reactance drop of 15 per cent.

The ampere-turns in the armature at full load usually are approximately equal to the ampere-turns in the shunt field at no load when the field current is adjusted to give normal voltage at no load and unity power-factor.

The series field at full load has generally about one-half as many ampere-turns as the shunt field at no load.

The effective power loss at no load, unity power-factor, normal direct voltage, would be about 4.5 per cent of the full-load effective power, and it would increase with the load, approximately in proportion to the ohmic drop.

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* For this practical method the authors are largely indebted to an article by Mr. Jens Bache-Wiig in the Electric Journal, for Nov., 1910.

Example 7. A modern compound synchronous converter was designed to have unity power-factor and normal voltage at the average load, which would probably be about three-quarters of the rated full load, when a reactance was used in series with it. Compute the power-factor and voltage at:

(*a*) No load.
(*b*) Half load.
(*c*) Full load.
(*d*) At 50 per cent overload.

Data summary:

	Per cent
Armature ampere-turns at full load	100
Shunt field amp.-turns at no load, unity power-factor	100
Series field amp.-turns at full load	50
Reactance drop, total at full-load current	15
Resistance drop, total at full-load current	5
Effective power at no load	4 5

(*a*) **At No Load.** With the converter designed to have unity power-factor at $\frac{3}{4}$ load, the shunt field rheostat must be so set that the sum of the ampere-turns in the shunt field and the series field at $\frac{3}{4}$ load equals 100 per cent, or the value which it was found the shunt field alone must have in order to produce unity power-factor at no load, with rated voltage.

At three-quarters load the series field would have the value

$$0.75 \text{ of } 50 \text{ per cent} = 37.5 \text{ per cent.}$$

Thus the shunt field would be required to have only

$$100 \text{ per cent} - 37.5 \text{ per cent} = 62.5 \text{ per cent.}$$

That is, at $\frac{3}{4}$ load we would set the shunt field rheostat so that it produced 62.5 per cent of the zero-load field. The series coils would then produce 37.5 per cent.

At no load the fields would be under-excited and a lagging current would flow in the armature. The reactive component of this current would be the same fraction of the full-load current that the excitation is below that required for unity power-factor. The fields are under-excited 100 − 62.5 or 37.5 per cent. Thus a lagging current of 37½ per cent of the full-load current will flow in the armature. The current component in phase with the voltage will be too small at no load to be considered.*

Since a full-load current causes 5 per cent resistance drop, this 37½ per cent current would cause 0.375 × 5 per cent or 1.88 per cent drop.

This resistance drop will be in phase with the 37½ per cent current. But as this current is wholly reactive, this resistance drop lags 90° behind the impressed voltage. So small a value at right angles to the voltage does not appreciably affect the ring voltage as is seen from Fig. 333, in which the vector E_R represents the ring voltage (100 per cent) and I_X the reactive current through the armature at no load. I_XR then represents the resistance drop due to this reactive current. Note that no attempt is made to draw the vectors to scale. The smaller values could not be represented if drawn to the same scale as the larger. For the sake of clearness, the resultant armature current is not drawn. In each case it is merely the vector sum of the power current and the reactive current.

A full-load current causes a 15 per cent drop due to reactance. Therefore 37½ per cent current will cause 37½ × 15 per cent or 5.63 per cent drop due to reactance. This reactance drop always leads the current by 90°, thus the vector I_XX represents the amount and phase relation of this reactance drop. Note that it is in phase with the ring voltage E_R. The power component of the current would be 4.5 per cent, and the IR drop in phase with this current would be 0.045 × 5.0 per cent or 0.225 per cent, which is negligible. Of course the reactance drop, 0.045 × 15 per cent or 0.675 per cent would also be negligible, in comparison with E_R which is 100 per cent.

---

* Armature ampere-turns at full load equal shunt-field ampere turns at no load, unity power-factor. The component of armature current which lags 90° behind the voltage produces a purely demagnetizing effect in a generator, or a purely magnetizing effect in a synchronous motor. Now, if the shunt field excitation is 50 per cent below the value that will produce 100 per cent power-factor at zero load, we have seen that the machine will take a lagging reactive component of current, and this component will grow in (effective) value until by its magnetizing action it has restored the flux to its former value, or rather until it has made the counter e.m.f. again just about equal to the impressed e.m.f. As the shunt field ampere-turns had been reduced 50 per cent (below unity power-factor value), enough lagging reactive armature amperes must flow to restore this 50 per cent of reduced field ampere-turns; and as full-load armature current produces armature ampere-turns equal to field ampere-turns, it will require lagging reactive armature amperes equal to 50 per cent of rated full-load amperes, in order to restore the equilibrium or make the counter e.m.f. equal to the impressed e.m.f.

In order to have a ring voltage of 100 per cent at zero load, the secondary transformer voltage must be $100 + 5.63 + 0.23$ or 105.9 per cent. That is, when there is no load on the transformer, its secondary voltage would be 105.9 per cent of the voltage at full load. As the given values of resistance and reactance included the armature of the converter, the terminal voltage between d-c. brushes at no load will also be 100 per cent of full-load voltage.

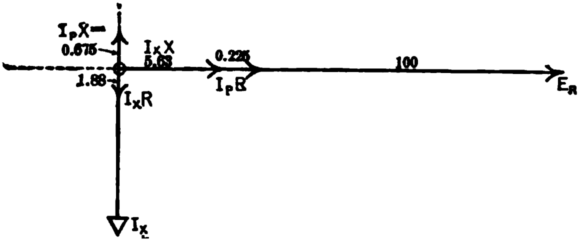

FIG. 333. Vector diagram at no load for current and voltage conditions for the compound-wound converter of Example 7. E_R = ring voltage. I_PR = resistance voltage drop due to power component of current. I_XR = resistance voltage drop due to reactive component I_X of current. I_PX = reactance drop due to power component of current. I_XX = reactance drop due to reactive component of current. The transformer voltage E_T (not shown in diagram) = $I_PX \oplus I_XX \oplus I_PR \oplus I_XR \oplus E_R = 105.9$.

Power-factor at No Load. Power-factor equals the relation between effective and apparent power, or between power component of no-load current and total current at no load.

$$\text{Power-factor at no load} = \frac{4.5}{\sqrt{37.5^2 + 4.5^2}} = 12 \text{ per cent.}$$

We have seen in Chapter I that any power-factor below 0.20 is practically the same as zero power-factor in its effect. Fig. 333 is, therefore, practically correct in that the current lags 90° behind the voltage. The omission of the power current from the diagram does not materially change the amount or the phase relations of the armature current.

As there is only a small current drawn from the line under these conditions, the low power-factor does not appreciably affect the line power-factor or voltage.

(*b*) **Half Load.** At one-half load (meaning half of rated load d-c. watts output), the power component of the current input is 50 per cent, and the *IR* drop due to it will be 0.50×5 per cent $= 2.5$

per cent. In Fig. 334, vector I_PR represents the drop in phase with the power component of the current, I_P. The reactive drop due to this power current equals 0.50 × 15 per cent = 7.5 per cent and is represented by the vector I_PX leading the current I_P by 90°.

The series-field amperes now equal 0.50 × 50 per cent = 25 per cent.

The shunt-field ampere-turns are constant, therefore the total field ampere-turns = 62.5 + 25 = 87.5 per cent.

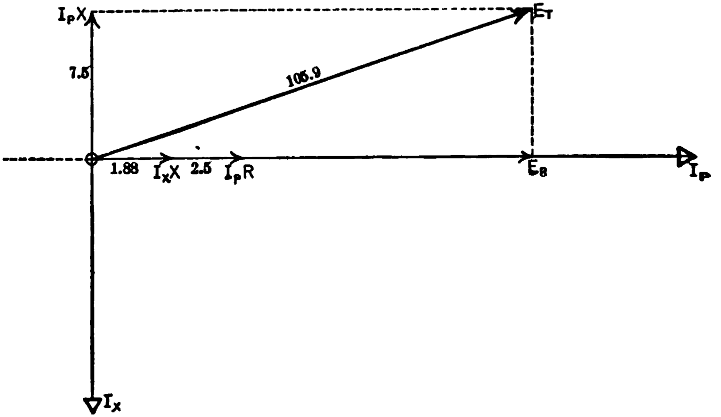

FIG. 334. Vector diagram for half-load conditions of converter of Example 7. E_T (constant at 105.9) = $I_PX \oplus I_XX \oplus I_PR \oplus E_R$. $E_R = 101.2$.

The converter is thus 100 − 87.5 or 12.5 per cent under-excited. Therefore a 12.5 per cent lagging reactive current flows in the armature which is represented by the vector I_X lagging 90° behind I_P in Fig. 334.

The IR drop in this direction would be 0.125 × 5 per cent or 0.625 per cent, too small to affect the computation, as it is at 90° to E_R.

The reactance drop, 90° ahead of the reactive current, equals 0.125 × 15 per cent = 1.875 per cent and is represented by the vector I_XX.

Thus the transformer voltage E_T in Fig. 334 is practically made up of the ring voltage E_R, the reactive drop due to the power current I_PX, the reactive drop due to the reactive current I_XX and the ohmic drop due to the power current I_PR. But E_T has the same value as at no load, namely, 105.9 per cent.

Therefore,

$$E_T^2 = (I_P X)^2 + (I_X X + I_P R + E_R)^2,$$

or

$$105.9^2 = 7.5^2 + (4.38 + E_R)^2,$$

$$(4.38 + E_R) = \sqrt{105.9^2 - 7.5^2},$$

$$E_R = 105.6 - 4.4$$

$$= 101.2.$$

Power-factor. Assuming that the watts lost increase in proportion to the ohmic drop due to the power current, then losses will be 4.5 + 2.5 = 7 per cent. The power output is 50 per cent of full-load output, therefore

$$\text{power-factor (approx.)} = \frac{(50 + 7)}{\sqrt{(50 + 7)^2 + 12.5^2}}$$

$$= \frac{\text{total watts input}}{\sqrt{(\text{watts input})^2 + (\text{reactive volt amp.})^2}}$$

$$= \frac{57.0}{58.4}$$

$$= 0.976.$$

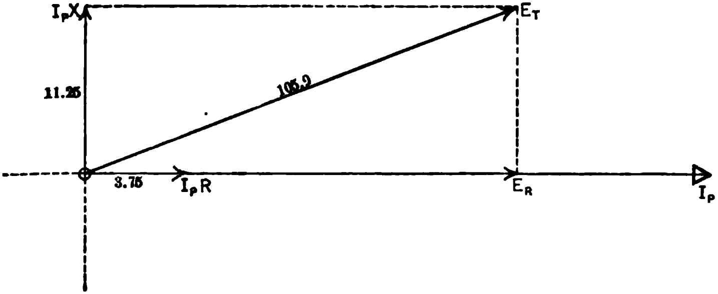

FIG. 335. Three-quarters load conditions of converter of Example 7. $E_R = 101.5$.

Three-quarters Load.

At three-quarters load:

The IR drop in phase with the power current = 0.75 × 5 per cent = 3.75 per cent and is represented by vector I_PR in Fig. 335.

The reactance drop due to power current = 0.75 × 15 per cent = 11.25 per cent and is represented by vector I_PX.

There is no reactive current component because at this load the fields are adjusted for unity power-factor.

From Fig. 335:

$$
\begin{aligned}
E^2_T &= 11.25^2 + (3.75 + E_R)^2,\\
(3.75 + E_R) &= \sqrt{105.9^2 - 11.25^2},\\
E_R &= 105.2 - 3.75\\
&= 101.5 \text{ per cent.}
\end{aligned}
$$

(*c*) **Full Load.**

At full load:

The ohmic drop due to power current = 5 per cent.
The reactive drop due to power current = 15 per cent.
The series field = 50 per cent.
Total field = 62.5 + 50 = 112.5 per cent.
Over-excitation = 12.5 per cent.

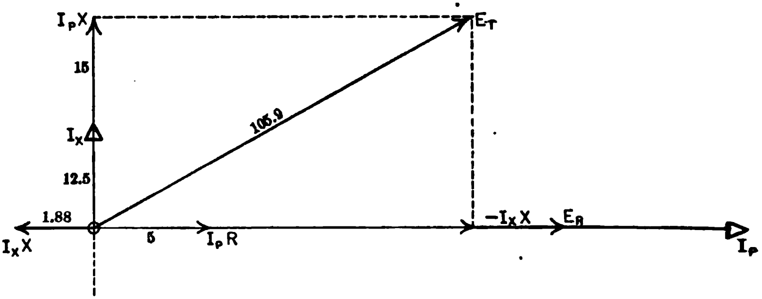

Fig. 336. Full-load conditions of converter of Example 7. $E_R = 101.6$.

Leading reactive current = 12.5 per cent.

Ohmic drop due to reactive current = 0.125 × 5 per cent = 0.625 per cent.

Reactive drop due to reactive current = 0.125 × 15 per cent = 1.87 per cent.

This is 90° ahead of the reactive current and is represented by the vector $I_X X$ in Fig. 336.

The ring voltage from Fig. 336 can be found by the following equation:

$$
\begin{aligned}
105.9^2 &= (E_R + 5 - 1.88)^2 + (15 + 0.63)^2,\\
(E_R + 3.12) &= \sqrt{105.9^2 - 15.63^2},\\
E_R &= 104.7 - 3.12\\
&= 101.6 \text{ per cent.}
\end{aligned}
$$

$$\text{Power-factor (approx.)} = \frac{(100 + 4.5 + 5)}{\sqrt{(100 + 4.5 + 5)^2 + (12.5)^2}}$$

$$= \frac{\text{total watts input as per cent of rated watts output}}{\text{total volt-amperes input as per cent of rated watts output}}$$

$$= 99.5 \text{ leading.}$$

(*d*) **Overload of 50 per cent.**

At 50 per cent overload:

The ohmic drop due to power current = 1.50 × 5 per cent = 7.5 per cent ($= I_PR$).

The reactive drop due to power current = 1.50 × 15 per cent = 22.5 per cent (I_PX, Fig. 337).

The series field = 1.50 × 50 per cent = 75 per cent.

Total field = 62.5 + 75 = 137.5.

Over-excitation = 137.5 − 100 = 37.5 per cent.

Reactive **leading** current = 37.5 per cent.

Ohmic drop due to reactive current = 0.375 × 5 per cent = 1.875 per cent ($= I_XR$).

Reactive drop due to reactive current = 0.375 × 15 per cent = 5.625 per cent ($= I_X\mathrm{X}$).

The voltage at the rings is found from the equation obtained from Fig. 337.

$$E^2_T = (E_R + 7.5 - 5.63)^2 + (22.5 + 1.88)^2.$$

$$(E_R + 1.87) = \sqrt{105.9^2 - 24.38^2}.$$

$$E_2 = 103.0 - 1.87$$

$$= 101.1 \text{ per cent.}$$

$$\text{Power-factor (approx.)} = \frac{(150 + 4.5 + 7.5)}{\sqrt{(150 + 4.5 + 7.5)^2 + 37.5^2}}$$

$$= 97.5 \text{ per cent } \textbf{leading.}$$

The relation of load to power-factor and voltage at the direct-current brushes of this synchronous converter is readily seen from the curves in Fig. 338. The direct-current voltage at no load is plotted as 600 volts, the usual railway substation voltage. By plotting these curves for various combinations of reactances, and series-field strengths, it is possible to ascertain the most desirable value for the reactance and the series field, when a certain load is specified at which unity power-factor must be obtained. The following problems show the results of varying the resistance, reactance, series-field turns, or unity power-factor load for a given converter.

Prob. 32–9. Plot curves as in Fig. 338, for the machine of Example 7, with the specification that the power-factor shall be unity at full load instead of at $\frac{3}{4}$ load. What is usually the disadvantage of this specification?

Prob. 33–9. Change the ampere-turns in the series field of the converter of Example 7 to 75 per cent and plot curves as in Fig. 338.

Prob. 34–9. Show the effect on the power-factor and voltage regulation from no load to 50 per cent overload, if the total reactance had been 25 per cent instead of the 15 per cent of Example 7.

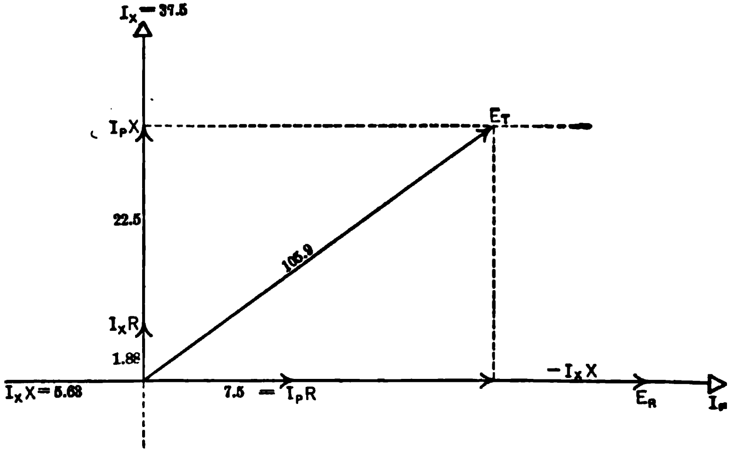

FIG. 337. Conditions in converter of Example 7 at an overload of 50 per cent. $E_R = 101.1$.

Prob. 35–9. Let the ohmic resistance of the converter in Example 7 be 10 per cent instead of 5 per cent and replot the curves of Fig. 338.

Prob. 36–9. Point out the various sources of inaccuracy in this method of computing the power-factor and voltage regulation of a converter, and explain why they do not lead to considerable errors.

141. Use of Commutating Poles on Converters. In Art. 132 it is stated that there is practically no armature reaction in a synchronous converter. This is strictly true at low commutator speeds and moderate current output per square inch of brush surface. Recently, however, the number of poles has been cut down to get more distance between brush holders. This has correspondingly increased the necessary commutator speed. Modern machines must also stand momentary loads equal to twice the normal load.

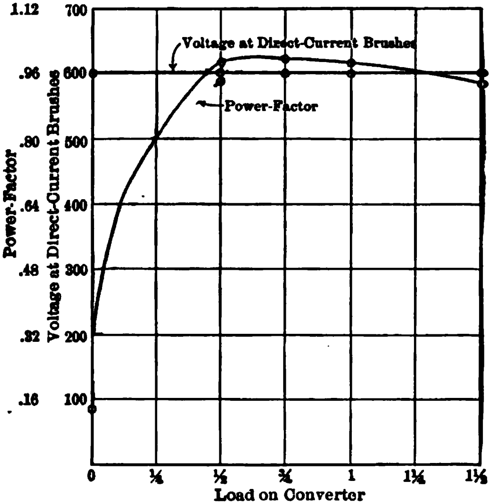

FIG. 338. Curves showing the relation between load and power-factor on the converter of Example 7. The voltage at the direct-current brushes is practically constant.

Under these severe conditions, an armature reaction is apparent and the field is shifted slightly, but still not enough to cause poor commutation at the lower speeds. By means of commutating poles with from 25 per cent to 40 per cent as many ampere-turns as the armature has, the shift of the field flux has been neutralized as in direct-current dynamos and the sparking practically eliminated. A great increase in the output per pound weight and an accompanying decrease in the cost per kilowatt output have resulted. In fact, the limiting factor which determines the capacity of a machine is now the amount of heat which the armature coils can radiate without too great a rise in temperature, rather than the amount of current which the brushes can take from the commutator without sparking. Commutating poles are not used in regulating-pole converters.

142. Hunting. Damping Grids. Synchronous converters like synchronous motors are likely to hunt when any sudden change in the load or in the line voltage takes place. In fact, owing to its slight armature reaction, a converter is much more sensitive to these sudden changes, and has an even greater tendency to hunt than a synchronous motor. A 60-cycle converter is more sensitive than a 25-cycle, the reason being that for the same speed it must have more poles, and hence less space between brushes. Accordingly, the same mechanical displacement out of the synchronous position would mean greater electrical phase-displacement in a 60-cycle converter, and hence greater synchronizing current.

In converters, hunting is likely to cause not only falling out of step, but also destructive sparking at the direct-current brushes, even "flashing over" from brush to brush. The flux set up by the armature rotates with respect to the armature at synchronous speed, in the opposite direction to the rotor, and thus stands still with respect to the field poles. But when the armature swings back and forth, behind and ahead of the synchronous position, its conductors cut the field flux correspondingly, and may set up large momentary circulatory currents in those conductors which are short-circuited by the brushes.

Hunting is effectually prevented under normal conditions by the use of damping grids or squirrel-cage windings of heavy copper, set in the pole-faces as shown in Fig. 240 and 297. These grids form closed circuits for the eddy currents induced by oscillation of the armature flux, which offer an opposing torque to the swinging of the rotor. However, they lower the efficiency of the converter somewhat more than one per cent. No converter, however, should be expected to operate satisfactorily on a line having much over 10 per cent line drop, unless the line is especially free from periodic variations in voltage and frequency such as are often caused by the pulsation of the prime movers. A large line drop means great line resistance, and great line resistance means a limiting of the synchronizing current, and thus a limiting of the force which tends to keep the rotor in synchronism.

Damping grids are never made to enclose any part of the commutating poles, as they would tend to choke any sudden increase or decrease of the flux in these poles. This would cause the commutating action of the poles to lag, and sparking would occur until the commutating flux could change properly.

143. Starting Synchronous Converters. There are three common methods of starting synchronous converters.

(*a*) **By Means of a Small Induction Motor** mounted on the same shaft. This induction motor has one fewer pair of poles than the converter, and consequently is able to raise the speed above the synchronous speed of converter. This enables the converter to be synchronized readily.

(*b*) **As a Direct-current Motor.** The converter is brought up to speed as a direct-current motor and synchronized by controlling the strength of the shunt-field current. It would require but a comparatively low direct voltage to start the motor and bring it up to synchronous speed, but the direct voltage must also be high enough to force sufficient current through the field coils to produce unity power-factor, as soon as the converter is thrown on the alternating-current line. Otherwise large reactive currents will flow in the armature.

(*c*) **By the Induction-motor Effect on the Converter Itself.** This is the most common and cheapest method of starting. Half-voltage taps are generally brought out from the secondary of the transformer. When the double-throw switch is thrown to the starting position it merely connects these half-voltage taps to the rings. This supplies enough current to start* the armature, but not enough pressure to produce excessive currents in it. If the pole-faces were not laminated strong enough eddy currents would be induced in the iron of the poles by this armature current to start the rotor. But although the pole-faces are laminated, and the eddy currents are very small, still the damping grids which are built into the pole-faces to prevent hunting serve as circuits for the induced eddy currents which produce sufficient torque to bring the rotor practically up to synchronous speed.

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* See Chap. VII, Introduction, Art. 94 and 95.

The procedure in starting a converter by the induction-motor action is as follows:

(1) Open the "field-break-up" switch marked F.B.S. in Fig. 328. Attach the voltmeter to indicate the voltage of the direct-current brushes.

(2) Close the double-throw line-switch to the starting position and the armature begins to rotate. With the "field-break-up" switch open, the direct-current voltmeter indicates the voltage set up by the armature conductors cutting the flux induced in the field by the reactive currents drawn from the line by the armature. The voltmeter needle will at first swing back and forth until the machine is running nearly in synchronism. In about one-half minute the steady indication of the direct-current voltmeter in one direction will show that the converter is running at synchronous speed.

(3) If the converter is self-exciting the direct-current voltmeter may now indicate that the polarity of the brushes is the reverse of what is desired. In that case, close the "field-break-up" switch in the reverse direction (which is usually downward). This connects the shunt field directly across the armature in the "reverse" direction,—that is, so that the flux due to the shunt coils opposes and tends to neutralize the flux set up by the armature current. Since the armature is now rotating in synchronism, the flux produced by the armature current will be stationary and tend to keep the poles energized. If the shunt-field current is strong enough to overcome this flux set up in the poles by the armature currents, the induced e.m.f. becomes zero, and the voltmeter needle will indicate this. The armature flux being thus pushed out of the poles into the spaces between the poles

immediately seeks the next poles, which of course have the opposite polarity. Thus the poles produced by the armature flux, which it must be remembered are stationary in space when the armature is rotating at synchronous speed, have been forced to leave one pole and enter the next. The armature synchronous position has, therefore, been changed by the amount of one pole. It is customary to say that the armature has "slipped a pole."

The voltmeter will now begin to indicate in the reverse direction, because all the poles have reversed. If the field current is allowed to flow, it will flow in the direction opposite to that in which it has been flowing (because the voltage is reversed) and will tend to neutralize the new polarity of the poles just as the first field current neutralized the old polarity. Thus the same process of "slipping a pole" would occur again. Therefore, at the instant the voltmeter begins to show a reversed polarity, the field switch must be thrown up into the running position.

(4) It may happen, however, that when the field switch is thrown down to make the armature slip a pole, the flux caused by the armature current is too strong and the reversed field current cannot overcome it. The armature will continue to rotate, but in a very unstable condition, without slipping a pole.

In this case, throw the field switch back to normal position, open the starting switch an instant, and let the armature fall slightly below synchronous speed. Then, close the starting switch to starting position again and wait for the machine to come up to synchronism again. If the polarity is still wrong, repeat opening and closing of starting switch until the polarity becomes correct and machine is running in synchronism.

Then throw the starting switch to running position, adjust the shunt field rheostat so that the direct voltage and the power-factor are normal.

If the converter is separately excited from direct-current busbars, the polarity cannot become reversed, so the field-break-up switch need not be of the double-throw type.

144. Field-break-up Switch. The field-break-up switch mentioned in the previous article is used to open the field windings at several points, usually four or five. During the starting of a converter, until the armature gets up to synchronous speed, the revolving magnetic flux set up by the alternating current in the armature windings is cutting the field windings at such a rate as to induce high e.m.f.'s in the separate field coils. In fact, the field coils are acting like the secondary windings in a transformer during this time. As these field coils have a large number of turns and are normally in series with one another, the induced e.m.f.'s add up and produce a voltage across the field terminals, sometimes as high as from six to seven thousand volts. To prevent the production of this high voltage, which is likely to puncture the insulation, the field coils are disconnected from one another by the field-break-up switch. The result is that as the switch opens the field at five points there are not much more than a thousand volts across the insulation at any place.

Prob. 37-9. Make a sketch of the complete electrical connections necessary for a self-excited shunt-wound three-phase converter to start on the alternating-current side from half voltage taps on the transformers. Show alternating-current bus-bars, oil switches, transformers and connections, shunt field, field rheostat, break-up and reversing switch, direct-current voltmeter and direct-current bus-bars.

145. Brush-raising Device. In addition to a field-break-up switch, a commutating-pole converter is usually supplied with a brush-raising device. While the armature is running below synchronism, the rotating field set up by the alternating current in the armature windings not only cuts the field coils, but also the armature coils and induces a voltage in them. The commutating poles are always immediately over the armature conductors which are short-circuited by the brushes. Thus a magnetic path of low reluctance is offered to the flux set up about these short-circuited conductors. The cutting of this strong flux induces voltage enough to set up short-circuit currents which burn the brushes and roughen the commutator.

The brush-raising device marked *R* in Fig. 339 and 340 is operated by the lever *L* and is used to lift all the brushes, except two small pilot brushes, during the process of starting the converter. The two small brushes, one positive and one negative, are to indicate polarity and to supply current to excite the field. These brushes are so narrow that they short-circuit fewer conductors and the sparking at them is very slight.

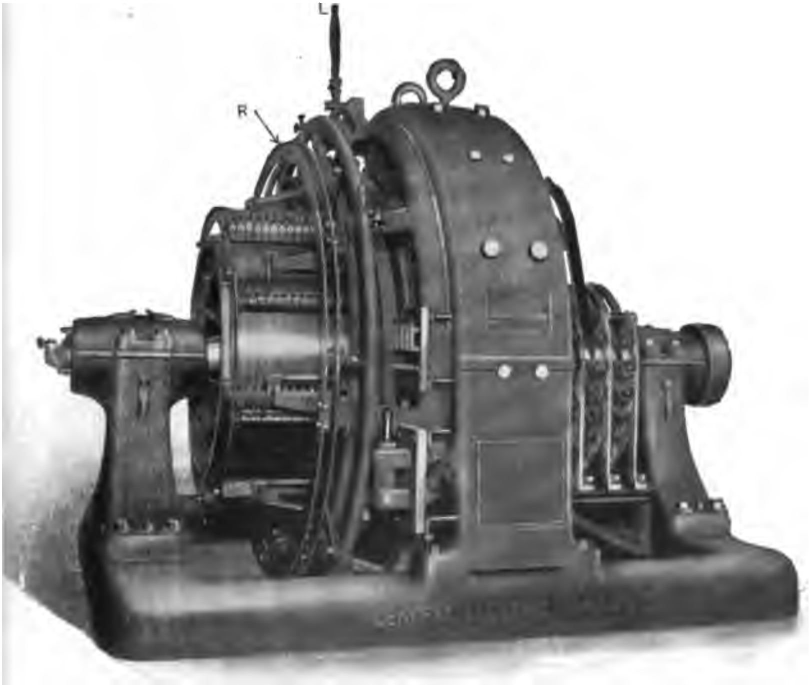

FIG. 339. Brush-raising device. By throwing over the lever *L*, the brushes are raised from the commutator during the process of starting. *The General Electric Co.*

The alternating current required for starting a commutating-pole converter with the brushes raised is less than that required to start converters not using the commutating poles with the brushes down, and all sparking is eliminated.

146. Equalizer Connections. All converter armature windings which at any instant occupy the corresponding position under poles of like polarity are joined together by heavy copper straps called equalizers. Thus if the machine were a 12-pole converter all six conductors which at a certain instant lay each exactly under the middle of a north pole would be joined together by an equalizer. This brings all these conductors and their respective commutator segments to exactly the same potential. Correspondingly situated conductors may vary somewhat in the voltage which is induced in them, due to unequal air gaps or unequal pole strengths. Unequal resistances of conductors and connections might cause unequal voltage drops in the conductors.

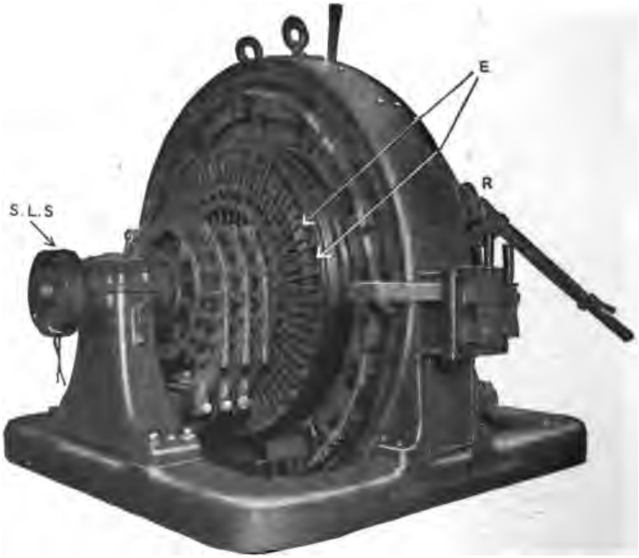

FIG. 340. The arrows marked *E* indicate the equalizer connections. The device marked *S. L. S.* is the "Speed Limiting Switch." *The General Electric Co.*

Thus the voltage across all pairs of brushes would not be exactly the same, and since all the positive brushes and all the negative brushes are in parallel, heavy equalizing currents would circulate between brushes of the same polarity in order to bring all correspondingly placed conductors to the same potential. The equalizing connections perform this duty and prevent the local currents from flowing through the brushes and producing sparking.

Fig. 340 shows these equalizing connections at *E*. The copper strips running back radially are the connections to the rings. The equalizer connections are the conductors banded together and secured by a V-shaped clamp over the edge of the armature core.

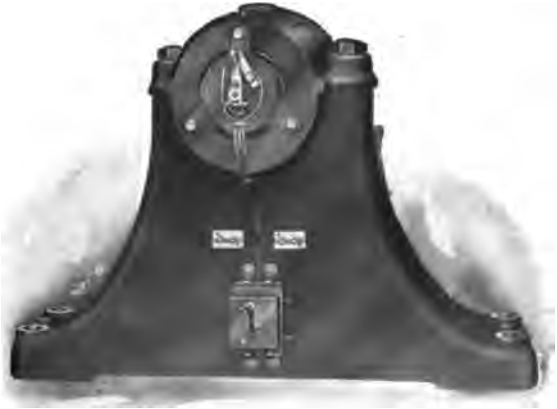

FIG. 341. Electromagnetic end-play device. *The General Electric Co.*

147. End-play Device. In order that the brushes do not wear grooves in the commutator and collector-rings, the armature is made to play back and forth along the line of its shaft. On large converters an electromagnet pulls on the end of the shaft and draws the armature slightly out of its field. This motion breaks the current in the electromagnet automatically, and the field pulls the armature back. This makes the contact again, which sets up another current in the electromagnet. The electromagnet and contact maker is called an **end play device** and is usually mounted at one end of the shaft, as shown in Fig. 341. It causes the armature to play smoothly back and forth in the natural period of oscillation of the armature, so that the brushes wear evenly on the entire surface of the commutator and collecting-rings.

Another device, entirely mechanical, consists of a ball and ball-race pressed against the end of the shaft by a spring. The plane of the race is not quite perpendicular to the shaft. Thus when the rotating shaft carries the ball around to the position where the distance between the shaft and race is less than the diameter of the ball, the spring gives a longitudinal thrust to the shaft, and pushes the armature out of the center of the field. As in the magnetic device, the field then draws the armature back and the operation is repeated.

148. Flashing at the Direct-current Brushes. Under certain abnormal conditions an arc is likely to be formed and maintained between two direct-current brushes of unlike polarity, to the detriment of the brushes and rigging. This action is called "flashing" or "flashing-over," and is generally caused by a "ground" or "short-circuit" on the direct-current lines. The large current, which flows between the brushes and the commutator, at the instant of the short circuit, vaporizes some of the carbon and forms a gas around the brushes. The alternating current supplied at this instant is so large that it causes a drop in the alternating voltage. The direct-current circuit breakers now let go, and the alternating voltage surges up to an abnormal value. This sets up a high voltage between adjacent commutator segments and the carbon vapor offers a path of comparatively low resistance, so arcs are set up from segment to segment until the brushes are practically short-circuited by the vapor paths thus formed. The normal alternating voltage is sufficient to maintain this arc, now that it is once started, and unless the attendant cuts the converter from the line, the brush-holders and commutator may be severely injured.

It has been shown in Art. 138 that severe hunting also may cause "flashing over."

Although it would be possible to prevent most "flash-overs" by locking the alternating-current and the direct-current circuit breakers together, it is usually not considered desirable to disconnect the machine from the alternating-current line and take the trouble to go through the process of starting up and synchronizing again. It is much more desirable to use extra care in designing the converter in order to minimize the likelihood of flashing and to protect the converter on the alternating-current side by independent time-limit relays, which disconnect the converter from the alternating-current line only when the machine **continues** to draw an overload from the mains. For this reason 600 volts is usually considered the safe upper limit for the voltage of the direct-current brushes, although 1200-volt and even 1500-volt converters have been built for 25-cycle systems. Six hundred volts call for such a value of normal voltage between commutator segments as will give a high factor of safety against flashing.

In street-railway service it has been found that if the feeders are not tapped into the trolley lines too near the substations, the reactance and resistance of the line between the converter and any short circuit limits the size and sudden growth of the short-circuit current so that flash-overs are rare occurrences.

149. Motor-converter. In order to secure steadiness in operation and ease in starting, a synchronous converter is sometimes mounted on the same shaft with a large slow speed induction motor having the same number of poles as the converter. The two machines are connected in series with each other * or in "concatenation" as it is called. The stator windings of the induction motor are connected directly to the line. The rotor of the motor is of the wound type and is connected directly to the armature of the converter. No rings are necessary, because both rotors are on the same shaft. In this way the induction motor receives all the electrical power from the line, uses about half of it to drive the combination and sends the remainder of it **at half frequency** to the converter.

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* See Chap. VII, Art. 105.

That the alternating current delivered to the converter has one-half the frequency of the line may be seen from the following:

At the instant the induction motor is thrown on the line, as we have seen in Chapter VII, the frequency of the e.m.f. induced in the rotor equals the frequency of the line. As the rotor gets up speed, the frequency of this e.m.f. induced in the rotor becomes less and less until it finally becomes zero as the rotor attains synchronous speed, which it would approach very closely if unloaded. The value of this induced e.m.f. is also gradually decreasing in proportion as the speed increases, so that at synchronous speed it would be zero.

On the other hand, the value of the e.m.f. induced in the armature of the converter is zero at the instant the induction motor is thrown on the line. Thus a very large current is delivered by the rotor of the induction motor to the converter armature. But as the speed rises, the e.m.f. induced in the armature winding increases and cuts down this current. Thus the current dies out for two reasons, — the induced e.m.f. in the rotor of the induction motor, which maintains the current, is **decreasing** as the speed rises, and the e.m.f. induced in the armature windings, which opposes the current, is **increasing** as the speed rises. When the shaft has reached one-half synchronous speed, the e.m.f. induced in the armature windings has just risen to the value which the e.m.f. induced in the rotor winding has **fallen** to. If the speed increased still further, the e.m.f. induced in the armature would send a reverse current back to the induction motor converter rotor. The converter would thus be acting as a generator and in delivering power would tend to slow down. Its induced e.m.f. would fall again, and the induced e.m.f. in the rotor would rise in proportion. Equilibrium is thus maintained at practically half-speed.

The converter is thus running on a 30-cycle current if the line frequency is 60 cycles and is accordingly more stable. The objection to this arrangement is the greater initial cost and the low power-factor at which the induction motor must operate on account of running at one-half synchronous speed.

150. Inverted Converter. Speed-limiting Switch. The synchronous converter can be used to convert direct current into alternating current. When so used it is called an **inverted converter.**

The use of inverted converters is limited to two cases:

(*a*) When a generating station is called upon to supply a local direct-current load and a distant alternating-current load. A converter is then used at times **straight** to help the direct-current generators, and at times **inverted** to help the alternators.

(*b*) When a direct-current system wishes to supply a district at a distance from the central plant, one converter is used inverted at the central station to change the power to alternating current for high voltage transmission, and another at the distant district as a straight converter to convert it back to direct current for use.

The main difficulty encountered in operating a converter in the above manner is to keep the speed constant, and hence the frequency of the alternating current steady. The speed of a straight converter is fixed by the frequency of the system, but an inverted converter is merely a direct-current shunt motor and its speed depends upon the strength of the field flux. Now if the fields are excited from a constant-voltage source of direct current, the converter acts like an alternator. If the delivered alternating current leads the alternating voltage, then, as we saw in Art. 9 and 10, the armature reaction strengthens the field flux. This merely causes the voltage of an alternator to rise; but since the speed of a shunt motor depends upon the strength of the field, this increase of field strength will cause the converter to slow down, and make a corresponding decrease in the frequency of the current delivered. At unity power-factor the armature reaction neither strengthens nor weakens the field, so the speed is not altered.

When the converter delivers a lagging current, however, the field flux is weakened by the armature reaction, just as described in Art. 9 and 10 for the case of an alternator. This weakened field causes the converter speed to rise, and to increase the frequency. An increase in the frequency may increase the inductive reactance of the circuit and cause the current to lag still more. This causes a still further weakening of the field and an increase in the speed, and so on until the speed becomes dangerously high.* It is evident that for successful inverted operation of converters excited by a constant-voltage source of direct current, some kind of speed-limiting switch is necessary.

A common type of this switch is operated by a centrifugal governor. A weight is mounted on the end of the shaft in such a way that as the speed increases the weight is forced further from the shaft. Such a device, marked *S. L. S.*, is shown attached to the end of the shaft of the converter in Fig. 340. Before the speed reaches a dangerous value, the weight has been forced out so far that it hits and closes a switch which short-circuits a no-voltage release on the direct-current circuit breaker. The opening of this breaker disconnects the driving power from the converter. This switch is usually mounted on one end of the converter shaft and the end-play device on the other end, but they are sometimes combined.

The speed-limiting device does not cause the converter to run at uniform speed, — it merely prevents the speed from becoming dangerously high. By exciting the inverted converter from a shunt-wound generator connected to the converter shaft, the speed may be held very nearly constant. The field of the exciter is run very much below saturation and any slight increase in speed causes a large increase in voltage and thus an increased field current in the converter.

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* Inductive reactance equals $2\pi fL$. Thus if the frequency f increases, the total reactance is increased in exact proportion.

Of course if a converter is running inverted in parallel with alternators of large capacity, there is no danger of its racing. It then acts exactly like another alternator running in parallel. If the field is too weak it draws a lagging current from the other alternators; if too strong, a leading current, just as any other alternator would, as explained in Art. 25, 28 and 29.

On the other hand, when operating as a straight converter in parallel with other converters, there is danger of racing, if at any time the voltage of the alternating-current supply is lost (through a short circuit or otherwise) and at the same time the direct-current power is maintained across the brushes by some external source of supply, such as a storage battery. It then operates as an inverted converter, and if there is considerable inductive reactance in any circuit that may exist between the alternating-current rings, a lagging current will be pumped through it with the resulting danger of racing. It is essential, then, that all converters be equipped with a speed-limiting device, reverse-current breaker on the direct-current side, and low-voltage release coils on the alternating-current breakers. The whole danger lies in the converter becoming disconnected from the alternating-current source of power. When properly connected, the shunt field may be completely killed and still the converter would not race. As we have seen, it would then merely take a large lagging current from the alternating-current lines.

151. Parallel Operation of Converters. Converters operating in parallel are subject to the same conditions as direct-current generators operating in parallel. The same precautions must be taken for an equal distribution of the load among the various machines, as are taken to assure the proper distribution of load among direct-current generators operating in parallel. In the case of compound converters, the terminals from which the series-field leads start must all be connected through switches to an equalizing bus-bar. In putting a converter into service and in parallel with other converters, the switch to this equalizing bar should be thrown in before the main switches are closed. This allows the series field to receive some current from the machines already running. Then close the negative switch (the series field being on the negative side), the direct-current circuit breakers and the positive switch, in the order given. Adjust the rheostats in the fields of the machines until each converter takes its share of the load.

When converters are run in parallel on both the direct- and the alternating-current side, each converter should take its power through a separate set of transformers; otherwise, all the machines must have exactly the same armature resistance, lead resistance, brush resistance and brush setting, in order that each machine take its share of the load. These conditions are practically impossible. When the converters are operating in separate banks the distribution of the load may be changed by changes in the shunt field, the series field, the equalizer circuit and the setting of the brushes. While changing the brush setting will change the direct voltage across the converter and thus change the strength of the shunt field and the compounding, it is likely to cause sparking. It is usually better to have the brushes on all the machines set for sparkless commutation, and as nearly as possible at similar positions, and to rely upon the changes in other respects to distribute the load properly.

The regulation of the converters should be suited to the characteristics of the load. Thus converters in parallel with storage batteries should have a drooping voltage characteristic or poor regulation, otherwise the batteries will not carry their share of the load. In general, we have seen that the machine with the greatest compounding, or having the most rising or least drooping characteristic, always takes the greatest share of the load.

152. Efficiency and Losses. The accompanying table is from the "American Handbook for Electrical Engineers" and gives the efficiency and distribution of losses of a typical 60-cycle and a 25-cycle converter. The values are all in per cent of input at full load

TABLE III

Converter Losses and Efficiencies

Kw. rating	500	300
Frequency	25	60
Core loss	1.00	1.75
Armature RI^2	0.55	0.60
Shunt field RI^2	0.70	0.60
Brush RI^2	0.40	0.40
Bearing friction	0.55	1.50
Brush friction	0.30	0.65
Efficiency	96.50	94.50

153. Mechanical Structure of Converters. The transfer of electrical energy in a converter takes place in the conductors without going through the intermediate step of mechanical energy. Thus the mechanical construction does not have to be as heavy as for a motor or generator handling the same amount of electrical energy. The shaft, bearings, bracing of the windings, etc., have to be constructed strong enough only to withstand centrifugal forces and the small torque set up to overcome the friction, core loss, copper loss, windage, etc.

154. Three-wire System. Dobrowolsky Method. When a synchronous converter is supplying a three-wire direct-current system, the neutral wire is brought back to the neutral point of the transformer secondaries as in Fig. 8a, First Course. In this way, the unbalanced current carried by the neutral wire returns to the system at the neutral point and enters the armature winding through the collecting-rings.

An arrangement similar to this is often used in connection with a direct-current generator in order to get the advantage of three-wire distribution. The armature is tapped at points 180 electrical degrees apart and brought out to two collector-rings (like the tapping on the armature of a two-ring converter), and a single-coil auto-transformer, called a **balancer coil,** is connected across the two rings. Fig. 342 shows the balancer coil *AB* connected to a two-pole generator at the points *a* and *b* which are 180° apart. The rings are omitted to give clearness to the connections. The point *a* will always be as much below the voltage-level (potential) of the brush B_1 as the point *b* is above the voltage-level of the brush B_2, regardless of what position the armature may be passing through. The middle *N* of the auto-transformer or balance coil *AB* is always just as far below the voltage-level of *a* as it is above the voltage-level of the point *b*. Therefore we see that the point *N* is always just as far below the voltage-level of brush B_1 as it is above the voltage-level of brush B_2; in other words, it is electrically midway between the positive and the negative mains at all times, or is in fact a neutral point for the direct-current distributing system. The balancer coil has a low resistance. Just as the central point of the transformer

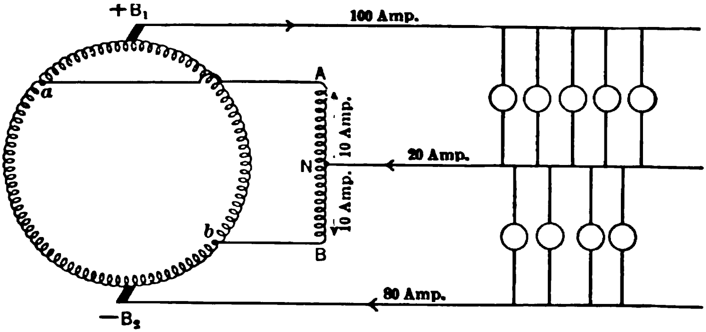


FIG. 342. Diagram of a Dobrowolsky three-wire direct-current generator. The balancer coil *AB* is connected to the two diametral taps *a* and *b* on the armature. The neutral wire comes to the middle point *N* of the balancer coil.

connected across the rings of a two-ring converter would be the neutral point of the converter, so the central point *N* of this balancer coil is the neutral point of the direct-current generator and will therefore allow a direct current to flow through it, but will oppose the flow of an alternating current. The alternating voltage induced in the armature conductors will always be sending an exciting current through the coil equal to the effective value of the induced voltage divided by the impedance of the coil at the frequency at which the voltage alternates. Thus, if the induced voltage had approximately a sine wave-form and was 240 volts at the direct-current brushes, the effective value across the balancer coil would be 0.707 of 240 or 169 volts. Assuming 300 ohms as the impedance (being practically all reactance, at the frequency of the machine), the exciting current would be $\frac{169}{300}$ or 0.58 ampere. Thus only this very small alternating current would be flowing although the machine was delivering 240 volts, because the reactance chokes back an alternating current.

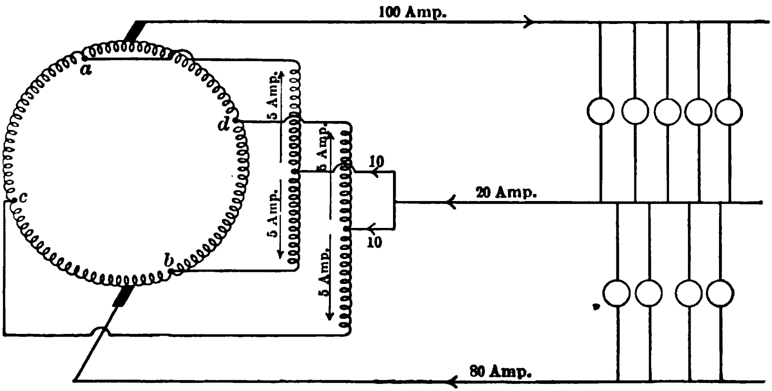


Fig. 343a. Diagram of a Dobrowolsky three-wire direct-current generator equipped with two balancer coils in order to distribute the out-of-balance current more uniformly throughout the armature windings.

Now assume the three-wire direct-current system is unbalanced as in Fig. 342, by the positive side taking 100 amperes and the negative only 80 amperes. The neutral which must then carry 20 amperes is connected at the exact central point *C* of the balancer coil. The low resistance coil offers very little opposition to the flow of direct current.

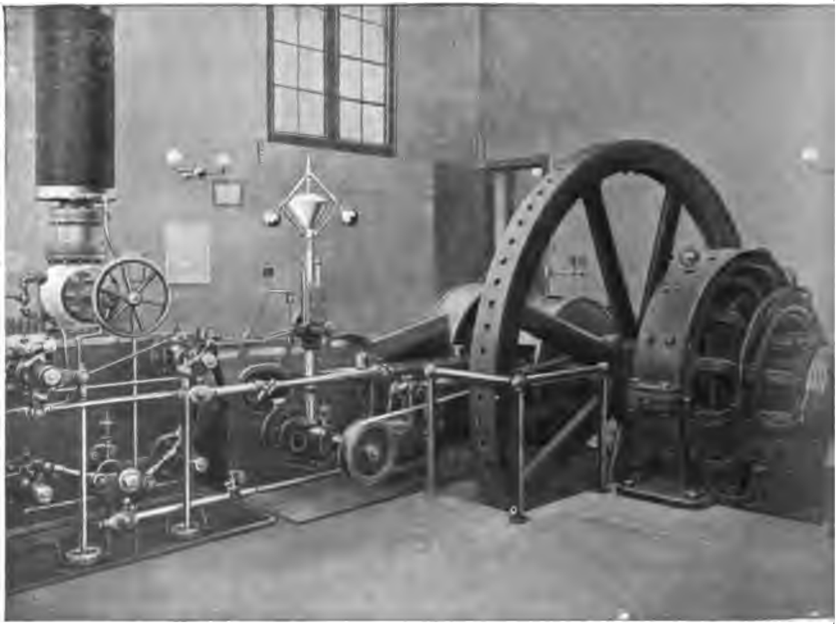


FIG. 343b. A photograph of the Dobrowolsky three-wire direct-current generator equipped with two balancer coils installed in the power plant of Wentworth Institute. *The Westinghouse Electric & Mfg. Co.*

Thus 10 amperes direct current flows in through one-half of the coil and 10 amperes through the other half, as in this way the least resistance is offered to the flow. These two currents flowing in opposite directions around the core exactly neutralize the magnetic effect of each other and so do not disturb the alternating exciting current.

In order to more evenly distribute the neutral current among the armature windings and thus decrease the heating, it is customary to use two balancer coils connected to the generator through four collecting-rings. In Fig. 343a the rings are omitted for the sake of clearness, but note that the neutral current enters the armature at four points *a*, *b*, *c*, *d*, instead of at two (*a* and *b*), as in Fig. 342. The appearance of a Dobrowolsky generator, equipped with four rings for two balancer coils is shown in Fig. 343b.

RECTIFIERS

For changing alternating-current power in small quantities into direct-current power there are several devices much less expensive than the motor-generator or the synchronous converter. These are called rectifiers and are of three types, (*a*) the mercury arc, (*b*) the electrolytic and (*c*) the vibrating.

155. Mercury-arc Rectifier. By far the most common rectifier is the mercury arc, which is widely used to rectify alternating currents for the purpose of charging storage batteries and operating arc lights. A picture of a mercury arc rectifier is shown in Fig. 344 and a connection-diagram in Fig. 345.

The glass tube containing the mercury is exhausted until a very low pressure is obtained. There are two wells, B and X, which contain mercury, and two positive graphite electrodes A and A', generally called the anodes.

The anodes A and A' are connected to opposite sides of the line from the transformer. A coil of high reactance but of low resistance ($T_1 - T_2$) is also connected across the transformer. The negative side of the battery to be charged, or of the arcs to be lighted, is connected to the middle point C of the reactance coil, and the positive side to the large mercury well at B. The small mercury pool at X, which is merely used to start the arc, is connected through a resistance R to one side of the transformer line. There is such a high resistance offered by the gap between the mercury wells that it would take many thousand volts to start a current through it, so a starting device is necessary. The tube is tilted until a bridge of mercury is formed across the space between *B* and *X*. This offers a path from wire *E* through resistance *R*, from *X* to *B*, through the battery to *C*, through half the reactance coil T_1, to the other side of the circuit *D*. An alternating current would therefore flow through this path. If the tube is now tilted back, an arc is formed which vaporizes some of the mercury and so charges it with electricity that the resistance is cut down between the points *A′* and *B* between and the points *A* and *B*. Now mercury vapor possesses the quality in common with almost all metallic vapors of allowing a current to pass easily in one direction and hardly at all in the other direction. Thus the current can now easily pass from either *A*

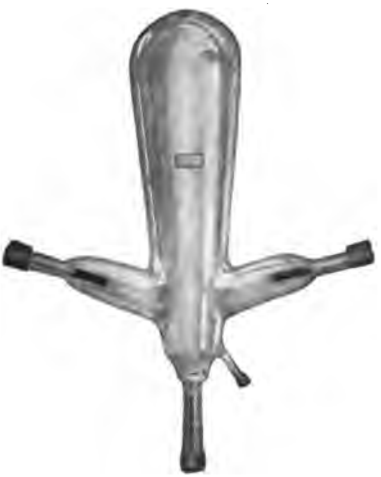


FIG. 344. The tube of the single-phase mercury arc rectifier. *The General Electric Co.*

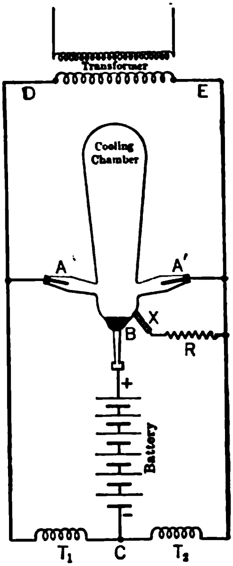


FIG. 345. Diagram of the connections for the single-phase mercury-arc rectifier of Fig. 344.

or A' to B, depending upon whether A or A' happens to be positive at this instant. If A' happens to be positive, a current is immediately set up through the vapor between A' and B, and flows from A' to B, through the battery to C, through T_1 to the other side (D) of the transformer B. At the next instant A' has become negative and A positive. Practically no current can flow back from B to A', but since A is now positive, a current flows from A to B through the vapor, then through the battery to C, through T_2 to the side E of the transformer. Thus the current through the battery is always in one direction.

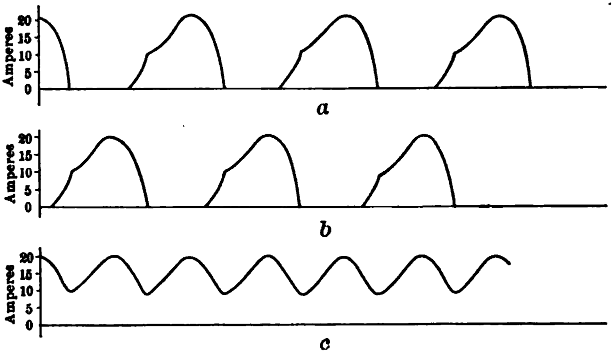


FIG. 346. Curve *a* represents the current in the lead wire at A of the rectifier of Fig. 345. Curve *b* is the current in the lead wire at A'. Curve *c* is the current in the lead wire at B. Note that curve *c* is merely the sum of curves *a* and *b*, and that none has negative values.

But the mercury arc has the properties of any other arc, — it requires a voltage to maintain it. Now when E is changing from positive to negative, or vice versa, there is an instant when the voltage is zero, and at this instant the arc tends to go out. The inductive reactance of the coils T_1 and T_2 is used for the purpose of preserving the arc. For, as we have seen, a current is set up in T_1 before the voltage from A' to B dies out. This current, during the decay of the voltage from A' to B, tends to keep up its strength because of the inductance of the coil T_1 and thus jumps through the vapor from A to B forming a local circuit, — from A to B, through the battery, through T_1 to A again. So, even before A becomes positive on account of the secondary transformer voltage, a current is already flowing from A to B, and it is merely increased by the rising positive voltage from A to B. Thus the currents overlap one another, as it were, and maintain a resultant current flowing through the tube continuously. In Fig. 346, curve a is a copy of an oscillogram taken of the current flowing into the tube at A, and curve b is an oscillogram of the current flowing into the tube at A'. Note that neither current has a negative loop. Curve c is an oscillogram of the current flowing out of the tube at B into the battery. Note that it is merely the sum of curves a and b and that it never falls to zero, although it pulsates and is full of ripples. Fig. 347 shows the direct-

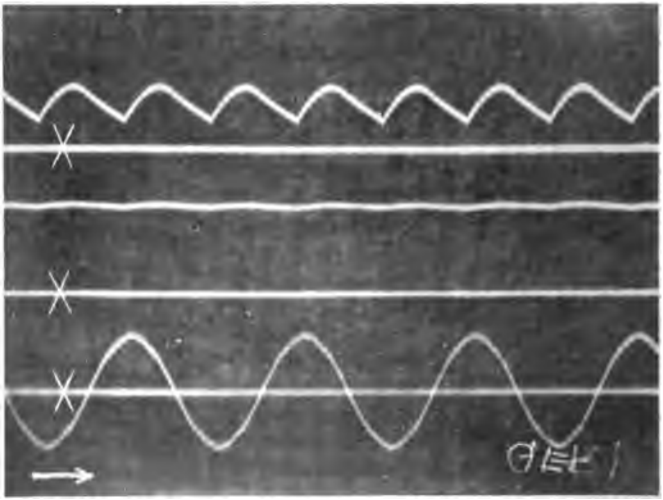


FIG. 347. Oscillograms taken on a General Electric mercury-arc rectifier, rated 60 cycles: volts, d-c. 30/118; a-c. 110/220; amperes, d-c. 30. Upper curve: direct current, 23.6 amperes. Middle curve: direct voltage, 99.4 volts. Lower curve: alternating voltage, 220 volts. Lines marked × are zero lines. *The General Electric Co.*

current curve, direct-voltage curve and the alternating-voltage curve obtained by an oscillograph, from a General Electric mercury-arc rectifier.

The current in the opposite direction (from mercury well to carbon anode) is not entirely shut off. A small "inverse" current will always flow in this direction, and it may at any instant become large enough to cause a short circuit, — for instance, when the tube gets old and the vacuum falls off. This allows the inverse current to assume very great proportions and destroys the rectifying action. In fact any cause which lowers the vacuum will allow the inverse current to be set up. Mercury will sometimes condense on the side of the tube and drop on the red hot carbon anodes This vaporizes the drop of mercury and instantly lowers the vacuum. The tube thus practically forms a short circuit on the alternating-current line, since the current can flow both ways through it with very little resistance. Precautions are therefore taken in designing the shape of the tube to prevent mercury globules from coming into contact with the carbon anodes.

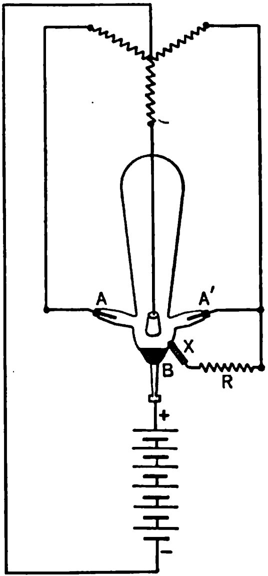
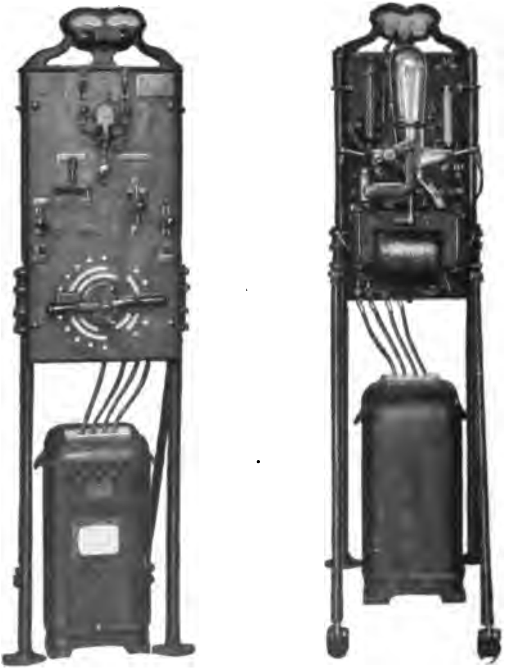


FIG. 348. Diagram of connections for a three-phase mercury-arc rectifier.

The reactance coils T_1 and T_2 may be omitted if a special transformer with large leakage reactance is used, having the secondary divided into two equal coils. The negative of the batteries is then brought to the juncture of these two secondary windings, which perform the duties of both reactance coils T_1 and T_2 and also of the secondary winding *DE*.

A tube constructed for use on a three-phase circuit works even better than a single-phase tube. Fig. 348 shows the connection for a three-phase tube. Note that the return from the battery is brought back to the neutral juncture of the three Y-connected transformer coils. The front and rear appearance of a mercury-arc battery-charging equipment is shown in Fig. 349.



Front Rear

FIG. 349. Front and rear views of a General Electric mercury-arc rectifying outfit for charging storage batteries.

156. Rating and Efficiency of Mercury-arc Rectifiers. The most common size of tube for charging storage batteries is built of glass with a maximum capacity of 30 amperes direct current, and with a minimum current of 5 or 6 amperes. If the current in these tubes is raised much above

30 amperes, the glass becomes too hot and soon breaks down by taking on a coating of mercury which short-circuits the terminals. If the current drops below 5 or 6 amperes, the arc "goes out" and the tube must be tilted up and started again. In other words, it takes 5 or 6 amperes to keep the mercury sufficiently vaporized and electrified to maintain an arc.

Other sizes in glass are rated at 10, 20, 40 and 50 amperes maximum. Steel tubes deliver as high as 300 amperes direct current. They can be built for practically any voltage by making the distance between the anode and the mercury well (or cathode) great enough. Tubes have been built to operate on 6000 volts. The direct voltage may be made any value between 20 per cent and 52 per cent of the alternating voltage of the transformer secondaries and the direct current 1½ to 2½ times the alternating current. The drop across the tube is always 14 volts in the battery-charging type and 25 volts in the series-lighting type, regardless of what the impressed voltage is. The efficiency then depends entirely upon the voltage at which the tube is operated; in fact, in those operating on high voltage, the tube loss is negligible, the efficiency depending upon the efficiency of the transformer. In practice, the combined efficiency of the transformers and tubes, etc., ranges from 80 per cent to 92 per cent. The power-factor is about 90 per cent. The regulation is excellent, depending entirely upon the drop in the transformers and reactance coils, since the drop in the bulb does not change at all with the load. The life depends upon the temperature at which the bulb is run, — a low temperature resulting in an indefinitely long life.

157. Electrolytic Rectifier. Many metals immersed in some solution offer a high resistance to the passage of an electric current when it is flowing from the metal to the solution, but yet offer a very low resistance when current flows from the liquid to the metal. In the case of aluminum, it is practical to make commercial use of this property for rectifying alternating-current power in relatively small amounts.

A plate of aluminum and a plate of lead are placed in an electrolyte, generally a solution of neutral ammonium sulphate. The aluminum will allow a current to flow from the lead through the solution to the aluminum plate with not much resistance, but offers a high resistance to the flow from the aluminum through the electrolyte to the lead plate. Such a cell is called an **electrolytic rectifier.**

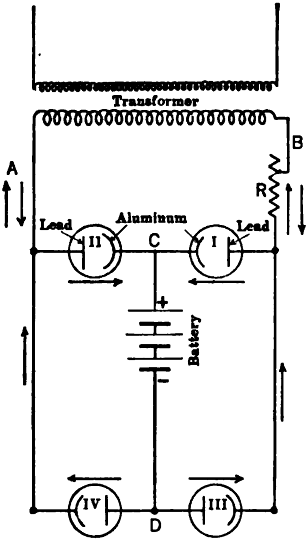


Fig. 350. Diagram of connection of four aluminum cells to be used as an electrolytic rectifier.

An arrangement of four cells is generally used when an alternating current is to be rectified for charging storage batteries, as in Fig. 350. When the side *B* of the transformer is positive, the current can flow through cell I from the lead to the aluminum, but not much current can get through cell II as it would have to pass from the aluminum to the lead. Therefore, it is forced through the battery to point *D*. From here, it can flow through cell IV, from lead to aluminum again, and reach the other side *A* of the transformer. Similarly, when *A* becomes positive, the current flows from *A* through cell II, through the battery and cell III to the side *B* of the transformer.

The efficiency of this arrangement depends upon the temperature of the electrolyte; being about 30 per cent for an equipment for charging a 6-volt battery, if the temperature does not rise above 30° C., but becoming very small as the aluminum cells get hot. In order to operate at the proper temperature, the aluminum plates should each have an area of about 2 square inches for each ampere which is delivered to the battery. Thus to charge a battery at the rate of 10 amperes, each aluminum cell should have an aluminum plate of about 20 square inches and a lead plate of about the same area, and should hold nearly a quart of electrolyte. The amount of current is regulated by the resistance *R*. Oscillograms for an electrolytic rectifier, furnished by the General Electric Co., are shown in Fig. 351.

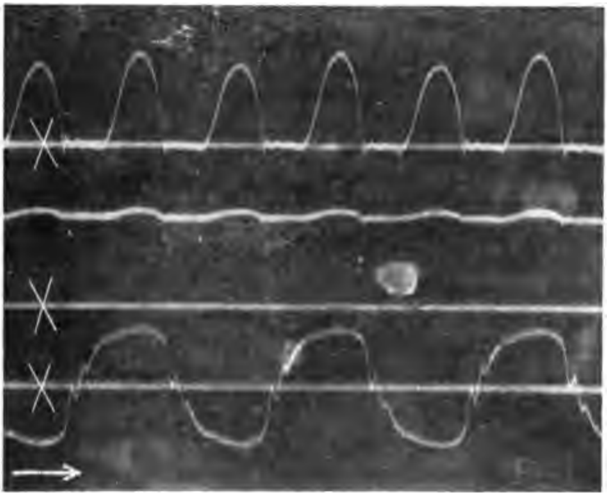


FIG. 351. Oscillograms made by the General Electric Co. on a Faria Electrolytic Rectifier, rated 30 amperes, 110 volts. Upper curve: direct current. Middle curve: direct voltage. Lower curve: alternating voltage. The lines marked X are zero lines.

158. Vibrating Rectifier. For supplying from three to eight amperes direct current for charging storage batteries from an alternating-current circuit, a vibrating rectifier like that in Fig. 352 is sometimes used. The operation may be understood by referring to Fig. 353. Across one-half of the transformer secondary are connected in series two magnets called the a-c. magnets. Note that the winding on one is reversed so that at any given instant they both present the same polarity to the magnet called the d-c. magnet. During one-half the cycle they will present a north pole to the d-c. magnet and during the other half a south pole. The d-c. magnet which is free to move about its center is energized from the battery which is being charged and its poles are permanent during the charging process. Thus at any instant one end will be attracted to an a-c. magnet and the other end will be repelled by the other magnet. As the a-c. magnets reverse their polarity, the other end of the d-c. magnet is drawn up and the first repulsed. Thus the d-c. magnet vibrates in synchronism with the alternations in the alternating-current line. As it vibrates it makes and breaks contacts at the points *X* and *Y*.

FIG. 352. A vibrating rectifier. *The Westinghouse Electric & Mfg. Co.*

Assume that the end marked 5 of the transformer is positive at a given instant. This makes contact points *Y* positive at that instant, and the two a-c. magnets will be presenting north poles to the d-c. magnet. Now if the battery terminal marked *M* happens to be the positive, then the left end of the d-c. magnet will be a north pole and will be repulsed by the a-c. magnet, and the right-hand end will be attracted. This makes a contact at *Y* and opens the circuit at *X*. The a-c. current from 5, therefore, enters the conducting strip on the d-c. magnet at *Y*, flows through this strip to the side of battery circuit marked *M*, through the battery to *N* and then back to the transformer. At the next instant, the current dies out and a spring brings the d-c. magnet back to the position in Fig. 353. The contact is thus broken at the instant when the current is zero, and no sparking results. Then the end 5 of the transformer becomes negative and the current flows in the opposite direction through the a-c. magnet coils, making them present south poles to the d-c. magnet. The left end of the d-c. magnet is, therefore, attracted up and the right end is repulsed down, making a contact at *X*. This connects *M* through the d-c. magnet and contact *X* with the end 3 of the transformer, which, we have seen, is now positive. Thus the lead *M* is still positive, and an intermittent direct current is delivered to the batteries. Note that it makes no difference whether the positive or the negative side of the battery is connected to *M* or *N*. If the negative side is connected to *M*, instead of the positive, as in the above explanation, it merely magnetizes the d-c. magnet in the opposite direction and the rectifying takes place in the reverse order so that the current is always delivered to the proper battery-lead. Fig. 354 shows the alternating voltage, direct voltage and direct current as taken by an oscillograph on a General Electric Co. vibrating rectifier.

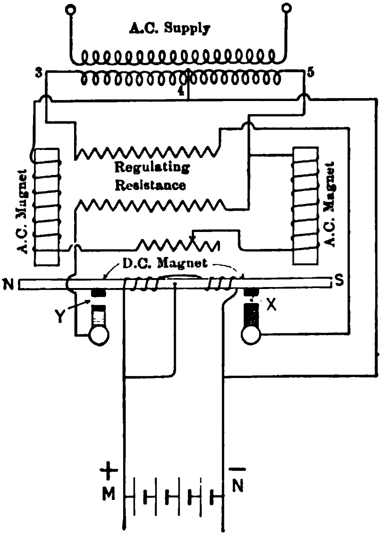


FIG. 353. Diagram of connections of a vibrating rectifier. *From the Electric Journal.*

159. Difference between Currents Delivered by Rectifiers and by Converters. The current from a rectifier cannot be used in a circuit containing inductance because its pulsations would set up eddy currents and would also cause hysteresis loss. It would, therefore, not be well to excite the field of an alternator with the current delivered by a rectifier. The curves of direct voltage as shown in the oscillograms of Fig. 347, 351 and 354 are smoothed out and sustained because in every case batteries are being charged. The current delivered by a converter, on the other hand, is usually as steady as that delivered by a direct-current generator and can be used for any purpose calling for direct-current power.

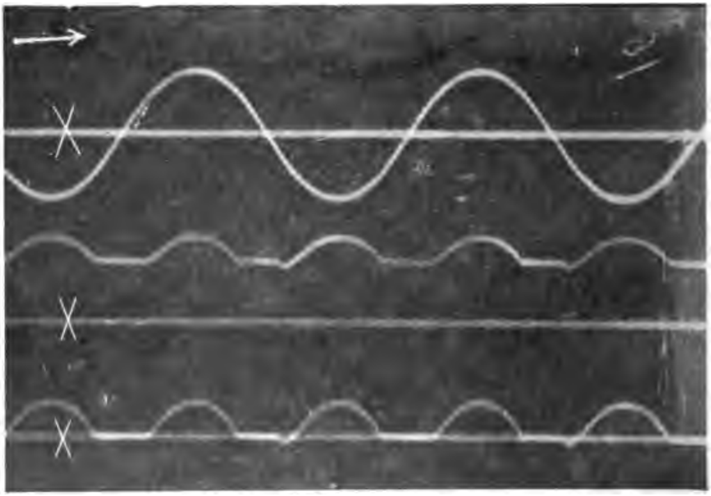


FIG. 354. Oscillograms taken on a General Electric Vibrating Rectifier, rated 115 volts, 6.8 amperes (alternating current) and 4.7 amperes, direct current. Upper curve: alternating voltage. Middle curve: direct voltage. Lower curve: direct current. The rectifier is charging a three-cell battery.

SUMMARY OF CHAPTER IX

A SYNCHRONOUS CONVERTER is a synchronous motor, the revolving armature of which is fitted with both a commutator and collecting-rings. Alternating-current power supplied to the rings drives the machine as a synchronous motor and direct-current power can be taken from the armature by means of brushes bearing on the commutator.

A COMPARISON OF THE SYNCHRONOUS CONVERTER with a motor-generator capable of converting the same amount of power from alternating to direct current shows that the synchronous converter:

(a) Is more efficient if not too much auxiliary apparatus is needed to control voltage, etc.

(b) Weighs less and occupies less space.

(c) Costs less. — about $11.00 per kilowatt for the larger units.

(d) In the smaller sizes, is hard to start and unstable on a line where sudden changes in load or voltage occur. Induction motor-generators have the advantage in these two particulars.

(e) The direct voltage cannot be controlled separately from the power-factor as in a synchronous motor-generator.

(f) Cannot be used to improve power-factor or the regulation of line without great loss in capacity as can a synchronous motor-generator.

THE RATIO OF THE ALTERNATING E.M.F. to the direct e.m.f. of a synchronous converter is practically fixed, as follows:

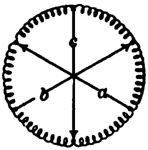
Number of phases.	Under ideal conditions.	Under actual conditions.	
		Full load, straight.	Full load, inverted.
One, two, six (diametral)...	0.707	0.71	0.675
Three or six (double delta).	0.612	0.62	0.580

THE RATIO OF THE ALTERNATING CURRENT PER RING TO THE DIRECT CURRENT is as follows for unity power-factor and 100 per cent efficiency. Correction can easily be made for other power-factors and efficiencies.

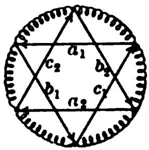
Single-phase (two rings)	$I_1 = 1.41\, I_d$
Two-phase (four rings)	$I_2 = 0.707\, I_d$
Three-phase (three rings)	$I_3 = 0.943\, I_d$
Six-phase (six rings)	$I_6 = 0.471\, I_d$

I_d = direct current per main lead.

SIX-PHASE CONVERTERS ARE OPERATED ON THREE-PHASE systems usually by connecting the secondaries of the transformers to the armature through the rings, either DIAMETRALLY or in DOUBLE DELTA, as shown diagrammatically below.



Diametral.



Double delta.

THE CURRENTS FLOWING IN THE ARMATURE COILS are combinations of the direct currents SUPPLIED TO the brushes and the alternating currents SUPPLIED BY the rings. There is thus a motor effect and a generator effect on the armature. In a single-phase converter these effects alternate and tend to make the machine unstable, but in a polyphase converter they are both continuous.

THE CAPACITY OF A POLYPHASE CONVERTER is greater than the capacity of the same machine, based on the same heating effect, if operated as a direct-current generator. Since the alternating currents and the direct currents in the armature windings partly neutralize each other, the heating of the armature coils is less in a converter than in the same machine run as a direct-current generator and delivering the same power.

The following table shows the capacities of the same machine operated with various numbers of rings, in comparison with its capacity as a direct-current generator with the same average heating.

RELATIVE CAPACITY OF CONVERTERS

	D-C. generator.	Converters.					
		2-ring.	3-ring.	4-ring.	6-ring diametral.	6-ring double delta.	12-ring.
Relative output, unity power-factor.........	100	85	134	165	197	197	224
87% power-factor	....	....	99	115	129	129	135

AT UNITY POWER-FACTOR, THE TAP COIL IS THE HOTTEST part of the armature and the mid-coil the coolest.

When the power-factor becomes less than unity, the hottest spot moves to one side of the tap coil. The average heating is greatly increased for a slight decrease in the power-factor, and consequently the capacity is greatly decreased.

THE RATING OF SYNCHRONOUS CONVERTERS, the output marked on the name plates, is based upon the maximum load which they can deliver without exceeding the standard maximum temperature rise under standard conditions of test. Converters built for railway service are usually rated so that they can deliver twice their normal load for one minute, and one and one-half normal load for two hours, without exceeding the specified standard temperature rise and with no serious sparking at the brushes.

THE REGULATION OF A CONVERTER is excellent, usually being about 2 per cent from no load to full load due to the armature impedance, and only 5 per cent when the drop due to the total impedance of the line, transformers and armature is considered.

THE DIRECT VOLTAGE of a single-phase converter tends to pulsate, due to the alternate generator and motor effect of the current in the windings. This pulsation is practically eliminated when the converter is operated as a polyphase machine, — the more phases, the steadier the direct voltage.

THE VOLTAGE AT THE DIRECT-CURRENT BRUSHES may be controlled by:

(a) Extra taps on the transformers. The ring-connections are shifted to higher voltage taps as the brush voltage tends to lower.

(b) Induction regulators, as explained in Chapter IV, Art. 63.

(c) Synchronous boosters. A small alternating-current generator is mounted on the same shaft. Its alternating-current terminals are connected in series with the transformer secondaries across the rings. The field coils are in series with the direct-current leads of the converter. The alternating e.m.f. of the booster is therefore proportional to the direct current delivered by the converter.

(d) Regulating or split poles, placed on the converter frame between the main poles. These poles add to the direct e.m.f. and to the alternating e.m.f. But the alternating e.m.f. added is nearly 90° out of phase with the e.m.f. produced in the armature winding by the flux from the main poles, and therefore the resultant alternating e.m.f. is not increased as much as the direct e.m.f. By decreasing the flux of main poles sufficiently, the alternating e.m.f. may be kept constant, and equal to the impressed e.m.f. This lessening of the flux of the main poles, however, does not decrease the direct e.m.f. as much as the flux from the regulating poles increases it. Thus the direct voltage may be raised or lowered 20 per cent without changing the alternating e.m.f.

(e) Compound field windings with series reactance.

The series coils carry the direct current delivered by the converter. Therefore as the direct-current load changes, the field strength changes and a corresponding change is produced in the phase relations between the alternating current and voltage supplied to the rings. A reactance is connected in series with the transformer supplying the rings. The transformer voltage must therefore overcome the drop in the reactance and the induced counter e.m.f. at the rings. When there is a large leading component in the alternating current, the drop across the reactance has practically a 180° phase relation to the ring e.m.f. and therefore a larger part of the transformer voltage is applied to rings than when there is a smaller leading component in the current. Thus the voltage applied to the rings rises when the current has a sufficiently large leading component and vice versa. The induced direct voltage rises accordingly. This scheme is practicable for producing a change of from 5 to 10 per cent in the induced direct voltage; that is, of producing nearly flat-compounding for an internal drop of from 5 to 10 per cent from no load to full load. The reactance is usually so proportioned as to produce unity power-factor at average load (generally $\frac{3}{4}$ full load). At all loads below this, the power-factor is lagging; at all loads above, it is leading. For practical method of determining the voltage at the direct-current brushes and the power-factor at various loads, see Art. 140.

COMMUTATING POLES ARE OFTEN USED ON CONVERTERS to increase the commutating facilities. This enables the converters to run at higher speeds and still commutate without excessive sparking at the brushes. This results in fewer poles with less weight and cost per kilowatt output.

CONVERTERS, LIKE SYNCHRONOUS MOTORS, ARE LIKELY TO HUNT unless their pole-faces are equipped with squirrel-cage windings, to act as DAMPERS.

SYNCHRONOUS CONVERTERS CAN BE STARTED in the following ways:

(a) By means of a small induction motor with fewer poles than the converter, mounted on the same shaft.

(b) From the direct-current bus-bars, as a direct-current motor.

(c) By the induction-motor effect between the armature and the squirrel-cage windings on the poles. See detailed directions for starting by this method in Art. 143.

A FIELD-BREAK-UP SWITCH is connected in the field circuit so that the field may be opened at several points. This prevents the e.m.f. induced in the field coils by the rotating armature flux from becoming great enough to puncture the insulation.

A BRUSH-RAISING DEVICE is attached to a converter having commutating poles to prevent the brushes (during the process of starting) from forming short circuits in those coils immediately under the commutating poles. The alternating flux in the armature cuts these coils, and the commutating poles form paths of low reluctance. Therefore destructively large currents would flow in these coils if they were short-circuited by the brushes remaining on the commutator.

EQUALIZER CONNECTIONS join together all armature windings which occupy corresponding positions under like poles, so that any equalizing currents can flow through these equalizers, rather than through the brushes, in order to bring all corresponding points to the same potential. This cuts down the tendency to spark and to heat the commutator.

END-PLAY DEVICES are often installed to give the armature a lateral motion along the shaft, and prevent the brushes from wearing grooves in the commutator.

FLASH-OVERS BETWEEN BRUSHES of opposite polarity may take place when a short-circuit occurs in the direct-current line. Due to the heated carbon vapors liberated from the brushes, the arc may persist and damage the brush rigging and commutator unless the converter is " cut-out " of service. To prevent flash-overs, resistance should be inserted in the direct-current leads near the converter.

A MOTOR-CONVERTER is a concatenated arrangement of a low-speed induction motor (with a wound rotor) mounted on the same shaft with the converter. The stator windings of the induction motor are connected to the line. The rotor windings are tapped directly to the armature windings of the converter. This arrangement has the advantage of enabling the converter to operate on a frequency of half that of the line, and results in a greater stability.

A CONVERTER IS SAID TO BE INVERTED when it is receiving direct-current power and delivering alternating current. This use is sometimes resorted to when a power station is called upon to deliver direct-current power to a local district and at the same time to supply power to distant consumers. An inverted converter acts like a shunt motor. It is difficult, therefore, to maintain the speed of the converter constant and the frequency steady. A leading current strengthens the fields and causes a decrease in the speed. A lagging current may weaken the field to such an extent that the increase in speed is likely to burst the armature.

A SPEED LIMITING DEVICE is therefore installed which disconnects the converter from the line as the speed approaches the danger point.

CONVERTERS ARE OPERATED IN PARALLEL under the same conditions as direct-current generators. More factors, however, enter in to disturb the distribution of the load among the paralleled machines. When converters are run in parallel on both the direct-current and the alternating-current sides, each transformer should take its power from a separate bank of transformers. The characteristics of the converters should be suited to the nature of the load.

THE MECHANICAL STRUCTURE OF CONVERTERS is much lighter than that of a motor or of a generator handling the same amount of power, because the transfer of energy takes place in the conductors, not having to go through the process of being changed into mechanical energy.

IN THE DOBROWOLSKY THREE-WIRE DIRECT-CURRENT GENERATOR one or more balancer coils of low resistance and high reactance are connected diametrally to the armature windings through collector-rings. The third wire is brought to the middle points of these coils which are of the same potential as the neutral point of the armature windings. The direct current from the neutral wire can flow through the balancer coils because of the low resistance, but the alternating e.m.f. induced in armature windings between the taps is able to force only a very small exciting current through the balancer coils on account of the high reactance.

RECTIFIERS ARE USED TO CONVERT small amounts of alternating-current power to direct-current power. The common types are:

(a) The mercury arc,
(b) The electrolytic,
(c) The vibrating.

THE MERCURY ARC RECTIFIER is the most common type, and is used generally for charging storage batteries and operating direct-current arc lights. The glass tube containing a small amount of mercury is exhausted to a very low pressure. The passage of an electric spark through the tube vaporizes some of the mercury and allows the passage of a current in one direction only through the tube, when an alternating voltage is impressed upon its terminals. Almost no reverse current will flow as long as a good vacuum is maintained. Thus, by its use, a direct current (but of a fluctuating nature) may be obtained from an alternating-current source of supply. The voltage drop across the tubes is constant, being 14 volts in one type and 25 volts in the other. Thus the efficiency depends entirely upon the voltage at which the tubes are operating. The efficiency of tube and transformer usually ranges from 80 per cent to 92 per cent. The power-factor is about 90 per cent. The regulation is excellent and the life indefinite, if run at a low temperature. The common glass tube has a maximum capacity of 30 amperes and a minimum of 5 amperes. The capacity of steel tubes runs as high as 300 amperes. They can be built to run on any voltage and be adapted to either constant-current or constant-voltage operation.

ELECTROLYTIC RECTIFIERS, commonly made by immersing plates of lead and plates of aluminum in ammonium sulphate, are sometimes used to rectify small currents (as high as ten amperes). The cell offers a high resistance to the flow of a current from the aluminum through the electrolyte to the lead, but a low resistance to the passage of a current in the opposite direction. The efficiency is about 30 per cent when the temperature is below 30° C. The temperature can be kept low if not more than $\frac{1}{2}$ ampere per square inch of aluminum plate area is rectified.

IN A VIBRATING RECTIFIER an electromagnetic reversing switch is operated by the alternating current, so that it reverses the alternating-current connections to a given pair of terminals in synchronism with the alternations of the e.m.f. Thus the terminals are maintained always at the same polarity. The common type will supply from three to eight amperes.

THE DIRECT CURRENT DELIVERED BY RECTIFIERS DIFFERS from that delivered by synchronous converters mainly in that it always pulsates more or less, while the current from a converter is always steady. Rectifiers are therefore not suitable for supplying current to inductive circuits.

PROBLEMS ON CHAPTER IX

Prob. 38–9. The following data are reported in the Electric Journal, Vol. IX, page 609. In a pipe and tube plant at Etna, Pa., the average power-factor of the load on the generators was 0.52 lagging. A motor-generator converter was added having a 1000-kv-a. synchronous motor. When this set is running at full load with 0.209 power-factor leading, the power-factor of the main generator becomes 98 per cent.

(*a*) What was the total kilovolt-ampere output of the generators before the motor-generator converter was added?

(*b*) What was the effective power output of the generator before the converter was added?

Prob. 39–9. After the motor-generator converter of Prob. 38 was added to the plant:

(*a*) What was the apparent output (kv-a.) of the generators?

(*b*) What was the effective power output (kw.) of the generators?

Prob. 40–9. At an efficiency of 82 per cent for the motor-generator converter of Prob. 38, what direct-current power was delivered by the set?

Prob. 41–9. The transformers in Fig. 355 are diametrally connected to the six-ring converter.

(*a*) What is the voltage between adjacent rings?

(*b*) What is the voltage ratio of the transformers?

(*c*) Taps are brought out to three auxiliary bus-bars. What is the voltage between these auxiliary bus-bars when the converter is running?

(*d*) What is the phase-difference between the voltage of the auxiliary bus-bars in (*c*) and the secondary voltage of the transformers?

(*e*) What is the voltage of the auxiliary bus-bars when the switches *a*, *b* and *c* are open? (From the Electric Journal, June, 1911.)

Prob. 42–9. (*a*) In Fig. 312, what is the phase angle between the induced e.m.f. in part ac_1 of the armature and the impressed voltage of coil C_1?

(*b*) What is the phase angle between the induced e.m.f. of ac_1 and the impressed voltage of coil B_2?

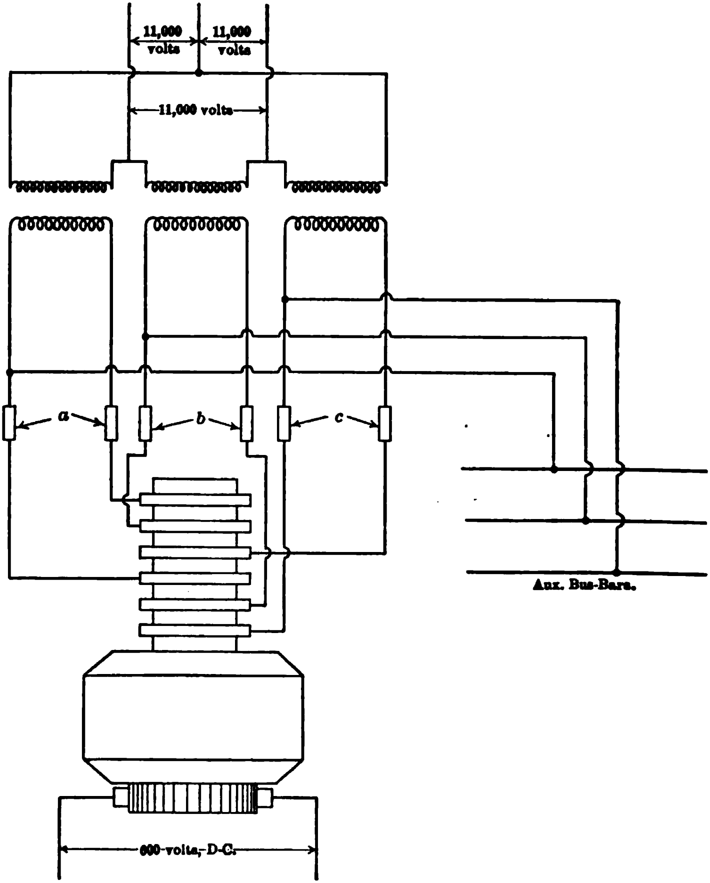


FIG. 355. Diagram of a converter connected diametrally to transformers. Auxiliary bus-bars are tapped to every other ring.

(*c*) From parts (*a*) and (*b*) of the problem determine the phase angle between the voltage of transformer *C* and transformer *B*. Does it check with the three-phase relation?

Prob. 43–9. Fig. 356 shows three transformers delta-connected to a three-phase transmission line and interconnected-star to a three-ring converter. This arrangement is common practice in conversion for a three-wire direct-current system.

If the direct voltage to neutral is 110 volts at full load and the transformers have a voltage ratio of 10 to 1 with the secondaries in series, find:

(*a*) Voltage between tapping points on the converter.
(*b*) Voltage across each secondary coil of the transformers.
(*c*) Voltage from tapping points on converter to neutral.
(*d*) Voltage between alternating-current mains.

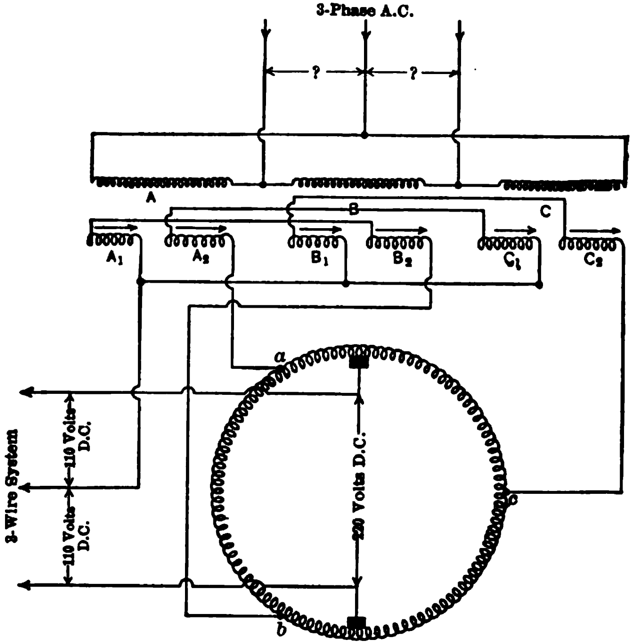


FIG. 356. Diagram of a converter with the taps brought out to the transformer secondaries which have an interconnected-star arrangement for the purpose of furnishing a neutral for the three-wire direct-current system.

Prob. 44–9. With converter of Prob. 43 running at unity power-factor, the efficiency is 90 per cent when 500 amperes are being delivered at the direct-current brushes.

(*a*) On this balanced load, what power is each transformer delivering?

(*b*) What is the phase angle between the current and the impressed voltage of each coil?

(*c*) What current flows in each secondary coil of the transformers?

Prob. 45–9. Connect to a 6-ring converter the double-tee connection of transformers shown in Fig. 357.

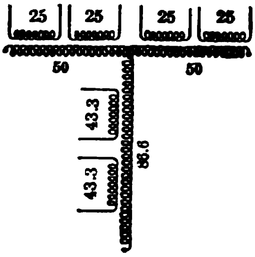


FIG. 357. Transformers connected in double-tee. The percentage of turns in each branch is indicated by the numbers on the coils.

Prob. 46–9. A power customer has to make a choice from the two equipments enumerated below. Either will serve his purpose equally. Data from Elec. Journal, Apr., 1915.

(I) Rotary converter — 300 kw., 3-phase, 60-cycle, 275 volts, direct-current, compound-wound, 900 r.p.m., self-starting from a-c. side with three 110 kv-a. transformers, 2300/170 – 85 volts, at from $4280 to $4350.

(II) Synchronous motor — 350 kv-a., 3-phase, 60-cycle, 2300 volts, rotating field, direct-connected to 300 kw. — 275 volts, commutating pole, compound-wound, direct-current generator, 900 r.p.m. and direct-connected exciter, at from $4650 to $4895.

EFFICIENCIES OF EQUIPMENTS

At per cent load (kv-a.)...	50	75	100	125
Converter..............	88.9	91.8	92.7	93
Transformers............	97.0	97.6	97.8	97.7
Motor-generator.........	82.4	86	87.2	87.0

What percentage saving is made on the investment if equipment I is chosen?

Prob. 47–9. The load-factor for the equipments in Prob. 46 would be about 20 per cent and the average load would be 50 per cent throughout 250 days of 9 hours each. Cost of electrical energy is $1\frac{1}{4}$ cents per kw-hr.

(*a*) Compute the yearly saving in power cost based on average load, if converter is used.

(*b*) What is the yearly saving as a percentage of investment?

Prob. 48–9. The resistance of a converter line and transformers is 10 per cent. Unity power-factor is desired at one-half load. Other specifications are as in Example 7. What inductive reactance should be used in order to produce practically flat-compounding from no load to full load?

APPENDIX A

TABLE I

NATURAL SINES, COSINES, TANGENTS AND COTANGENTS

0° to 3°

	sin	cos	tan	cot	
0	.000000	1.00000	.00000	∞	90°
5′	1454	1.00000	145	687.55	55′
10′	2909	1.00000	291	343.77	50′
15′	4363	0.99999	436	229.18	45′
20′	5818	998	582	171.89	40′
25′	7272	997	727	137.51	35′
30′	.008727	996	.00873	114.59	30′
35′	.010181	995	.01018	98.218	25′
40′	1635	993	164	85.940	20′
45′	3090	991	309	76.390	15′
50′	4544	989	455	68.750	10′
55′	5998	987	600	62.499	5′
1°	.017452	.99985	.01746	57.290	89°
5′	.01891	982	.01891	52.882	55′
10′	.02036	979	.02036	49.104	50′
15′	181	976	182	45.829	45′
20′	327	973	328	42.964	40′
25′	472	969	473	40.436	35′
30′	618	966	619	38.188	30′
35′	763	962	764	36.178	25′
40′	.02908	958	.02910	34.368	20′
45′	.03054	953	.03055	32.730	15′
50′	199	949	201	31.242	10′
55′	345	944	346	29.882	5′
2°	.03490	.99939	.03492	28.636	88°
5′	635	934	638	27.490	55′
10′	781	929	783	26.432	50′
15′	.03926	923	.03929	25.452	45′
20′	.04071	917	.04075	24.542	40′
25′	217	911	220	23.695	35′
30′	362	905	366	22.904	30′
35′	507	898	512	22.164	25′
40′	653	892	658	21.470	20′
45′	798	885	803	20.819	15′
50′	.04943	878	949	20.206	10′
55′	.05088	870	.05095	19.627	5′
3°	.05234	.99863	.05241	19.081	87°
10′	524	847	533	18.075	50′
20′	.05814	831	.05824	17.169	40′
30′	.06105	813	.06116	16.350	30′
40′	395	795	408	15.605	20′
50′	685	776	700	14.924	10′
	cos	sin	cot	tan	

87° to 90°

4° to 10°

	sin	cos	tan	cot	
4°	.06976	.99756	.06993	14.301	86°
10′	.07266	736	.07285	13.727	50′
20′	556	714	578	13.197	40′
30′	.07846	692	.07870	12.706	30′
40′	.08136	668	.08163	12.251	20′
50′	426	644	456	11.826	10′
5°	.08716	.99619	.08749	11.430	85°
10′	.09005	.99594	.09042	11.059	50′
20′	295	567	335	10.712	40′
30′	.0585	540	629	.385	30′
40′	.9874	511	.09923	10.078	20′
50′	.10164	482	.10216	9.7882	10′
6°	.10453	.99452	.10510	9.5144	84°
10′	.10742	421	.10805	.2553	50′
20′	.11031	390	.11099	9.0098	40′
30′	320	357	394	8.7769	30′
40′	609	324	688	.5555	20′
50′	.11898	290	.11983	.3450	10′
7°	.12187	.99255	.12278	8.1443	83°
10′	476	219	574	7.9530	50′
20′	.12764	182	.12869	.7704	40′
30′	.13053	144	.13165	.5958	30′
40′	341	106	461	.4287	20′
50′	629	067	.13758	.2687	10′
8°	.13917	.99027	.14054	7.1154	82°
10′	.14205	.98986	351	6.9682	50′
20′	493	944	648	.8269	40′
30′	.14781	902	.14945	.6912	30′
40′	.15069	858	.15243	.5606	20′
50′	356	814	540	.4348	10′
9°	.15643	.98769	.15838	6.3138	81°
10′	.15931	723	.16137	.1970	50′
20′	.16218	676	435	6.0844	40′
30′	505	629	.16734	5.9758	30′
40′	.16792	580	.17033	.8708	20′
50′	.17078	531	333	.7694	10′
10°	.17365	.98481	.17633	5.6713	80°
10′	651	430	.17933	.5764	50′
20′	.17937	378	.18233	.4845	40′
30′	.18224	325	534	.3955	30′
40′	509	272	.18835	.3093	20′
50′	.18795	218	.19136	.2257	10′
	cos	sin	cot	tan	

80° to 86°

11° to 20°

	sin	cos	tan	cot	
11°	.19081	.98163	.19438	5.1446	79°
10′	366	107	.19740	5.0658	50′
20′	652	.98050	.20042	4.9894	40′
30′	.19937	.97992	345	.9152	30′
40′	.20222	934	648	.8430	20′
50′	507	875	.20952	.7729	10′
12°	.20791	.97815	.21256	4.7046	78°
10′	.21076	754	560	.6382	50′
20′	360	692	.21864	.5736	40′
30′	644	630	.22169	.5107	30′
40′	.21928	566	475	.4494	20′
50′	.22212	502	.22781	.3897	10′
13°	.22495	.97437	.23087	4.3315	77°
10′	.22778	371	393	.2747	50′
20′	.23062	304	.23700	.2193	40′
30′	345	237	.24008	.1653	30′
40′	627	169	316	.1126	20′
50′	.23910	100	624	.0611	10′
14°	.24192	.97030	.24933	4.0108	76°
10′	474	.96959	.25242	3.9617	50′
20′	.24756	887	552	.9136	40′
30′	.25038	815	.25862	.8667	30′
40′	320	742	.26172	.8208	20′
50′	601	667	483	.7760	10′
15′	.25882	.96593	.26795	3.7321	75°
10′	.26163	517	.27107	.6891	50′
20′	443	440	419	.6470	40′
30′	.26724	363	.27732	.6059	30′
40′	.27004	285	.28046	.5656	20′
50′	284	206	360	.5261	10′
16°	.27564	.96126	.28675	3.4874	74°
10′	.27843	.96046	.28990	.4475	50′
20′	.28123	.95964	.29305	.4124	40′
30′	402	882	621	.3759	30′
40′	680	799	.29938	.3402	20′
50′	.28959	715	.30255	.3052	10′
17°	.29237	.95630	.30573	3.2709	73°
10′	515	545	.30891	.2371	50′
20′	.29793	459	.31210	.2041	40′
30′	.30071	372	530	.1716	30′
40′	348	284	.31850	.1397	20′
50′	625	195	.32171	.1084	10′
18°	.30902	.95106	.32492	3.0777	72°
10′	.31178	.95015	.32814	.0475	50′
20′	454	.94924	.33136	3.0178	40′
30′	.31730	832	460	2.9887	30′
40′	.32006	740	.33783	.9600	20′
50′	282	646	.34108	.9319	10′
19°	.32557	.94552	.34433	2.9042	71°
10′	.32832	457	.34758	.8770	50′
20′	.33106	361	.35085	.8502	40′
30′	381	264	412	.8239	30′
40′	655	167	.35740	.7980	20′
50′	.33929	.94068	.36068	.7725	10′
20°	.34202	.93969	.36397	2.7475	70°
10′	475	869	.36727	.7228	50′
20′	.34748	769	.37057	.6985	40′
30′	.35021	667	388	.6746	30′
40′	293	565	.37720	.6511	20′
50′	565	462	.38053	.6279	10′
	cos	sin	cot	tan	

70° to 79°

21° to 30°

	sin	cos	tan	cot	
21°	.35837	.93358	.38386	2.6051	69°
10′	.36108	253	.38721	.5826	50′
20′	379	148	.39055	.5605	40′
30′	650	.93042	391	.5386	30′
40′	.36921	.92935	.39727	.5172	20′
50′	.37191	827	.40065	.4960	10′
22°	.37461	.92718	.40403	2.4751	68°
10′	730	609	.40741	.4545	50′
20′	.37999	499	.41081	.4342	40′
30′	.38268	388	421	.4142	30′
40′	537	276	.41763	.3945	20′
50′	.38805	164	.42105	.3750	10′
23°	.39073	.92050	.42447	2.3559	67°
10′	341	.91936	.42791	.3369	50′
20′	608	822	.43136	.3183	40′
30′	.39875	706	481	.2998	30′
40′	.40141	590	.43828	.2817	20′
50′	408	472	.44175	.2637	10′
24°	.40674	.91355	.44523	2.2460	66°
10′	.40939	236	.44872	.2286	50′
20′	.41204	.91116	.45222	.2113	40′
30′	469	.90996	573	.1943	30′
40′	734	875	.45924	.1775	20′
50′	.41998	753	.46277	.1609	10′
25°	.42262	.90631	.46631	2.1445	65°
10′	525	507	.46985	.1283	50′
20′	.42788	383	.47341	.1123	40′
30′	.43051	259	.47698	2.0965	30′
40′	313	133	.48055	809	20′
50′	575	.90007	414	655	10′
26°	.43837	.89879	.48773	2.0503	64°
10′	.44098	752	.49134	353	50′
20′	359	623	495	204	40′
30′	620	493	.49858	2.0057	30′
40′	.44880	363	.50222	1.9912	20′
50′	.45140	232	587	768	10′
27°	.45399	.89101	.50953	1.9626	63°
10′	658	.88968	.51319	486	50′
20′	.45917	835	.51688	347	40′
30′	.46175	701	.52057	210	30′
40′	433	566	427	.9074	20′
50′	690	431	.52798	1.8940	10′
28°	.46947	.88295	.53171	1.8807	62°
10′	.47204	158	545	676	50′
20′	460	.88020	.53920	546	40′
30′	716	.87882	.54296	418	30′
40′	.47971	743	.54673	291	20′
50′	.48226	603	.55051	165	10′
29°	.48481	.87462	.55431	1.8040	61°
10′	735	321	.55812	1.7917	50′
20′	.48989	178	.56194	796	40′
30′	.49242	.87036	577	675	30′
40′	495	.86892	.56962	556	20′
50′	.49748	748	.57348	437	10′
30°	.50000	.86603	.57735	1.7321	60°
10′	252	457	.58124	205	50′
20′	503	310	513	.7090	40′
30′	.50754	163	.58905	1.6977	30′
40′	.51004	.86015	.59297	864	20′
50′	254	.85866	.59691	753	10′
	cos	sin	cot	tan	

60° to 69°

31° to 37°

	sin	cos	tan	cot	
31°	.51504	.85717	.60086	1.6643	59°
10′	.51763	567	483	534	50′
20′	.52002	416	.60881	426	40′
30′	250	264	.61280	319	30′
40′	498	.85112	.61681	212	20′
50′	745	.84959	.62083	107	10′
32°	.52992	.84805	.62487	1.6003	58°
10′	.53238	650	.62892	1.5900	50′
20′	484	495	.63299	798	40′
30′	730	339	.63707	697	30′
40′	.53975	182	.64117	597	20′
50′	.54220	.84025	528	497	10′
33°	.54464	.83867	.64941	1.5399	57°
10′	708	708	.65355	301	50′
20′	.54951	549	.65771	204	40′
30′	.55194	389	.66189	108	30′
40′	436	228	.66608	.5013	20′
50′	678	.83066	.67028	1.4919	10′
34°	.55919	.82904	.67451	1.4826	56°
10′	.56160	741	.67875	733	50′
20′	401	577	.68301	641	40′
30′	641	413	.68728	550	30′
40′	.56880	248	.69157	460	20′
50′	.57119	.82082	.69588	370	10′
35°	.57358	.81915	.70021	1.4281	55°
10′	596	748	455	193	50′
20′	.57833	580	.70891	106	40′
30′	.58070	412	.71329	.4019	39′
40′	307	242	.71769	1.3934	20′
50′	543	.81072	.72211	848	10′
36°	.58779	.80902	.72654	1.3764	54°
10′	.59014	730	.73100	680	50′
20′	248	558	547	597	40′
30′	482	386	.73996	514	30′
40′	716	212	.74447	432	20′
50′	.59949	.80038	.74900	351	10′
37°	.60182	.79864	.75355	1.3270	53°
10′	414	688	.75812	190	50′
20′	645	512	.76272	111	40′
30′	.60876	335	.76733	.3032	30′
40′	.61107	.79158	.77196	1.2954	20′
50′	337	.78980	.77661	876	10′
	cos	sin	cot	tan	

53° to 59°

38° to 45°

	sin	cos	tan	cot	
38°	.61566	.78801	.78129	1.2799	52°
10′	.61795	622	.78598	723	50′
20′	.62024	442	.79070	647	40′
30′	251	261	.79544	572	30′
40′	479	.78079	.80020	497	20′
50′	706	.77897	498	423	10′
39°	.62932	.77715	.80978	1.2349	51°
10′	.63158	531	.81461	276	50′
20′	383	347	.81946	203	40′
30′	608	.77162	.82434	131	30′
40′	.63832	.76977	.82923	.2059	20′
50′	.64056	791	.83415	1.1988	10′
40°	.64279	.76604	.83910	1.1918	50°
10′	501	417	.84407	847	50′
20′	723	229	.84906	778	40′
30′	.64945	.76041	.85408	708	30′
40′	.65166	.75851	.85912	640	20′
50′	386	661	.86419	571	10′
41°	.65606	.75471	.86929	1.1504	49°
10′	.65825	280	.87441	436	50′
20′	.66044	.75088	.87955	369	40′
30′	262	.74896	.88473	303	30′
40′	480	703	.88992	237	20′
50′	697	509	.89515	171	10′
42°	.66913	.74314	.90040	1.1106	48°
10′	.67129	.74120	.90569	1.1041	50′
20′	344	.73924	.91099	1.0977	40′
30′	559	728	.91633	913	30′
40′	773	531	.92170	850	20′
50′	.67987	333	.92709	786	10′
43°	.68200	.73135	.93252	1.0724	47°
10′	412	.72937	.93707	661	50′
20′	624	737	.94345	599	40′
30′	.68835	537	.94896	538	30′
40′	.69046	337	.95451	477	20′
50′	256	.72136	.96008	416	10′
44°	.69466	.71934	.96569	1.0355	46°
10′	675	732	.97133	295	50′
20′	.69883	529	.97700	235	40′
30′	.70091	325	.98270	176	30′
40′	298	.71121	.98843	117	20′
50′	505	.70916	.99420	058	10′
45°	.70711	.70711	1.00000	1.0000	45°
	cos	sin	cot	tan	

45° to 52°

TABLE II

1 cu. ft. of water weighs 62.5 lb.
1 horse power = 33,000 ft. lb. per minute.
1 kilowatt = 1.34 horse power.
1 B.t.u. = 780 ft. lb.
1 cu. in. of copper weighs 0.3195 lb.
1 cu. in. of aluminum weighs 0.0963 lb.
1 cu. ft. of $\left\{\begin{matrix}\text{solid}\\ \text{broken}\end{matrix}\right\}$ anthracite coal weighs approx. $\left\{\begin{matrix}\text{100 lb.}\\ \text{60 lb.}\end{matrix}\right.$
1 cu. ft. of $\left\{\begin{matrix}\text{solid}\\ \text{broken}\end{matrix}\right\}$ bituminous coal weighs approx. $\left\{\begin{matrix}\text{84 lb.}\\ \text{52 lb.}\end{matrix}\right.$
1 barrel of crude oil = 41 gal. = 310 lb.
1 ton = 2000 lb.
Resistance per mil-foot of copper at 20° C. . . . 10.371 ohms.[*]

TEMPERATURE COEFFICIENTS OF RESISTANCE[*]

	At 0° C.	At 20° C.
Commercial Copper	0.00427	0.00393
Commercial Aluminum	. . .	0.0039

[*] *Circular of the Bureau of Standards, No.* 31.

APPENDIX B

TABLE I

PROPERTIES OF ANNEALED COPPER WIRE

B. & S. gage.	Area in circ. mils. d^2	Diameter in mils. d		Number of strands in cable.	Resistance per mile at 20° C. or 68°F. (approx.).	Weight per mile in pounds (approx.).	
		Solid.	Stranded.			Solid.	Stranded.
14	4,107	64.05	73	7	13.3	65.6	70
12	6,530	80.81	92	7	8.40	104	108
10	10,380	101.9	116	7	5.27	166	172
8	16,510	128.5	146	7	3.31	264	269
6	26,250	162.0	184	7	2.08	420	428
5	33,100	181.9	206	7	1.65	530	544
4	41,740	204.3	232	7	1.31	667	682
3	52,630	229.4	260	7	1.04	841	866
2	66,370	257.6	292	7	0.824	1062	1,087
1	83,690	289.3	333	7	0.656	1337	1,368
0	105,540	324.9	375	7	0.518	1687	1,730
00	133,080	364.8	419	7	0.412	2127	2,190
000	167,810	409.6	470	7	0.328	2682	2,740
0000	211,600	460.0	528	19	0.259	3381	3,470
	250,000	500.0	575	19	0.217	....	4,090
	300,000	547.7	630	19	0.185	....	4,890
	350,000	591.6	681	19	0.159	....	5,740
	400,000	632.5	729	37	0.137	....	6,570
	450,000	670.8	773	37	0.122	....	7,470
	500,000	707.1	815	37	0.111	....	8,210
	550,000	741.6	855	37	0.100	....	9,020
	600,000	774.6	893	37	0.0898	....	9,850
	650,000	806.2	929	37	0.0845	....	10,660
	700,000	836.7	964	37	0.0793	....	11,480
	750,000	866.0	998	61	0.0739	....	12,320
	800,000	894.4	1031	61	0.0687	....	13,130
	900,000	948.7	1093	61	0.0633	....	14,850
	1,000,000	1000	1151	61	0.0528	....	16,420
	1,250,000	1118	1289	61	0.0438	....	20,300
	1,500,000	1225	1413	91	0.0304	....	24,600
	1,750,000	1323	1526	91	0.0311	....	28,700
	2,000,000	1414	1631	127	0.0275	....	32,800

TABLE II

PROPERTIES OF ALUMINUM WIRE

Aluminum Cables

The commercial sizes of stranded aluminum cables, made by the Aluminum Company of America, are not even circular mil and B. & S. sizes, but are of such cross sections as to give the same conductivity as even circular mil and B. & S. sizes of copper cables of 97 per cent conductivity. In the following table the first four columns are taken from a pamphlet entitled "Instructions for Installation and Maintenance of Aluminum Electrical Conductors," issued by the Aluminum Company of America in 1914.

(American Handbook for Electrical Engineers)

B. & S. gage or circular mils.		Usual number of strands.	Diameter of bare cable, inches.	Ohms per mile at 20° C. or 68° F.		Weight in pounds per mile.	
Copper (97 per cent) equivalent.	Aluminum 61 per cent.			Solid.	Stranded.	Solid.	Stranded.
6	41,740	7	15/64	2.147	2.194	200	203.3
5	52,630	7	17/64	1.703	1.740	253	256.1
4	66,370	7	19/64	1.350	1.380	319	323.1
3	83,640	7	21/64	1.071	1.094	402	406.6
2	105,530	7	23/64	0.8486	0.868	507	513.2
1	133,220	7	13/32	0.6720	0.689	640	647.3
0	167,800	7	15/32	0.5342	0.546	805	818.4
00	211,950	7	33/64	0.4229	0.433	1017	1030
000	266,800	7	37/64	0.3360	0.343	1281	1297
0000	336,420	7	41/64	0.2667	0.272	1617	1638
250,000	397,500	19	23/32	0.2253	0.2306	1907	1927
300,000	477,000	19	25/32	0.1879	0.1921	2290	2318
350,000	556,500	19	55/64	0.1611	0.1645	2670	2703
400,000	636,000	37	29/32	0.1409	0.1440	3050	3089
450,000	715,500	37	31/32	0.1254	0.1280	3440	3474
500,000	795,000	37	1 1/64	0.1127	0.1152	3820	3865
550,000	874,500	37	1 1/16	.....	0.1048	....	4250
600,000	954,000	37	1 7/64	.....	0.0959	....	4631
650,000	1,033,500	37	1 5/32	.....	0.0886	....	5016
700,000	1,113,000	37	1 13/64	.....	0.0823	....	5412
750,000	1,192,500	37	1 1/4	.....	0.0768	....	5797
800,000	1,272,000	61	1 9/32	.....	0.0720	....	6189
850,000	1,351,500	61	1 21/64	.....	0.0678	....	6563
900,000	1,431,000	61	1 23/64	.....	0.0640	....	6954
950,000	1,515,000	61	1 13/32	.....	0.0607	....	7355
1,000,000	1,590,000	61	1 7/16	.....	0.0577	....	7719

TABLE III

(From American Handbook for Electrical Engineers)

60-CYCLE REACTANCE OF SOLID NON-MAGNETIC WIRES*

Ohms per Mile of Each Wire of a Single-phase or of a Symmetrical Three-phase Line

Size of wire, cir. mils. or A.W.G.	Diam. of wire, inches.	Inches between wires, center to center.							
		1	3	6	9	12	18	24	30
1,000,000	1.0000	0.1145	0.2478	0.3319	0.3811	0.4158	0.4652	0.5003	0.5270
750,000	0.8660	0.1319	0.2652	0.3493	0.3985	0.4336	0.4826	0.5176	0.5448
500,000	0.7071	0.1565	0.2898	0.3739	0.4230	0.4581	0.5074	0.5421	0.5693
350,000	0.5916	0.1782	0.3115	0.3955	0.4449	0.4795	0.5289	0.5640	0.5908
250,000	0.5000	0.1986	0.3319	0.4158	0.4652	0.5003	0.5493	0.5844	0.6115
0000	0.4600	0.2087	0.3420	0.4260	0.4754	0.5101	0.5595	0.5945	0.6213
000	0.4096	0.2228	0.3561	0.4403	0.4893	0.5244	0.5734	0.6085	0.6356
00	0.3648	0.2368	0.3701	0.4543	0.5033	0.5384	0.5877	0.6224	0.6496
0	0.3249	0.2509	0.3842	0.4682	0.5176	0.5523	0.6017	0.6364	0.6635
1	0.2893	0.2650	0.3985	0.4826	0.5316	0.5666	0.6156	0.6507	0.6778
2	0.2576	0.2791	0.4124	0.4965	0.5455	0.5806	0.6300	0.6647	0.6918
4	0.2043	0.3072	0.4403	0.5248	0.5738	0.6099	0.6579	0.6929	0.7201
6	0.1620	0.3353	0.4686	0.5527	0.6021	0.6368	0.6861	0.7208	0.7480
8	0.1285	0.3635	0.4969	0.5810	0.6600	0.6650	0.7140	0.7491	0.7762
10	0.1019	0.3917	0.5248	0.6089	0.6382	0.6933	0.7423	0.7774	0.8045
12	0.08081	0.4196	0.5531	0.6371	0.6865	0.7212	0.7706	0.8053	0.8324
14	0.06408	0.4479	0.5813	0.6654	0.7144	0.7495	0.7985	0.8335	0.8607
16	0.05082	0.4762	0.6092	0.6933	0.7427	0.7774	0.8268	0.8618	0.8886

Size of wire, cir. mils. or A.W.G.	Feet between wires, center to center.								
	3	4	5	6	8	10	15	20	25
1,000,000	0.5493	0.5844	0.6115	0.6334	0.6684	0.6956	0.7446	0.7796	0.8068
750,000	0.5666	0.6017	0.6288	0.6507	0.6858	0.7129	0.7619	0.7970	0.8241
500,000	0.5915	0.6262	0.6533	0.6756	0.7103	0.7374	0.7868	0.8215	0.8486
350,000	0.6130	0.6481	0.6752	0.6971	0.7321	0.7593	0.8083	0.8433	0.8705
250,000	0.6334	0.6684	0.6956	0.7174	0.7525	0.7796	0.8286	0.8637	0.8909
0000	0.6435	0.6786	0.7057	0.7276	0.7627	0.7898	0.8388	0.8739	0.9010
000	0.6575	0.6925	0.7196	0.7416	0.7766	0.8038	0.8528	0.8878	0.9150
00	0.6718	0.7065	0.7336	0.7559	0.7906	0.8177	0.8671	0.9018	0.9289
0	0.6858	0.7204	0.7476	0.7698	0.8049	0.8317	0.8810	0.9161	0.9429
1	0.6997	0.7348	0.7619	0.7838	0.8188	0.8460	0.8950	0.9301	0.9572
2	0.7140	0.7487	0.7759	0.7981	0.8328	0.8599	0.9093	0.9440	0.9712
4	0.7419	0.7770	0.8041	0.8260	0.8611	0.8882	0.9372	0.9723	0.9994
6	0.7702	0.8049	0.8320	0.8543	0.8893	0.9161	0.9655	1.001	1.027
8	0.7985	0.8332	0.8603	0.8826	0.9172	0.9444	0.9938	1.028	1.056
10	0.8264	0.8614	0.8886	0.9105	0.9455	0.9727	1.022	1.057	1.084
12	0.8547	0.8893	0.9165	0.9387	0.9734	1.001	1.050	1.085	1.112
14	0.8826	0.9176	0.9448	0.9670	1.002	1.029	1.078	1.113	1.140
16	0.9108	0.9459	0.9730	0.9949	1.030	1.057	1.106	1.141	1.168

* The reactances given in this table also apply, with a practically negligible error, to ordinary stranded wires of the *same cross-section*.

For 25-cycle reactance multiply values in table by $\frac{25}{60}$ or 0.417.

TABLE IV

(American Handbook for Electrical Engineers)

Capacitance to Neutral* of Smooth Round Wires

Microfarads per Mile of Each Wire of a Single-phase or of a Symmetrical Three-phase Line

Size of wire, A.W.G.	Diam. of wire, inches.	Inches between wires, center to center.							
		1	3	6	9	12	18	24	30
0000	0.4600	0.06332	0.03490	0.02741	0.02438	0.02261	0.02051	0.01924	0.01836
000	0.4096	0.05802	0.03336	0.02647	0.02364	0.02197	0.01998	0.01877	0.01793
00	0.3648	0.05366	0.03198	0.02559	0.02293	0.02136	0.01947	0.01832	0.01752
0	0.3249	0.04995	0.03069	0.02477	0.02227	0.02078	0.01899	0.01790	0.01713
1	0.2893	0.04676	0.02951	0.02400	0.02165	0.02024	0.01854	0.01749	0.01676
2	0.2576	0.04400	0.02842	0.02328	0.02106	0.01972	0.01810	0.01710	0.01640
4	0.2043	0.03937	0.02645	0.02195	0.01997	0.01876	0.01729	0.01638	0.01573
6	0.1620	0.03566	0.02475	0.02077	0.01898	0.01789	0.01655	0.01571	0.01512
8	0.1285	0.03262	0.02326	0.01971	0.01809	0.01710	0.01587	0.01510	0.01455
10	0.1019	0.03006	0.02194	0.01875	0.01728	0.01637	0.01524	0.01453	0.01402
12	0.08081	0.02787	0.02076	0.01788	0.01654	0.01570	0.01466	0.01400	0.01353
14	0.06408	0.02599	0.01970	0.01709	0.01586	0.01509	0.01412	0.01351	0.01307
16	0.05082	0.02434	0.01874	0.01636	0.01523	0.01452	0.01362	0.01305	0.01264

Size of wire, A.W.G.	Feet between wires, center to center.								
	3	4	5	6	8	10	15	20	25
0000	0.01769	0.01674	0.01607	0.01556	0.01482	0.01429	0.01342	0.01286	0.01246
000	0.01730	0.01639	0.01574	0.01525	0.01454	0.01403	0.01319	0.01265	0.01227
00	0.01692	0.01604	0.01543	0.01496	0.01427	0.01378	0.01297	0.01245	0.01207
0	0.01656	0.01572	0.01512	0.01467	0.01401	0.01354	0.01275	0.01225	0.01189
1	0.01621	0.01540	0.01483	0.01440	0.01376	0.01330	0.01255	0.01206	0.01171
2	0.01587	0.01510	0.01455	0.01413	0.01352	0.01308	0.01235	0.01187	0.01153
4	0.01525	0.01453	0.01402	0.01363	0.01306	0.01265	0.01196	0.01152	0.01120
6	0.01467	0.01400	0.01353	0.01317	0.01263	0.01225	0.01160	0.01118	0.01088
8	0.01413	0.01351	0.01307	0.01273	0.01223	0.01187	0.01126	0.01087	0.01058
10	0.01363	0.01306	0.01264	0.01233	0.01186	0.01152	0.01094	0.01057	0.01030
12	0.01316	0.01263	0.01224	0.01194	0.01150	0.01118	0.01064	0.01029	0.01003
14	0.01273	0.01223	0.01187	0.01159	0.01117	0.01087	0.01036	0.01002	0.009777
16	0.01232	0.01185	0.01151	0.01125	0.01085	0.01057	0.01008	0.009768	0.009536

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* The capacitance *between* wires equals one-half the values given in this table.

TABLE V

60-Cycle Charging Current in Line of Smooth Round Wires

(Adapted from American Handbook for Electrical Engineers)

Charging current on each line wire, in amperes per mile = (current from table) $\times \dfrac{\textit{(volts to neutral)}}{1{,}000{,}000}$.

Amperes in Each Wire of Line *One Mile Long*, for Each 1,000,000 Volts to Neutral

Size of wire, A.W.G.	Diam. of wire, inches.	Inches between wires, center to center.							
		1	3	6	9	12	18	24	30
0000	0.4600	23.87	13.16	10.33	9.191	8.524	7.732	7.253	6.922
000	0.4096	21.87	12.58	9.979	8.912	8.283	7.532	7.076	6.760
00	0.3648	20.23	12.06	9.647	8.645	8.053	7.340	6.907	6.605
0	0.3249	18.83	11.57	9.338	8.396	7.834	7.159	6.748	6.458
1	0.2893	17.63	11.13	9.048	8.162	7.630	6.990	6.594	6.319
2	0.2576	16.59	10.71	8.777	7.940	7.434	6.824	6.447	6.183
4	0.2043	14.84	9.972	8.275	7.529	7.073	6.518	6.175	5.930
6	0.1620	13.44	9.331	7.830	7.155	6.745	6.239	5.923	5.700
8	0.1285	12.30	8.769	7.430	6.820	6.447	5.983	5.693	5.485
10	0.1019	11.33	8.271	7.069	6.515	6.171	5.745	5.478	5.286
12	0.08081	10.51	7.827	6.741	6.236	5.919	5.527	5.278	5.101
14	0.06408	9.798	7.427	6.443	5.979	5.689	5.323	5.093	4.927
16	0.05082	9.176	7.065	6.168	5.742	5.474	5.135	4.920	4.765

Size of wire, A.W.G.	Feet between wires, center tc center								
	3	4	5	6	8	10	15	20	25
0000	6.669	6.311	6.058	5.866	5.587	5.387	5.059	4.848	4.697
000	6.522	6.179	5.934	5.749	5.482	5.289	4.973	4.769	4.626
00	6.379	6.047	5.817	5.640	5.380	5.195	4.890	4.694	4.550
0	6.243	5.926	5.700	5.531	5.282	5.105	4.807	4.618	4.483
1	6.111	5.806	5.591	5.429	5.188	5.014	4.731	4.547	4.415
2	5.983	5.693	5.485	5.327	5.097	4.931	4.656	4.475	4.347
4	5.749	5.478	5.286	5.139	4.924	4.769	4.509	4.343	4.222
6	5.531	5.278	5.101	4.965	4.762	4.618	4.373	4.215	4.102
8	5.327	5.093	4.927	4.799	4.611	4.475	4.245	4.098	3.989
10	5.139	4.924	4.765	4.648	4.471	4.343	4.124	3.985	3.883
12	4.961	4.762	4.614	4.501	4.336	4.215	4.011	3.879	3.781
14	4.799	4.611	4.475	4.369	4.211	4.098	3.906	3.778	3.686
16	4.645	4.467	4.339	4.241	4.090	3.985	3.800	3.683	3.595

For 25 cycles the charging current equals $\frac{25}{60}$ or 0.417 of the values in this table.

TABLE VI

CAPACITANCE TO NEUTRAL* OF STANDARD STRANDS

(American Handbook for Electrical Engineers)

Microfarads per Mile of each Conductor of a Single-phase or of a Symmetrical Three-phase Line

Size of cable, C.M. or A.W.G.	Diam. of strand, inches	Inches between conductors, center to center.							
		1	3	6	9	12	18	24	30
1,000,000	1.152	......	0.0554	0.0383	0.0325	0.0294	0.0260	0.0240	0.0226
750,000	0.998	......	0.0506	0.0361	0.0309	0.0281	0.0249	0.0231	0.0218
500,000	0.814	0.134	0.0452	0.0333	0.0289	0.0264	0.0236	0.0219	0.0208
350,000	0.681	0.0955	0.0413	0.0312	0.0273	0.0251	0.0225	0.0210	0.0200
250,000	0.575	0.0776	0.0383	0.0295	0.0260	0.0240	0.0216	0.0202	0.0192
0000	0.528	0.0713	0.0369	0.0286	0.0253	0.0234	0.0212	0.0198	0.0189
000	0.470	0.0644	0.0352	0.0276	0.0245	0.0227	0.0206	0 0193	0.0184
00	0.418	0.0590	0.0336	0.0266	0.0238	0.0221	0.0201	0.0189	0.0180
0	0.373	0.0544	0.0322	0.0258	0.0231	0.0214	0.0196	0.0184	0.0176
1	0.332	0.0506	0.0309	0.0249	0.0224	0.0209	0.0191	0.0180	0.0172
2	0.292	0.0470	0.0296	0.0241	0.0217	0.0203	0 0186	0.0175	0.0168
4	0.232	0.0417	0.0275	0.0227	0.0205	0.0193	0.0177	0.0168	0.0161
6	0.184	0.0376	0.0256	0.0214	0.0195	0.0184	0.0169	0.0161	0.0154

Size of cable, C.M. or A.W.G.	Feet between conductors, center to center.								
	3	4	5	6	8	10	15	20	25
1,000,000	0.0216	0.0202	0.0193	0.0185	0.0175	0.0168	0.0156	0.0148	0.0143
750,000	0.0209	0.0196	0.0187	0.0180	0.0470	0.0163	0.0152	0.0145	0.0140
500,000	0.0200	0.0188	0.0179	0.0173	0.0164	0.0157	0.0147	0.0140	0.0135
350,000	0.0192	0.0181	0.0173	0.0167	0.0159	0.0153	0.0143	0.0136	0.0132
250,000	0.0185	0.0175	0.0168	0.0162	0.0154	0.0148	0.0139	0.0133	0.0129
0000	0.0182	0.0172	0.0165	0.0160	0.0152	0.0146	0.0137	0.0131	0.0127
000	0.0178	0.0168	0.0161	0.0156	0.0149	0.0143	0 0135	0.0129	0.0125
00	0.0174	0.0165	0.0158	0.0153	0.0146	0.0141	0.0132	0.0127	0.0123
0	0.0170	0.0161	0.0155	0.0150	0.0143	0.0138	0.0130	0.0125	0.0121
1	0.0166	0.0158	0.0152	0.0147	0.0141	0.0136	0.0128	0.0123	0.0119
2	0.0162	0.0154	0.0149	0.0144	0.0138	0.0133	0.0126	0.0121	0.0117
4	0.0156	0.0148	0.0143	0.0139	0.0135	0.0129	0.0122	0.0117	0.0114
6	0.0150	0.0143	0.0138	0.0134	0.0129	0.0125	0.0118	0.0114	0.0111

---

* The capacitance *between* conductors equals one-half the values given in this table.

TABLE VII

60-Cycle Charging Current in Line of Standard Strands

(Adapted from American Handbook for Electrical Engineers)

Charging current on each line wire, in amperes per mile = *(current from table)* $\times \dfrac{(\textit{volts to neutral})}{1{,}000{,}000}$

Approximate Amperes in Each Wire of One-mile Line for Each 1,000,000 volts to neutral

Size of cable, C.M. or A.W.G.	Diam. of strand, inches.	Inches between cables, center to center.							
		1	3	6	9	12	18	24	30
1,000,000	1.152	......	20.9	14.4	12.3	11.1	9.80	9.05	8.52
750,000	0.998	......	19.1	13.6	11.6	10.6	9.39	8.71	8.22
500,000	0.814	50.5	17.0	12.6	10.9	9.95	8.90	8.26	7.84
350,000	0.681	36.0	15.6	11.8	10.3	9.46	8.48	7.92	7.54
250,000	0.575	29.3	14.4	11.1	9.80	9.05	8.14	7.62	7.24
0000	0.528	26.9	13.9	10.8	9.54	8.82	7.99	7.46	7.13
000	0.470	24.3	13.3	10.4	9.24	8.56	7.77	7.28	6.94
00	0.418	22.2	12.7	10.0	8.97	8.33	7.58	7.13	6.79
0	0.373	20.5	12.1	9.73	8.71	8.07	7.39	6.94	6.63
1	0.332	19.1	11.6	9.39	8.44	7.88	7.20	6.79	6.48
2	0.292	17.7	11.2	9.09	8.18	7.65	7.01	6.60	6.33
4	0.232	15.7	10.4	8.56	7.73	7.28	6.67	6.33	6.07
6	0.184	14.2	9.65	8.07	7.35	6.94	6.37	6.07	5.81

Size of cable, C.M. or A.W.G.	Feet between cables, center to center.								
	3	4	5	6	8	10	15	20	25
1,000,000	8.14	7.62	7.28	6.97	6.60	6.33	5.88	5.58	5.39
750,000	7.88	7.39	7.05	6.79	6.41	6.15	5.73	5.47	5.28
500,000	7.54	7.09	6.75	6.52	6.18	5.92	5.54	5.28	5.09
350,000	7.24	6.82	6.52	6.30	5.99	5.77	5.39	5.13	4.98
250,000	6.97	6.60	6.33	6.11	5.81	5.58	5.24	5.01	4.86
0000	6.86	6.48	6.22	6.03	5.73	5.50	5.16	4.94	4.79
000	6.71	6.33	6.07	5.88	5.62	5.39	5.09	4.86	4.71
00	6.56	6.22	5.96	5.77	5.50	5.32	4.98	4.79	4.64
0	6.40	6.07	5.84	5.66	5.39	5.20	4.90	4.71	4.56
1	6.26	5.96	5.73	5.54	5.32	5.13	4.83	4.64	4.49
2	6.11	5.81	5.62	5.43	5.20	5.01	4.75	4.56	4.41
4	5.88	5.58	5.39	5.24	5.01	4.86	4.60	4.41	4.30
6	5.66	5.39	5.20	5.05	4.86	4.71	4.45	4.30	4.18

For 25 cycles the charging current equals $\frac{25}{60}$ or 0.417 of the values in this table.

TABLE VIII

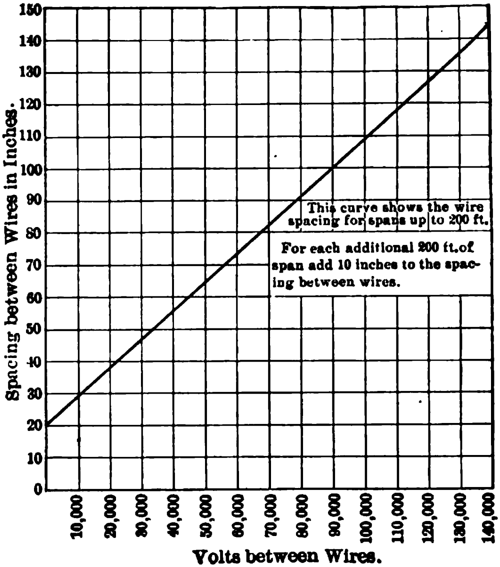
HEMP-CENTER COPPER CABLE *

Made of 6 wires around a hemp center .

(*The American Brass Co.*)

Size of cable, A.W.G.	Diameter of wire, inch.	Outside diameter, inch.
0000	0.1879	0.564
000	0.1672	0.502
00	0.1489	0.447
0	0.1326	0.398
1	0.1181	0.354
2	0.1052	0.316
3	0.0937	0.281
4	0.0835	0.250

* Standard Handbook for Electrical Engineers.



Curve *I*, Appendix B. Spacing of conductors for spans up to 200 ft. From Still's "Overhead Electric Power Transmission."

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